MATEMATIK ANALIZ VA MATEMATIK FIZIKA TENGLAMALARINING ZAMONAVIY MASALALARI

XALQARO ILMIY-AMALIY KONFERENSIYA
MATERIALLARI TO‘PLAMI

СОВРЕМЕННЫЕ ЗАДАЧИ МАТЕМАТИЧЕСКОГО АНАЛИЗА И УРАВНЕНИЙ МАТЕМАТИЧЕСКОЙ ФИЗИКИ

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МЕЖДУНАРОДНОЙ НАУЧНО-ПРАКТИЧЕСКОЙ КОНФЕРЕНЦИИ

MODERN PROBLEMS OF MATHEMATICAL ANALYSIS AND EQUATIONS OF MATHEMATICAL PHYSICS

PROCEEDINGS OF THE
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Termiz, 21-22 sentabr 2026-yil

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TERMIZ DAVLAT UNIVERSITETI

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V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

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TENGLAMALARINING ZAMONAVIY MASALALARI

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TERMEZ STATE UNIVERSITY

V. I. ROMANOVSKIY INSTITUTE OF MATHEMATICS

OF THE ACADEMY OF SCIENCES OF THE REPUBLIC OF UZBEKISTAN

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5 – SHO‘BA. SONLI USULLAR VA MATEMATIK MODELLASHTIRISH

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6 – SHO‘BA. SUN’IY INTELLEKTNING ISTIQBOLLI TEXNOLOGIYALARI

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Termiz davlat universiteti “Matematik analiz” kafedrasi professori, fizika-matematika fanlari doktori, M. Mirsaburovning ilmiy faoliyati haqida

Mirsaburov Miraxmat 1951-yil 30 iyunda Surxondaryo viloyati Termiz tumanida tug‘ilgan.

M.Mirsaburov 1968-yilda Termiz shaxridagi 10-sonli umumiy o‘rta ta’lim maktabini muvaffaqiyatli tamomlab, shu yili Toshkent davlat universitetining(xozirgi Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universiteti) mexanika-matematika fakultetiga o‘qishga kirdi va 1973-yilda universitetni matematik analiz ixtisosligi bo‘yicha tugatib, Termiz davlat pedagogika institutining Matematika kafedrasida o‘z pedagogik faoliyatini boshladi.

1978-yilda O‘zbekiston Fanlar Akademiyasi V.I.Romanovskiy nomli matematika instituti aspiranturasiga o‘qishga kirdi va 1981-yilda O‘zbekiston Fanlar Akademiyasining akademigi M. S. Salohiddinov rahbarligida “Краевые задачи для вырождающихся гиперболических уравнений и уравнений смешанного типа с сингулярными коэффициентами” mavzusida nomzodlik dissertatsiyasini muvaffaqiyatli himoya qildi.

1981-yildan boshlab 1995-yilga qadar Termiz davlat universiteti matematik tahlil kafedrasida katta o‘qituvchi, kafedra mudiri lavozimlarida ishladi.

1995-1998 yillarda O‘zbekiston Fanlar Akademiyasi V. I. Romanovskiy nomli matematika institutida doktoranturada tahsil oldi va 2000- yilda “К теории вырождающихся уравнений и уравнений смешанного типа с сингулярными коэффициентами” mavzusidagi doktorlik dissertatsiyasini himoya qildi.

2000-2001 yillarda Termiz davlat universiteti differensial tenglamalar va geometriya kafedrasi mudiri lavozimida faoliyat yuritdi.

2001-2005 yillarda Termiz iqtisodiyot kolleji direktori bo‘lib ishladi. U respublikamizda o‘tkazilgan siyosiy tadbirlarda faol ishtirok etib kelmoqda. 2004-yil 27- dekabr kuni o‘tkazilgan O‘zbekiston Respublikasi Oliy Majlisi Qonunchilik palatasiga saylov o‘tkazuvchi 65-Termiz okrugi komissiyasi raisi vazifasida muvaffaqiyatli ish olib bordi.

2006-yilda M. Mirsaburovga Uzbekiston Respublikasi OAK tomonidan differensial tenglamalar va geometriya kafedrasi professori ilmiy unvoni berildi.

M.Mirsaburov tomonidan ko‘p yillar o‘z yechimini kutib yotgan xususiy hosilali tenglamalar sohasidagi muammolar muvaffaqiyatli hal etilgan. Xususiy hosilali differensial tenglamalar sohasida ishlayotgan mutaxassislarga yaxshi ma’lumki, 1923-yilda italiyalik olim F.Trikomi

birinchi bo‘lib aralash turdagi tenglama uchun korrekt (to‘g‘ri) masala qo‘yadi, bu ishdan so‘ng aralash turdagi tenglamalar uchun chegaraviy masalalarga olimlar tomonidan katta e’tibor beriladi, uning natijalari aerodinamika sohasidagi ko‘p masalalarni yechishda qo‘llanilib kelinmoqda.

F.Trikomi masalasi butun dunyo olimlari tomonidan asosan uch yo‘nalishda rivojlantirilmoqda. Bular:

1. F.Trikomi masalasi umumiyroq tenglamalar uchun o‘rganiladi.
2. F.Trikomi masalasi chegaraviy shartlar shakli o‘zgargan holda tadqiq yetiladi.
3. Trikomi tenglamasi uchun spektral masalalar o‘rganiladi.

Bu yo‘nalishlarda o‘nlab doktorlik, yuzlab nomzodlik dissertatsiya ishlari bajarilgan.

70-yillarga kelib, olimlar orasida aralash turdagi tenglamalar uchun barcha masalalar deyarli o‘z yechimini topgandek tasavvur paydo bo‘ladi. Aralash turdagi tenglamalar uchun chegaraviy masalalarning keyingi rivojida yangi g‘oya zaruriyati tug‘ildi hamda A.V.Bitsadze va A.A.Samarskiylarning elliptik turdagi tenglamalar uchun yangi nolokal masalasi bu borada turtki , impuls vazifasini o‘tadi.

Elliptik tipdagi tenglama uchun tariflangan Bitsadze-Samarskiy masalasi keyinchalik turli shakllarda aralash turdagi tenglamalar, yuqori tartibli karrali xarakteristikali tenglamalar uchun qo‘yildi va bu masalalarning korrekt ekanligi isbotlandi. Faqat bir turdagi, ya’ni giperbolik turdagi tenglama uchun Bitsadze-Samarskiy masalasini o‘rganish ko‘p yillar, ya’ni 30 yil davomida o‘z yechimini kutib yotdi.

✓ M. Mirsaburov 1998 yilda singulyar koeffitsiyentli buziluvchan giperbolik turdagi tenglama uchun Bitsadze-Samarskiy masalasini ta’rifladi va bu masalani korrekt ekanligini isbotladi. Olingan natija **“Доклады РАН 1999, №1, с 10-12” ва “Дифференциальные уравнения” 2002, т.38 №2, с. 271-276** jurnallarida chop etildi. (M.S.Salaxitdinov bilan hamkorlikda).

✓ M. Mirsaburov tomonidan Trikomi masalasi yangi ko‘rinishda ta’riflandi, ya’ni butun chegaraviy xarakteristika ixtiyoriy ravishda ikkiga bo‘linib, birinchi bo‘lakda F.Trikomi sharti, ikkinchi bo‘lakda va soha ichida yotuvchi xarakteristikada Bitsadze-Samarskiy sharti berildi. Bu masala aralash turdagi tenglamalar uchun qo‘yilgan masalalar tarixida o‘zida aralash sohaning giperbolik qismida lokal va nolokal shartlarni birlashtirgan birinchi masala bo‘ldi.

Mazkur masalani o‘rganish jarayoni ko‘pgina nostandart holatlarni yuzaga keltirdi, jumladan:

F.Trikomining siljishli singulyar integral tenglamasiga duch kelindi va uning yechimi S. G. Mixlin tomonidan takomillashtirilgan Karleman usulida amalga oshiriladi;

aralash turdagi tenglamalar uchun barcha chegaraviy masalalar o‘ng tomoni regulyar operatordan iborat singulyar integral tenglamaga olib kelinar va standart usulda regulyarizatsiyalanar edi. M.Mirsaburov tomonidan o‘rganilayotgan masala o‘ng tomoni regulyar bo‘lmagan (bir nuqtada singulyar maxsuslikka ega bo‘lgan) tenglamalarga olib kelindi va u dastlab siljishli singulyar integral tenglama sifatida regulyarizatsiyalandi, ikkinchi martada Viner-Xopf integral tenglamasiga olib kelindi va unga Fredgolm integral tenglamalar nazariyasini qo‘llash mumkin ekanligi isbotlandi.

Bu natija Rossiya Fanlar Akademiyasining **“Дифференциальные уравнения”. 2001, т.37, №9, с.1281-1284** jurnalida chop etildi.

M. Mirsaburov bajarilishi 2012-2016 yillarga mo‘ljallangan fundamental tadqiqotlar davlat ilmiy texnika dasturi doirasida bajariladigan loyiha ilmiy rahbaridir. (грант Ф-4-32) Loyiha mavzusi: “Singulyar koeffitsiyentli Gellerstedt tenglamasi uchun Trikomi, siljishli masalalar va Bitsadze-Samarskiy shartlarini bir ta’rifda birlashtirgan masalalarning korrektligini o‘rganish”.

Shuningdek, bajarilishi 2022-2026 yillarga mo‘ljallangan (grant FZ-202009211) “Aralash turdagi tenglamalar uchun xarakteristika va buzilish chizig‘ida Frankl va Bitsadze Samarskiy

shartlari berilgan masalalar korrektligini noklassik singulyar integral tenglamalarga olib kelib o‘rganish” nomli ilmiy loyiha rahbaridir.

M. Mirsaburov Termiz davlat universitetida “Matematikaning dolzarb masalalari” deb nomlangan ilmiy-uslubiy seminar rahbari hisoblanadi va bu seminarda yosh tadqiqotchilarning ilmiy ishlari muntazam ravishda muhokama qilinib kelinmoqda. Prof M. Mirsaburovning butun pedagogik, ilmiy-ijodiy faoliyati Termiz davlat universiteti bilan uzluksiz bog‘liq.

1973-2011 yillar mobaynida o‘z faoliyatini matematika kafedrasi o‘qituvchisi lavozimidan boshlab, Termiz davlat universiteti ilmiy ishlar prorektori, fizika-matematika fakulteti dekani, “Ishlab chiqarish asoslari”, “Matematik tahlil”, “Differensial tenglamalar va geometriya” kafedralari mudiri vazifalarida ishladi. M. Mirsaburov 2008-2017 yillarda Surxondaryo viloyati pedagogik kadrlarni qayta tayyorlash va malakasini oshirish instituti “Tabiiy va aniq fanlar ta’limi” kafedrasi mudiri lavozimida ishladi.

Prof. M. Mirsaburovning jami 100 dan ortiq maqolasi davriy ilmiy jurnallarda chop etilgan, shulardan 50 dan ortiq maqolalari Scopous tizimidagi jurnallarda e’lon qilinib , bu jurnallar ingliz tilida Yevropa va AQShda qayta nashr qilingan.

M. Mirsaburov matematiklarning umumjaxon III kongressida (Novosibirsk-1998) o‘z ma’ruzasi bilan ishtirok etgan. U Moskva, Nalchik, Ashxabat, Chimkent shaharlarida bo‘lib o‘tgan xalqaro konferensiyalar ishtirokchisidir.

M. Mirsaburov O‘zbekiston Fanlar Akademiyasining akademigi M. S. Saloxiddinov bilan hammualliflikda **“Нелокальные задачи для уравнений смешанного типа с сингулярными коэффициентами”** nomli monografiya hamda magistrlar uchun “Singulyar koeffitsiyentli Gellerstedt tenglamasi uchun Trikomi masalasi”, “Ikkinchi tartibli singulyar koeffitsiyentli noklassik tenglamalar uchun korrekt qo‘yilgan masalalar” (B. Islomov, N. B. Islomov vilan xamkorlikda) nomli o‘quv qo‘llanmalar va bir qancha uslubiy qo‘llanmalar yozgan.

M. Mirsaburov 2017- 2024 yillarda Termiz davlat universiteti “Matematik analiz” kafedrasi mudiri lavozimida ishladi.

M. Mirsaburov 2024- yildan buyon Termiz davlat universiteti “Matematik analiz” kafedrasi professori lavozimida ishlab kelmoqda.

Professor M. Mirsaburov raxbarligida 10 nafardan ortiq yoshlar fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) ilmiy darajasini olish uchun dissertatsiyalarini muvaffaqiyatli himoya qilishgan.

Professor M. Mirsaburov singulyar koeffitsiyentli buziluvchan giperbolik, elliptik va aralash turdagi tenglamalar uchun lokal va nolokal masalalar nazariyasi sohasida o‘z ilmiy yo‘nalishi va maktabiga ega bo‘lgan, salohiyatli mutaxassislardan biri hisoblanadi.

Uning 53 yildan oshiq davrdagi samarali mehnatlari munosib taqdirlangan va 2011-yilda “Mustaqillikning 20 yilligi”, 2025-yilda Oliy ta‘lim a‘lochisi ko‘krak nishonlari bilan taqdirlangan.

Fizika matematika fanlari doktori, professor Miraxmat Mirsaburovni qutlug‘ 75 yoshi bilan tabriklab, USTOZ ga sihat-salomatlik va ilmiy- pedagogik ishlariga ulkan muvaffaqiyatlar tilaymiz.

Konferensiya tashkiliy qo‘mitasi

4 – SHO'BA. ALGEBRA, ANALIZ VA GEOMETRIYANING ZAMONAVIY MASALALARI

СЕКЦИЯ 4. СОВРЕМЕННЫЕ ПРОБЛЕМЫ АЛГЕБРЫ, АНАЛИЗА И ГЕОМЕТРИИ

UDC 517.572

ON REMOVABLE SINGULARITIES OF α-SEPARATELY HARMONIC FUNCTIONS

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Consider an arbitrary differential form of bidegree $(m-1,\, m-1)$ in a domain $G \subset \mathbb{C}^m$ of the form

$$\alpha = \left(\frac{i}{2}\right)^{m-1} \sum_{l,\, p=1}^{m} A_{lp}(\varsigma) d\varsigma\,[l] \wedge d\overline{\varsigma}\,[p]$$

subject to the following conditions:

(i) the form is closed, that is, $d\alpha = 0$.

(ii) the form is strictly positive in the sense that for every compact subdomain there exists a number $\delta > 0$ such that

$$\alpha - \delta\gamma^{n-1} \geq 0$$

in D, where $\gamma = dd^c\left(|\varsigma|^2\right)$.

(iii) all coefficients satisfy $A_{lp}(\varsigma) \in C^1(G)$, $i,\, p = \overline{1,m}$, here

$$d\varsigma\,[l] = d\varsigma_1 \wedge ... \wedge d\varsigma_{l-1} \wedge d\varsigma_{l+1} \wedge ... \wedge d\varsigma_m$$

and

$$d\bar{\varsigma}\,[p] = d\bar{\varsigma}_1 \wedge ... \wedge d\bar{\varsigma}_{p-1} \wedge d\bar{\varsigma}_{p+1} \wedge ... \wedge d\bar{\varsigma}_m$$

(see [1]).

Definition 1([2]). *The function $\upsilon(\varsigma) \in C^2(G)$ is called α-subharmonic, if $dd^c\upsilon \wedge \alpha \geq 0$ in domain $G \subset \mathbb{C}^m$.*

Definition 2([2]). *The function $\upsilon(\varsigma) \in C^2(G)$ is called α-harmonic, if $dd^c\upsilon \wedge \alpha = 0$ in domain $G \subset \mathbb{C}^m$.*

Let $G \times \Omega \subset \mathbb{R}^m \times \mathbb{R}^n$ be a domain, and let $M \subset G$, $K \subset \Omega$ be arbitrary subsets. Assume that a function $\vartheta(x, y)$ is initially defined only on the Cartesian product $M \times K$. Suppose that the following extension properties hold:

(i) for every fixed $x^0 \in M$, the function

$$y \mapsto \vartheta\left(x^0, y\right)$$

admits a harmonic continuation from K to the whole domain Ω;

(ii) for every fixed $y^0 \in K$, the function

$$x \mapsto \vartheta\left(x, y^0\right)$$

admits a harmonic continuation from M to the whole domain G. Under these assumptions, the corresponding harmonic extensions determine a function on the set

$$X = (M \times \Omega) \cup (G \times K).$$

Such a function is called *separately harmonic on* X.

In the particular case where $M = G$ and $K = \Omega$, the function $\vartheta(x, y)$ is said to be separately harmonic in $G \times \Omega$; that is, it is harmonic with respect to each group of variables while the other group is kept fixed.

It should be noted that the set X, in general, is not itself a domain. Nevertheless, the notion of separate harmonicity on such a set is meaningful. This concept parallels the definition of separately analytic functions in complex analysis: although separate analyticity does not immediately imply joint analyticity, it often leads to strong regularity results under appropriate dimensional assumptions. A similar phenomenon occurs in the harmonic setting, since harmonic functions share several structural properties with analytic functions.

More generally, if $Q \subset \mathbb{R}^m(x) \times \mathbb{R}^n(y)$, where $m, n \geq 2$, is an arbitrary domain (not necessarily representable as a Cartesian product), a function $\vartheta(x, y)$ defined on Q is called separately harmonic if the following conditions are satisfied:

(i) for every fixed x^0 such that the section $\{x = x^0\} \cap Q$ is nonempty, the function

$$y \mapsto \vartheta\left(x^0, y\right)$$

is harmonic in the variable y on this section;

(ii) for every fixed y^0 such that the section $\{y = y^0\} \cap Q$ is nonempty, the function

$$x \mapsto \vartheta\left(x, y^0\right)$$

is harmonic in the variable x on this section.

In this case, the function ϑ is called *separately harmonic in the domain* Q.

Let α' be a strictly positive, closed differential form of bidegree $(m-1, m-1)$ defined in a domain $G \subset \mathbb{C}^m$ and α'' be a strictly positive, closed differential form of bidegree $(n-1, n-1)$ defined in a domain $\Omega \subset \mathbb{C}^n$.

We consider the following operators on the space $C^2(G \times \Omega)$, $G \times \Omega \subset \mathbb{C}^m \times \mathbb{C}^n$:

$$d_\varsigma d^c_{\bar\xi}\vartheta(\varsigma,\eta) \wedge \alpha'(\varsigma) = \left[\frac{i}{2}\sum_{\nu,p=1}^{m} \frac{\partial^2\vartheta(\varsigma,\eta)}{\partial\varsigma_\nu \partial\bar z_p} d\varsigma_\nu \wedge d\bar\varsigma_p\right] \wedge \left[\left(\frac{i}{2}\right)^{m-1} \sum_{\nu,p=1}^{m} \alpha'_{\nu p}(\varsigma)\, d\varsigma[\nu] \wedge d\bar\varsigma[p]\right] =$$

$$= \left(\tfrac{i}{2}\right)^m \sum_{\nu,p=1}^{m} (-1)^{\nu+p+1} \alpha'_{\nu p}(\varsigma) \tfrac{\partial^2\vartheta(\varsigma,\eta)}{\partial\varsigma_\nu \partial\bar\varsigma_p} d\varsigma \wedge d\bar\varsigma,\ \forall \eta \in \Omega;$$

$$d_\eta d^c_{\bar\eta}\vartheta(\varsigma,\eta) \wedge \alpha''(\eta) = \left[\frac{i}{2}\sum_{\nu,p=1}^{n} \frac{\partial^2\vartheta(\varsigma,\eta)}{\partial\eta_\nu \partial\bar\varsigma_p} d\eta_\nu \wedge d\bar\eta_p\right] \wedge \left[\left(\frac{i}{2}\right)^{n-1} \sum_{\nu,p=1}^{n} \alpha''_{\nu p}(\eta)\, d\eta[\nu] \wedge d\bar\eta[p]\right] =$$

$$= \left(\tfrac{i}{2}\right)^m \sum_{\nu,p=1}^{m} (-1)^{\nu+p+1} \alpha''_{\nu p}(\eta) \tfrac{\partial^2\vartheta(\varsigma,\eta)}{\partial w_\nu \partial\bar\eta_p} d\eta \wedge d\bar\eta,\ \forall \varsigma \in G.$$

Definition 3([3]). *If a function* $\vartheta(\varsigma, \eta) \in C^2(G \times \Omega)$ *satisfies the following conditions:*
(i) for every fixed $\eta \in \Omega$,

$$d_\varsigma d^c_{\bar\xi}\vartheta(\varsigma,\eta) \wedge \alpha'(\varsigma) \geq 0;$$

(ii) for every fixed $\varsigma \in G$,

$$d_\eta d^c_{\bar{\eta}} \vartheta(\varsigma, \eta) \wedge \alpha''(\eta) \geq 0,$$

then it is called an α*-separately subharmonic function in the domain* $G \times \Omega$, *where* $\alpha = (\alpha', \alpha'')$.

Definition 4([3]). *If a function* $\vartheta(\varsigma, \eta) \in C^2(G \times \Omega)$ *satisfies the following conditions:*
(i) for every fixed $\eta \in \Omega$,

$$d_\varsigma d^c_{\bar{\varsigma}} \vartheta(\varsigma, \eta) \wedge \alpha'(\varsigma) = 0;$$

(ii) for every fixed $\varsigma \in G$,

$$d_\eta d^c_{\bar{\eta}} \vartheta(\varsigma, \eta) \wedge \alpha''(\eta) = 0,$$

then it is called an α*-separately harmonic function in the domain* $G \times \Omega$, *where* $\alpha = (\alpha', \alpha'')$

The following theorem holds.

Theorem. $G \subset \mathbb{C}^m$ *and* $\Omega \subset \mathbb{C}^n$ *be domains, and let* $M \Subset G$ *and* $K \Subset \Omega$ *be compact subsets. Assume that* $\alpha = (\alpha', \alpha'')$ *is a real-analytic strictly positive closed differential biform in* $G \times \Omega$. *If a function* $\vartheta(\varsigma, \eta)$ *is* α*-separately harmonic in* $(G \times \Omega) \backslash (M \times K)$, *then there exists a real-analytic function* $\tilde{\vartheta}(\varsigma, \eta)$, α*-separately harmonic in* $G \times \Omega$, *such that*

$$\tilde{\vartheta}(\varsigma, \eta)\Big|_{(G\times\Omega)\backslash(M\times K)} \equiv \vartheta(\varsigma, \eta).$$

It follows from this theorem that α-separately harmonic functions also do not admit compact singularities.

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Integral operatorlarning invariant qism fazolari

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Kirish. Kvant mexanikasi, qattiq jismlar nazariyasi va statistik fizika masalalari ko‘p hollarda differensial yoki integral tenglamalarning yechimlarini tadqiq qilish orqali hal qilinadi. Differensial tenglamalarni yechish esa ko‘pincha integral tenglamalarni yechishga olib keladi. Integral tenglamalar nazariyasi Neumann, Volterr, Liuville, Fredholm, Hilbert va Shmidt kabi olimlar tomonidan rivojlantirilgan (qarang [1–3]).

Operatorlar nazariyasida eng muhim muammolaridan biri spektr masalasidir. Chiziqli operator spektrini tekshirishning sinalgan usullaridan biri, bu operatorga nisbatan invariant qism fazolarni topishdir.

Faraz qilaylik, $\mathbb{T}^2 = (-\pi, \pi]^2$ ikki o‘lchamli tor, $L_2(\mathbb{T}^2)$ esa $\mathbb{T}^2$ da aniqlangan kvadrati bilan integrallanuvchi funksiyalarning Hilbert fazosi bo‘lsin.

Hilbert fazoda $L_2(\mathbb{T}^2)$ quyidagi integral operatorni kiritamiz:

$$(Af)(\mathbf{p}) = \int_{\mathbb{T}^2} K(\mathbf{p}, \mathbf{q}) f(\mathbf{q}) d\mathbf{q}.$$

A integral operator yadrosi $K(\mathbf{p}, \mathbf{q})$ ga quyidagi shartlar qo‘yiladi:

$K \in L_2(\mathbb{T}^2 \times \mathbb{T}^2)$ kvadri bilan integrallanuvchanlik sharti, ya’ni

$$\int_{\mathbb{T}^2} \int_{\mathbb{T}^2} |K(\mathbf{p}, \mathbf{q})|^2 d\mathbf{p} d\mathbf{q} < \infty. \tag{1}$$

K yadroning simmetriklik sharti

$$K(\mathbf{p}, \mathbf{q}) = \overline{K(\mathbf{q}, \mathbf{p})}. \tag{2}$$

(1) shart bajarilganda A operatorga Hilbert-Shmidt tipidagi operator deyiladi. (2) shart bajarilganda $A = A^*$ o‘z-o‘ziga qo‘shma operator bo‘ladi.

Hilbert fazosi $L_2(\mathbb{T}^2)$ ni juft funksiyalardan tashkil topgan $L_2^e(\mathbb{T}^2) := \{f \in L_2(\mathbb{T}^2) : \ f(-\mathbf{p}) = f(\mathbf{p})\}$ qism fazo va toq funksiyalardan tashkil topgan $L_2^o(\mathbb{T}^2) := \{f \in L_2(\mathbb{T}^2) : \ f(-\mathbf{p}) = -f(\mathbf{p})\}$ qism fazolarning to‘g‘ri yig‘indisi shaklida yozish mumkin:

$$L_2(\mathbb{T}^2) = L_2^e(\mathbb{T}^2) \oplus L_2^o(\mathbb{T}^2).$$

A integral operator yadrosi $K(\mathbf{p}, \mathbf{q})$ qanday shartlarni qanoatlantirganda, $L_2^e(\mathbb{T}^2)$ va $L_2^o(\mathbb{T}^2)$ qism fazolar A operatorga nisbatan invariant bo‘ladi. Bu savolga ijobiy javob bor.

Bu $K(\mathbf{p}, \mathbf{q})$ yadroning juftlik sharti ekan:

$$K(-\mathbf{p}, -\mathbf{q}) = K(\mathbf{q}, \mathbf{p}). \tag{3}$$

1-teorema. Agar (3) shart bajarilsa, u holda $L_2^e(\mathbb{T}^2)$ va $L_2^o(\mathbb{T}^2)$ qism fazolar A operatorga nisbatan invariant bo‘ladi.

A operatorning $L_2^e(\mathbb{T}^2)$ qism fazodagi qismi (restriction) ni A^e bilan, $L_2^o(\mathbb{T}^2)$ qism fazodagi qismini A^o bilan belgilaymiz. Agar A^e integral operatorning yadrosini $K^e(\mathbf{p}, \mathbf{q})$ bilan, A^o integral operatorning yadrosini $K^o(\mathbf{p}, \mathbf{q})$ bilan belgilasak, u holda

$$K(\mathbf{p}, \mathbf{q}) = K^e(\mathbf{p}, \mathbf{q}) + K^o(\mathbf{p}, \mathbf{q})$$

tenglik o‘rinli bo‘ladi. Demak, quyidagi tenglik ham o‘rinli

$$A = A^e \oplus A^o = A^e + A^o.$$

Operatorlar to‘g‘ri yig‘indisi spektri haqidagi teoremaga [2] hamda 1-teremadan

$$\sigma(A) = \sigma(A^e) \cup \sigma(A^o)$$

tenglik kelib chiqadi.

Shunday qilib, A operatorning spektrini topish masalasi, A^e va A^o operatorlarning spektrlarini alohida – alohida topish masalasiga keltirildi.

Ma'lumki, $L_2(\mathbb{T}^2)$ fazoni simmetrik funksiyalardan tashkil topgan $L_2^s(\mathbb{T}^2) := \{f \in L_2(\mathbb{T}^2) : f(p_1,p_2) = f(p_2,p_1)\}$ qism fazo va antisimmetrik funksiyalardan tashkil topgan $L_2^{as}(\mathbb{T}^2) := \{f \in L_2(\mathbb{T}^2) : \ f(p_1,p_2) = -f(p_2,p_1)\}$ qism fazolar to'g'ri yig'indisi shaklida yozish mumkin:

$$L_2(\mathbb{T}^2) = L_2^s(\mathbb{T}^2) \oplus L_2^{as}(\mathbb{T}^2). \tag{4}$$

A integral operator yadrosi $K(p_1,p_2,q_1,q_2)$ qanday shartlarni qanoatlantirganda, $L_2^s(\mathbb{T}^2)$ va $L_2^{as}(\mathbb{T}^2)$ qism fazolar A operatorga nisbatan invariant bo'ladi. Bu savolga ham ijobiy javob bor.

Bu $K(p_1,p_2,q_1,q_2)$ yadroning simmetriklik sharti ekan:

$$K(p_1,p_2,q_1,q_2) = K(p_2,p_1,q_2,q_1). \tag{5}$$

2-teorema. Agar (5) shart bajarilsa, u holda $L_2^s(\mathbb{T}^2)$ va $L_2^{as}(\mathbb{T}^2)$ qism fazolar A operatorga nisbatan invariant bo'ladi.

(4) tenglik va 2-teoremadan

$$\sigma(A) = \sigma(A^s) \cup \sigma(A^{as})$$

tenglik kelib chiqadi. Bu yerda $A^s = A|_{L_2^s}$ va $A^{as} = A|_{L_2^{as}}$.

Ma'lumki, panjaradagi ikki yoki uch zarrachali sistemaga mos Shryodinger operatorlari (energiya operatorlari) ning muhim spektrdan tashqarida yotuvchi xos qiymatlari Faddeev tipidagi integral operatorlarning xos qiymatlarini topish masalasiga keladi. Integral operatorlarning invariant qism fazolari uch bozonli sistemaga mos diskret Shryodinger operatorining xos qiymatlarini topishga tatbiq qiliingan [3].

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Idempotent real and complex measures

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The max-plus algebra is an algebraic structure consisting of real numbers where the standard operations of addition $(+)$ and multiplication $(\cdot)$ are replaced by the operation $(\oplus)$ of taking a maximum and the operation $(\odot)$ of standard addition, respectively. More precisely, let $\overline{\mathbb{R}}$ denote the set $\mathbb{R}\cup\{-\infty\}$, $\oplus$ a binary operation on $\overline{\mathbb{R}}$ with $x\oplus y = \max\{x,y\}$, and $\odot$ the binary operation

on $\overline{\mathbb{R}}$ with $x \odot y = x + y$. The resulting algebraic structure $\mathbb{R}_{\max} = (\overline{\mathbb{R}}, \oplus, \odot)$ is called a *max-plus semi-field* or simply a *max-plus algebra*. Further, by $\mathbb{R}_{\max}$ we mean both $\overline{\mathbb{R}}$ and $(\overline{\mathbb{R}}, \oplus, \odot)$. Note, $\mathbb{R}_{\max}$ is also called a commutative idempotent semiring or a dioid. In the max-plus algebra $\varepsilon = -\infty$ is the additive identity: $x \oplus \varepsilon = \varepsilon \oplus x = x$, for each $x \in \mathbb{R}_{\max}$ and the multiplicative identity is $e = 0$: $x \odot e = e \odot x = x$, for all $x \in \mathbb{R}_{\max}$ [1-2].

We put

$$\overline{\mathbb{C}} = \overline{\mathbb{R}} + i\overline{\mathbb{R}} = \{x + iy : x,\, y \in \overline{\mathbb{R}}\}$$

here i is the imaginary unit, i. e. $i^2 = -1$. On $\overline{\mathbb{C}}$, we introduce a partial order as

$$z_1 \leq z_2 \iff \begin{cases} x_1 \leq x_2, \\ y_1 \leq y_2. \end{cases}$$

We define a binary operation $\oplus : \overline{\mathbb{C}} \times \overline{\mathbb{C}} \to \overline{\mathbb{C}}$ as

$$z_1 \oplus z_2 = z_1 \vee z_2 = (x_1 \oplus x_2) + i(y_1 \oplus y_2),$$

and $\odot : \overline{\mathbb{C}} \times \overline{\mathbb{C}} \to \overline{\mathbb{C}}$ as

$$z_1 \odot z_2 = z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

Here $z_k = x_k + iy_k$, $k = 1, 2$. Next, we construct an algebraic structure on $\overline{\mathbb{C}}$ with respect to the binary operations $\oplus$ and $\odot$. The neutral element with respect to $\oplus$ is $\varepsilon = (-\infty) + i(-\infty)$ and the neutral element with respect to $\odot$ is $e = 0 + i0$.

The following conditions are satisfied on $\overline{\mathbb{C}}$:

1. $(z_1 \oplus z_2) \oplus z_3 = z_1 \oplus (z_2 \oplus z_3)$ for $\forall z_1, z_2, z_3 \in \overline{\mathbb{C}}$;
2. $z \oplus \varepsilon = \varepsilon \oplus z = z$ for $\forall z \in \overline{\mathbb{C}}$;
3. $z_1 \oplus z_2 = z_2 \oplus z_1$ for $\forall z_1, z_2 \overline{\mathbb{C}}$;
4. $(z_1 \odot z_2) \odot z_3 = z_1 \odot (z_2 \odot z_3)$ for $\forall z_1, z_2, z_3 \in \overline{\mathbb{C}}$;
5. $z_1 \odot (z_2 \oplus z_3) = z_1 \odot z_2 \oplus z_1 \odot z_3$ for $\forall z_1, z_2, z_3 \in \overline{\mathbb{C}}$;
6. $\forall z \in \mathbb{C}, \exists (-z) \in \mathbb{C} \implies z \odot (-z) = e$.

Remark 1. It is worth noting that $\overline{\mathbb{C}}$ has non-invertible elements with respect to $\odot$ distinguished from e. For example, $z = -\infty + ia$, $-\infty < a < \infty$, is non-invertible according to $\odot$.

So, then the resulting algebraic structure $\mathbb{C}_{\max} = (\overline{\mathbb{C}}, \oplus, \odot)$ is called a *max-plus complex semi-ring* or *idempotent complex semi-ring* (which is not a semi-field).

Let $\mathcal{P}(\Omega)$ denote the powerset of Ω and $\mathcal{A}$ be a directed set, i.e., $\mathcal{A}$ is equipped with a reflexive and transitive binary relation $\leq$ such that for every $\alpha, \beta \in \mathcal{A}$ there exists $\gamma \in \mathcal{A}$ satisfying

$$\alpha \leq \gamma,\ \beta \leq \gamma.$$

Definition 1. A set function

$$\mu : \mathcal{P}(\Omega) \longrightarrow \overline{\mathbb{R}}$$

is called an *idempotent real measure* on Ω if the following conditions hold:

1. $\mu(\varnothing) = -\infty$;

2. $\mu(A \cup B) = \max\{\mu(A), \mu(B)\}, \qquad \forall\, A, B \in \mathcal{P}(\Omega);$

3. $\mu\left(\bigcup\limits_{\alpha\in\mathcal{A}} A_\alpha\right) = \sup\limits_{\alpha\in\mathcal{A}} \mu(A_\alpha).$

for every increasing net $\{A_\alpha : \alpha \in \mathcal{A}\}$ of subsets of Ω.

Definition 2. A set function

$$\nu : \mathcal{P}(\Omega) \longrightarrow \overline{\mathbb{C}}$$

is called an *idempotent complex measure* on Ω if the following conditions hold:

1. $\nu(\varnothing) = \varepsilon;$

2. $\nu(A \cup B) = \nu(A) \vee \nu(B), \qquad \forall\, A, B \in \mathcal{P}(\Omega);$

3. $\nu\left(\bigcup\limits_{\alpha\in\mathcal{A}} A_\alpha\right) = \bigvee\limits_{\alpha\in\mathcal{A}} \nu(A_\alpha).$

for every increasing net $\{A_\alpha : \alpha \in \mathcal{A}\}$ of subsets of Ω.

Theorem. Let

$$\nu : \mathcal{P}(\Omega) \longrightarrow \overline{\mathbb{C}}$$

be a set function. For every $A \in \mathcal{P}(\Omega)$, write

$$\nu(A) = \mu_1(A) + i\mu_2(A),$$

where

$$\mu_1, \mu_2 : \mathcal{P}(\Omega) \longrightarrow \overline{\mathbb{R}}.$$

Moreover, $\nu = \mu_1 + i\mu_2$ is an idempotent complex measure if and only if μ_1 and μ_2 are idempotent real measures.

Proof. Suppose first that μ is an idempotent complex measure. Since

$$\nu(\varnothing) = \varepsilon = (-\infty) + i(-\infty),$$

we have

$$\mu_1(\varnothing) = -\infty, \qquad \mu_2(\varnothing) = -\infty.$$

Let A, $B \in \mathcal{P}(\Omega)$. By the definition of an idempotent complex measure,

$$\nu(A \cup B) = \nu(A) \vee \nu(B).$$

Writing

$$\nu(A) = \mu_1(A) + i\mu_2(A), \qquad \nu(B) = \mu_1(B) + i\mu_2(B),$$

we obtain

$$\nu(A \cup B) = (\mu_1(A) \vee \mu_1(B)) + i\,(\mu_2(A) \vee \mu_2(B)).$$

Hence,

$$\mu_1(A \cup B) = \mu_1(A) \vee \mu_1(B) = \max\{\mu_1(A), \mu_1(B)\}$$

and

$$\mu_2(A \cup B) = \mu_2(A) \vee \mu_2(B) = \max\{\mu_2(A), \mu_2(B)\}.$$

Now let

$$\{A_\alpha : \alpha \in \mathcal{A}\}$$

be an increasing net of subsets of Ω. By the definition of an idempotent complex measure,

$$\nu\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) = \bigvee_{\alpha\in\mathcal{A}} \nu(A_\alpha) = \bigvee_{\alpha\in\mathcal{A}} (\mu_1(A_\alpha) + i\mu_2(A_\alpha)) = \bigvee_{\alpha\in\mathcal{A}} \mu_1(A_\alpha) + i \bigvee_{\alpha\in\mathcal{A}} \mu_2(A_\alpha).$$

On the other side

$$\nu\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) = \mu_1\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) + \mu_2\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right).$$

Consequently,

$$\mu_1\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) = \sup_{\alpha\in\mathcal{A}} \mu_1(A_\alpha)$$

and

$$\mu_2\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) = \sup_{\alpha\in\mathcal{A}} \mu_2(A_\alpha).$$

Thus, μ_1 and μ_2 are idempotent real measures.

Conversely, suppose that μ_1 and μ_2 are idempotent real measures. Define

$$\nu(A) = \mu_1(A) + i\mu_2(A), \qquad A \in \mathcal{P}(\Omega).$$

Since

$$\mu_1(\varnothing) = \mu_2(\varnothing) = -\infty,$$

we have

$$\nu(\varnothing) = (-\infty) + i(-\infty) = \varepsilon.$$

For arbitrary A, $B \in \mathcal{P}(\Omega)$,

$$\begin{aligned}\nu(A\cup B) &= \mu_1(A\cup B) + i\mu_2(A\cup B)\\ &= \max\{\mu_1(A),\mu_1(B)\} + i\max\{\mu_2(A),\mu_2(B)\}\\ &= \nu(A)\vee\nu(B).\end{aligned}$$

Finally, for every increasing net $\{A_\alpha : \alpha\in\mathcal{A}\}$,

$$\begin{aligned}\nu\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) &= \mu_1\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right) + i\mu_2\left(\bigcup_{\alpha\in\mathcal{A}} A_\alpha\right)\\ &= \sup_{\alpha\in\mathcal{A}} \mu_1(A_\alpha) + i \sup_{\alpha\in\mathcal{A}} \mu_2(A_\alpha)\\ &= \bigvee_{\alpha\in\mathcal{A}} \mu_1(A_\alpha) + i \bigvee_{\alpha\in\mathcal{A}} \mu_2(A_\alpha)\\ &= \bigvee_{\alpha\in\mathcal{A}} \nu(A_\alpha).\end{aligned}$$

Hence, ν is an idempotent complex measure.

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TRIGONOMETRIK YIG'INDILAR VA ULARNING AYRIM SONLAR NAZARIYASI MASALALARIGA TADBIQLARI

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Sonlar nazariyasining rivojlanishida trigonometrik yig'indilar usuli alohida o'rin egallaydi. Bu usul XX asr boshlarida G. Xardi (G.H. Hardy) va J. Litlvud (J.E. Littlewood) tomonidan taklif etilgan "doira usuli" (*circle method*) hamda I.M. Vinogradov tomonidan yaratilgan trigonometrik yig'indilar usuli asosida shakllangan bo'lib, additiv sonlar nazariyasining ko'plab murakkab masalalarini yechishda universal vosita bo'lib xizmat qiladi. Trigonometrik yig'indilar usulining asosiy g'oyasi biror sonlar to'plamining arifmetik xossalarini o'rganishni shu to'plam bilan bog'liq bo'lgan trigonometrik yig'indini baholashga keltirishdan iborat.

1. Asosiy tushunchalar. Ushbu $exp(x) = exp^{2\pi ix}$ belgilashni kiritamiz. Umumiy ko'rinishda trigonometrik yig'indini

$$S(\alpha) = \sum_{n=1}^{N} exp(f(n)\alpha)$$

ko'rinishda ifodalanadi, bu yerda $f(n)$ berilgan butun qiymatli funksiya. Xususiy holda, quyidagi klassik yig'indilar keng qo'llaniladi:
1.Gauss yig'indisi:

$$G(a,q) = \sum_{x=0}^{q-1} e\left(\frac{ax^2}{q}\right);$$

2.Veyl yig'indisi:

$$S_k(\alpha) = \sum_{n=1}^{N} e(\alpha n^k), \quad k \geq 2;$$

3. Klosterman yig'indisi:

$$K(a,b;q) = \sum_{x \pmod q}{}' e\left(\frac{ax + bx^{-1}}{q}\right).$$

Bu yig'indilarning har biri modul q bo'yicha taqsimlanish xossalari va yig'indi hadlarining bir-birini neytrallash xossasi orqali sonlar nazariyasi masalalarini o'rganishda muhim ahamiyatga ega.

2. Sonlar nazariyasi masalalariga tadbiqlari

2.1. Waring muammosi. Har bir natural son N ni k-darajali natural sonlar yig'indisi sifatida

$$N = x_1^k + x_2^k + \cdots + x_s^k$$

ko'rinishida ifodalash mumkinligini isbotlashda Vinogradovning o'rtacha qiymat teoremasi hal qiluvchi rol o'ynaydi. Bu teorema

$$J_{s,k}(N) = \int_0^1 |S_k(\alpha)|^{2s} d\alpha$$

integralining N ga nisbatan yuqori bahosini beradi va natijada s ning k ga bog'liq minimal qiymati (Waring soni $G(k)$) uchun samarali baholar olishga imkon beradi.

2.2. Kvadratik chegirmalarning taqsimlanishi. Gauss yig'indilaridan foydalanib, q moduli bo'yicha kvadratik chegirmalarning intervallar bo'yicha taqsimlanishi o'rganiladi. Gauss yig'indisining

$$|G(a,q)| \le \sqrt{q}$$

ko'rinishidagi bahosi kvadratik chegirmalarning $[0,q)$ oralig'ida deyarli tekis taqsimlanganligini ko'rsatadi.

2.3. Sonlarning 1 modul bo'yicha tekis taqsimlanishi. Veyl kriteriysi (*Weyl's criterion*) asosida (αn) ketma-ketligining $[0,1)$ da tekis taqsimlanishi

$$\sum_{n=1}^{N} e(h\alpha n)$$

yig'indining barcha $h \neq 0$ uchun N ga nisbatan $o(N)$ tartibida bo'lishiga ekvivalent ekanligi ko'rsatiladi. Bu natija irratsional sonlarning arifmetik xossalarini o'rganishda asosiy vositalardan biridir.

2.4. Diofant tenglamalari yechimlari sonini baholash. Doira usuli yordamida chiziqli bo'lmagan Diofant tenglamalarining berilgan chegaradagi butun yechimlari soni katta (*major arcs*) va kichik (*minor arcs*) yoylar bo'yicha integrallarga ajratiladi. Katta yoylar bo'yicha olingan integral asimptotik formulaning bosh hadini, kichik yoylardagi integral esa qoldiq hadni beradi, uning kichikligi trigonometrik yig'indining katta yoylardagi bahosiga bog'liq.

2.5. Tub sonlarning taqsimlanishi bilan bog'liq masalalar. Vinogradov usuli tub sonlar bo'yicha trigonometrik yig'indi

$$\sum_{p \le N} e(\alpha p)$$

ni baholashga imkon berib, har qanday yetarlicha katta toq son uch tub sonning yig'indisi sifatida ifodalanishini (Goldbax-Vinogradov teoremasi) isbotlashda asosiy vosita bo mezon bo'lib xizmat qildi. Bu natija additiv sonlar nazariyasida trigonometrik yig'indilar usulining eng yorqin tadbiqlaridan biri hisoblanadi. Shuningdek, xuddi shu usul asosida tub sonlar orasidagi arifmetik progressiyalarga oid natijalar va tub sonlarning kichik intervallardagi taqsimlanishi ham o'rganiladi.

2.6. Minor yoylarda bahoning ahamiyati. Doira usulining muvaffaqiyati asosan kichik yoylarda trigonometrik yig'indi uchun N ga nisbatan nozik yuqori bahoni olishga bog'liq. Vinogradovning bu boradagi g'oyasi — yig'indini ratsional yaqinlashish a/q asosida guruhlash va Veyl tengsizligidan foydalanib bahoni qadam-baqadam kamaytirish — keyinchalik ko'plab boshqa additiv masalalarga (masalan, k-darajali sonlarning yig'indisi, aralash sistemalar) muvaffaqiyatli tatbiq etilgan.

Xulosa

Keltirilgan misollardan ko'rinadiki, trigonometrik yig'indilar usuli additiv va multiplikativ sonlar nazariyasining turli masalalarini yagona metodologik yondashuv asosida tadqiq etish imkonini beradi. Ushbu usulning yanada rivojlantirilishi va aniqroq baholarning olinishi Waring tipidagi masalalar, Diofant tenglamalari, kvadratik chegirmalarning taqsimlanishi hamda tub sonlarning taqsimlanishi bilan bog'liq zamonaviy tadqiqotlarda muhim ahamiyat kasb etadi. Kelgusi tadqiqotlarda ushbu usulni ko'p o'lchovli va aralash sistemalar uchun umumlashtirish dolzarb masalalardan biri bo'lib qolmoqda.

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UCHTA TUB O'ZGARUVCHILI CHIZIQLI TENGLAMALAR JUFTLIGI HAQIDA

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Additiv sonlar nazariyasining markaziy yo'nalishlaridan biri natural sonlarni tub sonlarning yig'indilari yoki chiziqli kombinatsiyalari ko'rinishida ifodalash bilan bog'liq[cite: 1]. 1742-yilda Goldbax tomonidan qo'yilgan binar va ternar gipotezalar nomi bilan sonlar nazariyasida mashhur gipotezaga ko'ra, har bir 2 dan katta juft sonni ikkita tub sonning yig'indisi ko'rinishida, har qanday beshdan katta toq sonni uchta tub sonlar yig'indisi ko'rinishida ifodalash mumkin[1]. I.M. Vinogradov 1937-yilda har bir yetarlicha katta toq son uchta tub son yig'indisi ekanini isbotladi[1]. Juft sonlar uchun masala hozirgacha ochiq, biroq X. Montgomeri va R. Von 1975-yilda maxsus to'plam uchun yuqoridan baho oldilar[2]: agar katta X uchun

$$E(X) := \{2 \le b \le X : b - \text{juft va } b = p_1 + p_2 \text{ tub } p_1, p_2 \text{ larda yechimsiz}\} \tag{1}$$

deb belgilansa, u holda shunday effektiv absolyut $\varepsilon > 0$ mavjudki, card $E(X) \ll X^{1-\varepsilon}$[1].

M.C. Liu va K.M. Tsang ushbu masalani sezilarli darajada umumlashtirib, b_1, b_2 butun sonlar juftligini bir vaqtning o'zida

$$b_\lambda = a_{\lambda 1}p_1 + a_{\lambda 2}p_2 + a_{\lambda 3}p_3, \quad \lambda = 1, 2 \tag{2}$$

ko'rinishda ifodalash masalasini o'rgandilar; bu yerda $a_{\lambda j}$ – berilgan butun koeffitsiyentlar, p_1, p_2, p_3 tub sonlar[1].

Triviallik va xoslikdan qutilish maqsadida sistema tenglamalarining koeffitsiyentlaridan tuzilgan matritsasining ikkinchi tartibli minorlari uchun $\Delta_{ij} := a_{1i}a_{2j} - a_{1j}a_{2i}$ belgilashlar, hamda ushbu $\Delta_{12}\Delta_{13}\Delta_{23} \neq 0$ va $(\Delta_{12}, \Delta_{13}, \Delta_{23}) = 1$ shartlarni o'rinli bo'lsin deb qabul qilamiz[1].

Теорема 1. (M.C. Liu, K.M. Tsang). Ba'zi musbat haqiqiy y_1, y_2, y_3 sonlar uchun $a_{\lambda 1}y_1 + a_{\lambda 2}y_2 + a_{\lambda 3}y_3 > 0$ $(\lambda = 1, 2)$ bo'lsin. U holda har qanday katta X uchun

$$\text{card } W(X) \gg X^2(B^2 \log\log B)^{-2} \tag{3}$$

va maxsus to'plam uchun card $E_1(X) \leq X^{2-\varepsilon}$ baho o'rinlidir[1].

Teorema Xardi-Litlvud doira metodining ikki o'lchovli varianti yordamida isbotlanadi[1]. $N = 18B^3X$ deb olib, logarifmik vazinli yechimlar soni $I(b) := \sum \Lambda(n_1)\Lambda(n_2)\Lambda(n_3)$ o'rganiladi[cite: 1]. Eksponensial yig'indi $S(y) := \sum \Lambda(n)e(ny)$ orqali $I(b)$ ikki o'lchovli integral ko'rinishida ifodalanadi[1]:

$$I(b) = \iint e(-x \cdot b)S(x_1')S(x_2')S(x_3')dx_1dx_2 = I_1(b) + I_2(b), \tag{4}$$

bu yerda I_1 – katta yoylar, I_2 – kichik yoylar bo'yicha integral[1]. Katta yoylar tahlilida Gallaxerning zichlik bahosi hal qiluvchi ahamiyatga ega.

Ishning asosiy natijasi uchta tub son o'zgaruvchisiga ega bo'lgan ikkita chiziqli tenglamadan iborat sistemaning yechilishi masalasi o'rganilib. M.C. Liu va K.M. Tsang tomonidan olingan asosiy natijalar bunday sistemani qanoatlantiruvchi juftliklar to'plamining quyi bahosi hamda maxsus to'plamning yuqori bahosi to'liq bayon etiladi, isbotning metodik sxemasi (Xardi-Litlvud doira metodi, Gallager zichlik bahosi, maxsus nol tahlili, singulyar qator va singulyar integral) yoritiladi[3]. Natijalarning Goldbax muammosidagi maxsus to'plamga tatbiqi hamda beshta tub o'zgaruvchili umumlashmasini ko'rsatishdan iborat.

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ENG KICHIK KVADRATIK CHEGIRMAMASLAR VA VINOGRADOV GIPOTEZASI

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Sonlar nazariyasining klassik va hozirgacha to'liq yechilmagan muhim masalalaridan biri berilgan tub modul bo'yicha eng kichik kvadratik chegirmamasning o'sish tartibini aniqlashdan iboratdir. Ushbu masala tub sonlar taqsimotining chuqur qonuniyatlari, Dirixle xarakterlari yig'indilarining asimptotik xatti-harakati hamda analitik sonlar nazariyasining fundamental usullari bilan uzviy bog'liqdir.

1. Asosiy tushunchalar va masalaning qo'yilishi. Aytaylik, $p > 2$ toq tub son va a butun son bo'lib, $(a, p) = 1$ shartni qanoatlantirsin. Agar

$$x^2 \equiv a \pmod{p}$$

taqqoslama butun yechimga ega bo'lsa, a soni p moduli bo'yicha *kvadratik chegirma*, aks holda *kvadratik chegirmamas* deb ataladi. Kvadratik chegirmalilik xossasi Lejandr simvoli $\left(\frac{a}{p}\right)$ yordamida ifodalanadi:

$$\left(\frac{a}{p}\right) = \begin{cases} 1, & \text{agar } a \text{ kvadratik chegirma bo'lsa,} \\ -1, & \text{agar } a \text{ kvadratik chegirmamas bo'lsa.} \end{cases}$$

Lejandr simvoli p moduli bo'yicha aniqlangan haqiqiy bosh bo'lmagan Dirixle xarakteridan iboratdir. n_p orqali p moduli bo'yicha eng kichik musbat kvadratik chegirmamasni belgilaylik:

$$n_p := \min\left\{ n \in \mathbb{Z}^+ : \left(\frac{n}{p}\right) = -1 \right\}.$$

Ushbu miqdorning $p \to \infty$ dagi yuqori va quyi chegaralarini topish analitik sonlar nazariyasining markaziy muammolaridan biridir.

2. Polya–Vinogradov tengsizligi va trivial baho. Dirixle xarakterlari yig'indilarini baholash bo'yicha 1918-yilda mustaqil ravishda J. Polya va I. M. Vinogradov tomonidan olingan tengsizlikka ko'ra, har qanday bosh bo'lmagan xarakter $\chi \pmod p$ va butun $M, N > 0$ sonlar uchun

$$\left| \sum_{n=M+1}^{M+N} \chi(n) \right| < c\sqrt{p} \ln p$$

baho o'rinlidir (c – absolut o'zgarmas). Agar $n_p > N$ deb olsak, $1 \le n \le N$ oraliqdagi barcha butun sonlar uchun $\chi(n) = 1$ bo'ladi va $\sum_{n=1}^{N} \chi(n) = N$ tenglik kelib chiqadi. Polya–Vinogradov tengsizligiga asosan $N < c\sqrt{p} \ln p$ ekanligi, ya'ni

$$n_p \ll \sqrt{p} \ln p$$

trivial bahosi olinadi.

3. Vinogradov gipotezasi va silliq sonlar. I. M. Vinogradov 1918-yilda ushbu bahoni jiddiy takomillashtirish mumkinligini ko'rsatib, quyidagi mashhur gipotezani ilgari surdi.

Vinogradov gipotezasi. Har qanday $\varepsilon > 0$ uchun

$$n_p \ll_\varepsilon p^\varepsilon,$$

ya'ni $n_p = p^{o(1)}$ munosabat o'rinlidir. Boshqacha aytganda, n_p miqdor p ning har qanday musbat darajasidan sekinroq o'sadi.

Ushbu gipotezani o'rganishda Vinogradov xarakterning to'liq multiplikativligi $\chi(ab) = \chi(a)\chi(b)$ xossasidan foydalangan. Agar $n_p > y$ bo'lsa, y dan oshmaydigan barcha tub sonlar uchun $\chi(q) = 1$ bo'ladi. Natijada tub bo'luvchilari faqat y dan oshmaydigan barcha sonlar (y-silliq sonlar) uchun ham majburiy tarzda $\chi(n) = 1$ bo'lib qoladi. Silliq sonlar soni $\Psi(x, y)$ ning xarakterlar yig'indisidagi ulushini hisoblash orqali Vinogradov trivial bahoni birinchi bo'lib yaxshilagan.

4. Shartli va Berjessning shartsiz baholari. Agar Dirixle L-funksiyalari uchun Umumlashgan Riman Gipotezasi (GRH) to'g'ri deb qabul qilinsa, N. K. Ankeni (Ankeny, 1952) juda kuchli natijani isbotladi:

$$n_p \ll (\log p)^2 \qquad \text{(GRH sharti ostida).}$$

Shartsiz holatda esa eng yaxshi natija D. A. Berjess (Burgess, 1957; 1962) tomonidan olingan bo'lib, u kichik oraliqlardagi xarakterlar yig'indilari uchun Veyl (A. Weil) natijalariga asoslangan holda quyidagi tengsizlikni isbotladi:

$$\sum_{n=M+1}^{M+N} \chi(n) \ll N^{1-\frac{1}{r}} p^{\frac{r+1}{4r^2}} \log p, \qquad (r = 1, 2, 3).$$

Ushbu tengsizlikdan n_p uchun hozirgacha eng yaxshi bo'lib turgan quyidagi rekord baho kelib chiqadi:

$$n_p \ll_\varepsilon p^{\frac{1}{4\sqrt{e}}+\varepsilon} \approx p^{0.1516...+\varepsilon}.$$

Bu natija 60 yildan ortiq vaqt davomida prinsipial jihatdan yaxshilanmay kelmoqda.

5. Quyi chegara va zamonaviy holat. Eng kichik kvadratik chegirmamasning quyi chegarasi bo'yicha X. Montgomeri (Montgomery, 1971) ehtimolli modellar va Erdyosh tipidagi tasodifiy ketma-ketliklar mulohazasiga tayangan holda, shunday cheksiz ko'p tub p sonlari mavjudligini ko'rsatdiki, ular uchun

$$n_p = \Omega\big(\log p \log\log\log p\big)$$

munosabat o'rinli bo'ladi.

Xulosa

Shunday qilib, analitik sonlar nazariyasida n_p ning aniq o'sish tartibi $\Omega(\log p \log\log\log p)$ hamda $O(p^{\frac{1}{4\sqrt{e}}+\varepsilon})$ chegaralari oralig'ida joylashgan. Berjess baholarini (*Burgess barrier*) yaxshilash, kichik oraliqlardagi xarakterlar yig'indisini yangi usullar orqali baholash hamda Vinogradov gipotezasini shartsiz isbotlash zamonaviy sonlar nazariyasining markaziy va istiqbolli yo'nalishlaridan biri bo'lib qolmoqda.

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NON-PERIODIC p-ADIC QUASI GIBBS MEASURES FOR A THREE-STATE SOS MODEL ON A CAYLEY TREE OF ORDER FOUR

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The theory of p-adic Gibbs distributions on trees provides a natural non-Archimedean analogue of the probabilistic formalism of statistical mechanics. In this setting the ultrametric structure of the field $\mathbb{Q}_p$ is compatible with the hierarchical geometry of a Cayley tree, and the compatibility equations for finite-volume distributions can be reduced to nonlinear recursive relations for boundary fields. Such recursions may possess several solutions and hence generate different p-adic Gibbs-type measures; see, for example, [1–5]. The purpose of this note is to isolate one part of the construction from the three-state p-adic SOS model and to describe non-periodic quasi Gibbs measures on the Cayley tree of order four. We deliberately restrict ourselves to this case and do not discuss the constructions for arbitrary orders, boundedness of the resulting measures, or the phase-transition statements obtained in the full article.

Recall briefly the non-Archimedean background. For a fixed prime p, every nonzero rational number can be represented in the form $x = p^r n/m$, where $r \in \mathbb{Z}$ and n, m are relatively prime to p. Its p-adic norm is $|x|_p = p^{-r}$, and the completion of $\mathbb{Q}$ with respect to this norm is denoted by $\mathbb{Q}_p$. The strong triangle inequality

$$|x + y|_p \le \max\{|x|_p, |y|_p\}$$

is the basic analytic feature of the field. The p-adic exponential

$$\exp_p(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

converges for $|x|_p < p^{-1/(p-1)}$. Hence, if the coupling constant J belongs to this ball, then $\theta = \exp_p(J)$ is well defined and belongs to the multiplicative neighborhood $\mathcal{E}_p$ of 1. This restriction is essential when the algebraic equations for the boundary law are solved: an algebraic root is physically admissible for the present construction only when it lies in the required p-adic domain.

To make the role of the recursion explicit, let V_n denote the ball of radius n around the root and W_n its boundary sphere. For a boundary field $z = (z_{i,x})$, the finite-volume distribution has the form

$$\mu_z^{(n)}(\sigma_n) = \frac{1}{Z_{n,z}} \exp_p\{-H_n(\sigma_n)\} \prod_{x \in W_n} z_{\sigma(x),x},$$

with the corresponding partition function $Z_{n,z}$. The family $\{\mu_z^{(n)}\}$ defines an infinite-volume measure precisely when it is compatible under marginalization from V_n to V_{n-1}. For the SOS interaction this Kolmogorov-type compatibility condition is equivalent to a local recursive equation at every vertex. Thus the global measure problem on an infinite tree is converted into the study of an algebraic-dynamical system for the boundary variables. This reduction is the main structural reason why Cayley trees are particularly convenient for explicit Gibbs-measure constructions.

Let $\Gamma^k = (V, L)$ be a Cayley tree of order k, and let $S(x)$ denote the set of direct successors of a vertex x with respect to a fixed root. Thus $|S(x)| = k$ for every non-root vertex. We consider the solid-on-solid (SOS) model with spin set

$$\Phi = \{0, 1, 2\}.$$

For a finite set V_n and a configuration $\sigma \in \Phi^{V_n}$ the Hamiltonian is

$$H_n(\sigma) = J \sum_{\langle x,y \rangle \in L_n} |\sigma(x) - \sigma(y)|_\infty, \tag{1}$$

where $J \in B(0, p^{-1/(p-1)})$. Put

$$\theta = \exp_p(J), \qquad \mathcal{E}_p = \left\{ u \in \mathbb{Q}_p : \ |u-1|_p < p^{-1/(p-1)} \right\}.$$

For a family of boundary vectors $\widetilde{z}_x = (\widetilde{z}_{0,x}, \widetilde{z}_{1,x}, \widetilde{z}_{2,x})$, the finite-volume p-adic distributions are defined by the usual exponential weight and a normalizing partition function. The compatibility condition for these distributions is equivalent to a system of recursive equations for the normalized boundary variables. After setting $z_{0,x} = 1$ and writing $z_x = z_{1,x}$, the three-state model reduces to the scalar recursion

$$z_x = \prod_{y \in S(x)} F_\theta(z_y), \quad where \quad F_\theta(z) = \frac{z + 2\theta}{\theta^2 + \theta z + 1}. \tag{2}$$

Every solution of (2) determines a unique p-adic quasi Gibbs measure for the model [2,3]. Consequently, the construction of new measures can be reduced to the construction of non-periodic solutions of (2).

For the Cayley tree of order two, a translation-invariant boundary law satisfies

$$z = F_\theta(z)^2. \tag{3}$$

In the parameter region considered in the source article, three translation-invariant solutions are used:

$$z^{(1)} = 1 + o[1], \qquad z^{(2)} = -2 + o[1], \qquad z^{(3)} = -2 + o[1]. \tag{4}$$

Here the notation $o[1]$ refers to a p-adic quantity whose norm is strictly less than one. The essential idea is to use the values in (4) not as a constant boundary law, but as labels distributed recursively over the vertices of Γ^4. Since each vertex has four direct successors, different local successor patterns lead to genuinely non-translation-invariant solutions.

We first use the pair $z^{(2)}, z^{(3)}$. If a vertex x carries the value $z^{(2)}$, assign $z^{(2)}$ to three successors and $z^{(3)}$ to the remaining successor. If x carries $z^{(3)}$, interchange the roles of the two values. The compatibility equation becomes

$$\begin{cases} z^{(2)} = F_\theta(z^{(2)})^3 F_\theta(z^{(3)}), \\ z^{(3)} = F_\theta(z^{(3)})^3 F_\theta(z^{(2)}). \end{cases} \tag{5}$$

Using (3) for $z^{(2)}$ and $z^{(3)}$, system (5) is reduced to the single condition

$$F_\theta(z^{(2)}) F_\theta(z^{(3)}) = 1. \tag{6}$$

Substitution of the explicit form of F_θ gives

$$(\theta - 1)(\theta + 1)\left[\theta^2 + \theta(z^{(2)} + z^{(3)}) + z^{(2)} z^{(3)} - 1\right] = 0. \tag{7}$$

The nontrivial factor yields

$$\theta_{1,2} = \frac{-(z^{(2)} + z^{(3)}) \pm \sqrt{(z^{(2)} - z^{(3)})^2 + 4}}{2},$$

and an admissible value is selected under the condition $\theta \in \mathcal{E}_p$. Thus the local rule above gives a non-constant solution of the compatibility equation whenever the corresponding p-adic root belongs to the admissible exponential domain.

A second construction combines $z^{(1)}$ and $z^{(2)}$. At a vertex carrying $z^{(1)}$, assign $z^{(1)}$ to two successors and $z^{(2)}$ to the other two; at a vertex carrying $z^{(2)}$, assign $z^{(2)}$ to all four successors. Then

$$\begin{cases} z^{(1)} = F_\theta(z^{(1)})^2 F_\theta(z^{(2)})^2, \\ z^{(2)} = F_\theta(z^{(2)})^4. \end{cases} \tag{8}$$

Because each $z^{(i)}$ already satisfies (3), the additional restriction in (8) is

$$F_\theta(z^{(2)})^2 = 1. \tag{9}$$

The equation $F_\theta(z^{(2)}) = \pm 1$ reduces to elementary quadratic equations in θ. Formally, the corresponding candidates are

$$\theta = 1, \quad 1 - z^{(2)}, \quad -1, \quad -1 - z^{(2)},$$

from which only those belonging to $\mathcal{E}_p$ are relevant. Replacing $z^{(2)}$ by $z^{(3)}$ gives a third construction with the same structure and condition

$$F_\theta(z^{(3)})^2 = 1. \tag{10}$$

There is also a mixed construction involving all three values. If x carries $z^{(1)}$, place $z^{(1)}$ on two successors and place $z^{(2)}$, $z^{(3)}$ on the remaining two successors. If x carries $z^{(2)}$, place $z^{(2)}$ on three successors and $z^{(3)}$ on one; for a vertex carrying $z^{(3)}$ use the symmetric rule. The resulting system is

$$\begin{cases} z^{(1)} = F_\theta(z^{(1)})^2 F_\theta(z^{(2)}) F_\theta(z^{(3)}), \\ z^{(2)} = F_\theta(z^{(2)})^3 F_\theta(z^{(3)}), \\ z^{(3)} = F_\theta(z^{(3)})^3 F_\theta(z^{(2)}). \end{cases} \tag{11}$$

Again, relation (3) reduces all three equalities in (11) to condition (6). Hence this assignment supplies another non-periodic solution distinct from the preceding local patterns.

Theorem. *Let $p > 3$ and assume that $p \equiv 1 \pmod 8$ or $p \equiv 3 \pmod 8$. For the three-state p-adic SOS model with $\theta \in \mathcal{E}_p$ on the Cayley tree of order four, the compatibility equation admits at least four non-periodic boundary laws of the types described above. Consequently, the model possesses at least four non-periodic p-adic quasi Gibbs measures.*

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Tensor product of idempotent measures

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Let us consider compact Hausdorff spaces X_1 and X_2, and projections $\pi_i\colon X_1 \times X_2 \to X_i$, $i = 1, 2$. For every $\varphi \in C(X_i)$ one has $\varphi \circ \pi_i \in C(X_1 \times X_2)$, $i = 1, 2$. For functions $\varphi_i \in C(X_i)$ under $\varphi_1 + \varphi_2$, $\varphi_1 \cdot \varphi_2$, $\max\{\varphi_1, \varphi_2\}$, and so on, we mean $\varphi_1 \circ \pi_1 + \varphi_2 \circ \pi_2$, $(\varphi_1 \circ \pi_1) \cdot (\varphi_2 \circ \pi_2)$, $\max\{\varphi_1 \circ \pi_1, \varphi_2 \circ \pi_2\}$, i. e.

$$\varphi_1 + \varphi_2 = \varphi_1 \circ \pi_1 + \varphi_2 \circ \pi_2,$$

$$\varphi_1 \cdot \varphi_2 = (\varphi_1 \circ \pi_1) \cdot (\varphi_2 \circ \pi_2),$$

$$\max\{\varphi_1, \varphi_2\} = \max\{\varphi_1 \circ \pi_1, \varphi_2 \circ \pi_2\}.$$

Hence, we can write $\varphi_1 + \varphi_2 \in C(X_1 \times X_2)$, $\max\{\varphi_1, \varphi_2\} \in C(X_1 \times X_2)$, and so on, for every pair $(\varphi_1, \varphi_2) \in C(X_1) \times C(X_2)$ of continuous functions.

For $\psi \in C(X_1 \times X_2)$ putting

$$\psi_{x_1}(x_2) = \psi(x_1, x_2), \qquad x_1 \text{ fixed},$$

$$\psi_{x_2}(x_1) = \psi(x_1, x_2), \qquad x_2 \text{ fixed},$$

we define functions $\psi_{x_i} \in C(X_j)$, $i, j \in \{1, 2\}$, $i \neq j$. It is clear that in the case when x_i varies over X_i then

$$\psi_{x_i} \circ \pi_j \in C(X_1 \times X_2).$$

Moreover,

$$\psi_{x_i} \circ \pi_j = \psi, \qquad i, j \in \{1, 2\}, \ i \neq j.$$

Let $I(X_i)$ be the space of idempotent probability measures on X_i, and $\mu_i \in I(X_i)$ an arbitrary idempotent probability measure, $i = 1, 2$. Take any continuous function $\psi \in C(X_1 \times X_2)$. Since $\psi_{x_i} \in C(X_j)$ for fixed $x_i \in X_i$, the value $\mu_j(\psi_{x_i})$ is well-defined. We put

$$\mu_j(x_i)(\psi) = \mu_j(\psi_{x_i}), \qquad i, j \in \{1, 2\}, \ i \neq j.$$

Proposition 1. For any $x_i \in X_i$ and $\mu_j \in I(X_j)$ the quantity $\mu_j(x_i)$ is an idempotent probability measure on $X_1 \times X_2$, $i, j \in \{1, 2\}$, $i \neq j$.

Theorem 1. For any $\mu_j \in I(X_j)$ and $\psi \in C(X_1 \times X_2)$ we have $(\mu_j(x_i)(\psi))_{x_i \in X_i} \in C(X_i)$, $i, j \in \{1, 2\}$, $i \neq j$.

Proof. The entry $(\mu_j(x_i)(\psi))_{x_i \in X_i}$ means that x_i varies over all X_i. It is enough to look at one case, for example, consider case $i = 1$, $j = 2$. Therefore, if x_1 varies across all X_1 we receive a function $\mu_2(\cdot)(\psi)\colon X_1 \to \mathbb{R}$.

Now fix any $x_1^0 \in X_1$. Since ψ is continuous on $X_1 \times X_2$, for every $\varepsilon > 0$ there exists an open neighbourhood $O_{x_1^0}$ in X_1 such that

$$|\psi(x_1, x_2) - \psi(x_1^0, x_2)| < \tfrac{\varepsilon}{2}$$

for each $(x_1, x_2) \in O_{x_1^0} \times X_2$. Then

$$|\psi_{x_1}(x_2) - \psi_{x_1^0}(x_2)| < \tfrac{\varepsilon}{2}, \qquad x_2 \in X_2.$$

Consequently,

$$\|\psi_{x_1} - \psi_{x_1^0}\| < \tfrac{\varepsilon}{2},$$
$$-\tfrac{\varepsilon}{2} < \psi_{x_1} - \psi_{x_1^0} < \tfrac{\varepsilon}{2},$$
$$\psi_{x_1^0} - \tfrac{\varepsilon}{2} < \psi_{x_1} < \psi_{x_1^0} + \tfrac{\varepsilon}{2}.$$

Since every idempotent probability measure is order-preserving and weakly additive,

$$\mu_2(\psi_{x_1^0}) - \varepsilon < \mu_2(\psi_{x_1^0}) - \tfrac{\varepsilon}{2} \le \mu_2(\psi_{x_1}) \le \mu_2(\psi_{x_1^0}) + \tfrac{\varepsilon}{2} < \mu_2(\psi_{x_1^0}) + \varepsilon.$$

That is why

$$|\mu_2(\psi_{x_1}) - \mu_2(\psi_{x_1^0})| < \varepsilon.$$

We have just shown that the inclusion $x_1 \in O_{x_1^0}$ yields

$$|\mu_2(x_1)(\psi) - \mu_2(x_1^0)(\psi)| < \varepsilon,$$

that means continuity of $\mu_2(\cdot)(\psi)\colon X_1 \to \mathbb{R}$.

Similarly, it turns out that $(\mu_1(x_2)(\psi))_{x_2 \in X_2} \in C(X_2)$ for each $\mu_1 \in I(X_1)$ and $\psi \in C(X_1 \times X_2)$.

Theorem 1 is proved.

So, using a given idempotent probability measure $\mu \in I(X)$ (or $\nu \in I(Y)$) and a function $\psi \in I(X \times Y)$ one can receive a function $\mu(\cdot)(\psi)\colon Y \to \mathbb{R}$ (or $\nu(\cdot)(\psi)\colon X \to \mathbb{R}$, respectively). This phenomenon gives opportunity to determine the value $\nu(\mu(\cdot)(\psi))$ (or $\mu(\nu(\cdot)(\psi))$, respectively). Future, we will apply these values to define a tensor product of idempotent probability measures μ and ν.

Putting

$$(\mu \circledast \nu)(\psi) = \mu\left(\left(\nu(y)(\psi)\right)_{y \in Y}\right) \tag{1}$$

we introduce a functional $\mu \circledast \nu\colon C(X \times Y) \to \mathbb{R}$.

The following statement is the main achievement of the present note.

Theorem 2. For idempotent probability measures $\mu \in I(X)$ and $\nu \in I(Y)$ the functional $\mu \circledast \nu\colon C(X \times Y) \to \mathbb{R}$ defined by (1) is an idempotent probability measure on $X \times Y$, i. e. $\mu \circledast \nu \in I(X \times Y)$.

Proposition 2. For idempotent probability measures $\mu \in I(X)$ and $\nu \in I(X)$ one has

$$\mu \circledast \nu = \nu \circledast \mu.$$

The functional $\mu \circledast \nu$ we will call as a tensor product of idempotent probability measures $\mu \in I(X)$ and $\nu \in I(Y)$.

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CRITICAL SETS AND DISCRETE SPECTRUM OF A TWO-BOSON LATTICE SCHRÖDINGER OPERATOR

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Discrete Schrödinger operators on lattices form an important class of models in the spectral theory of few-particle quantum systems. The location and number of bound states depend essentially on the interaction parameters and on the behavior of the Fredholm determinant near the edges of the essential spectrum [1–3]. We consider the zero-quasi-momentum fiber of a two-boson Schrödinger operator on the two-dimensional lattice with on-site, nearest-neighbor, and next nearest-neighbor interactions.

Let $\mathbb{T}^2 = (-\pi, \pi]^2$, $d\nu(p) = dp/(2\pi)^2$, and let $L^{2,\mathrm{e,s}}(\mathbb{T}^2)$ be the subspace of even functions which are symmetric with respect to the interchange of the coordinates. In this subspace,

$$H_\mu^{\mathrm{e,s}}(0) = H_0(0) + V_\mu^{\mathrm{e,s}}, \qquad \mu = (\mu_1, \mu_2, \mu_3) \in \mathbb{R}^3,$$

where

$$(H_0(0)f)(p) = \mathcal{E}_0(p)f(p), \qquad \mathcal{E}_0(p) = 4(2 - \cos p_1 - \cos p_2),$$

and

$$V_\mu^{\mathrm{e,s}} f = \sum_{j=1}^{3} \mu_j \langle \phi_j, f \rangle \phi_j,$$

with

$$\phi_1(p) = 1, \qquad \phi_2(p) = \cos p_1 + \cos p_2, \qquad \phi_3(p) = \cos 2p_1 + \cos 2p_2.$$

The form factors ϕ_1, ϕ_2, ϕ_3 are orthonormal and $\operatorname{rank} V_\mu^{\mathrm{e,s}} \le 3$. Moreover,

$$\sigma_{\mathrm{ess}}\big(H_\mu^{\mathrm{e,s}}(0)\big) = [0, 16].$$

Let $n_-(H_\mu^{\mathrm{e,s}}(0))$ and $n_+(H_\mu^{\mathrm{e,s}}(0))$ denote the numbers of eigenvalues in $(-\infty, 0)$ and $(16, +\infty)$, respectively. The min–max principle gives

$$n_-\big(H_\mu^{\mathrm{e,s}}(0)\big) + n_+\big(H_\mu^{\mathrm{e,s}}(0)\big) \le 3. \tag{1}$$

For $z \in \mathbb{R} \setminus [0, 16]$, set

$$a_{ij}(z) = \int_{\mathbb{T}^2} \frac{\phi_i(p)\phi_j(p)}{\mathcal{E}_0(p) - z}\, d\nu(p), \qquad A(z) = \big(a_{ij}(z)\big)_{i,j=1}^{3},$$

and

$$D_\mu = \operatorname{diag}(\mu_1, \mu_2, \mu_3).$$

The Lippmann–Schwinger equation reduces to

$$\big(I_3 + D_\mu A(z)\big)x = 0.$$

Hence $z \notin [0,16]$ is an eigenvalue of $H_\mu^{\mathrm{e,s}}(0)$ if and only if

$$\Delta_\mu^{\mathrm{e,s}}(z) := \det\big(I_3 + D_\mu A(z)\big) = 0. \tag{2}$$

At the lower and upper thresholds, respectively,

$$\Delta_\mu^{\mathrm{e,s}}(z) = \begin{cases} -\dfrac{P_0^-(\mu)}{4\pi}\ln\Big(-\dfrac{z}{32}\Big) + O(1), & z \nearrow 0, \\ \dfrac{P_0^+(\mu)}{4\pi}\ln\Big(\dfrac{z-16}{32}\Big) + O(1), & z \searrow 16. \end{cases} \tag{3}$$

The leading coefficients admit the factorization

$$P_0^\pm(\mu) = (\mu_1 \mp 8)\big[\mu_2 Q_0^\pm(\mu_3) \mp R_0^\pm(\mu_3)\big] - 16Q_0^\pm(\mu_3), \tag{4}$$

where

$$Q_0^\pm(\mu_3) = \frac{128-39\pi}{24\pi}\mu_3 \mp \frac{1}{4}, \qquad R_0^\pm(\mu_3) = \frac{32-9\pi}{3\pi}\mu_3 \mp \frac{1}{2}.$$

Thus the critical parameter sets corresponding to the two edges are

$$\Gamma^\pm = \{\mu \in \mathbb{R}^3 : P_0^\pm(\mu) = 0\}. \tag{5}$$

The function $Q_0^\pm$ has the unique zero

$$\mu_{3,0}^\pm = \pm\frac{6\pi}{128-39\pi}.$$

For $\mu_3 \neq \mu_{3,0}^\pm$, put

$$\mu_2^\pm(\mu_3) = \pm\frac{R_0^\pm(\mu_3)}{Q_0^\pm(\mu_3)}.$$

The graph of $\mu_2^\pm$ consists of two smooth unbounded branches with the vertical asymptote $\mu_3 = \mu_{3,0}^\pm$ and divides the (μ_2,μ_3)-plane into three open connected domains $D_0^\pm, D_1^\pm, D_2^\pm$. On their union, equation (5) is equivalent to

$$\mu_1 = \mu_1^\pm(\mu_2,\mu_3) := \pm 8 + \frac{16Q_0^\pm(\mu_3)}{\mu_2 Q_0^\pm(\mu_3) \mp R_0^\pm(\mu_3)}. \tag{6}$$

Hence

$$\Gamma^\pm = \Gamma_0^\pm \cup \Gamma_1^\pm \cup \Gamma_2^\pm,$$

where

$$\Gamma_\alpha^\pm = \big\{\big(\mu_1^\pm(\mu_2,\mu_3),\mu_2,\mu_3\big) : (\mu_2,\mu_3) \in D_\alpha^\pm\big\}, \qquad \alpha = 0,1,2.$$

These are pairwise disjoint smooth unbounded connected surfaces. Their complement has exactly four connected components, which we denote by

$$C_0^\pm,\ C_1^\pm,\ C_2^\pm,\ C_3^\pm,$$

numbered successively with respect to the oriented $\pm\mu_1$-direction.

The above geometric decomposition determines the discrete spectrum completely.

Theorem 1. For each choice of sign,

$$n_\pm\big(H_\mu^{\mathrm{e,s}}(0)\big) = \alpha, \qquad \mu \in C_\alpha^\pm, \qquad \alpha = 0,1,2,3.$$

Moreover,

$$n_{\pm}\big(H_{\mu}^{\mathrm{e,s}}(0)\big) = \alpha, \qquad \mu \in \Gamma_{\alpha}^{\pm}, \qquad \alpha = 0, 1, 2.$$

For a compact formulation of the result, set

$$\mathcal{R}_{\alpha}^{\pm} = C_{\alpha}^{\pm} \cup \Gamma_{\alpha}^{\pm}, \qquad \alpha = 0, 1, 2, 3,$$

where $\Gamma_{3}^{\pm} = \varnothing$.

Corollary 1. For each choice of sign and every $\alpha \in \{0, 1, 2, 3\}$,

$$n_{\pm}\big(H_{\mu}^{\mathrm{e,s}}(0)\big) = \alpha \quad \Longleftrightarrow \quad \mu \in \mathcal{R}_{\alpha}^{\pm}. \tag{7}$$

Thus the interaction parameter space is completely divided into spectral regions corresponding to $0, 1, 2$, and 3 discrete eigenvalues below and above the essential spectrum.

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Kasr tartibli hosila yordamida izotrop fazo differensial geometriyasini o'rganish

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Mazkur ishda uch o'lchamli izotrop fazoda sirtlarning differensial-geometrik xossalari kasr tartibli differensial hisoblash usuli yordamida etiladi. Klassik differensial geometriyada sirtlarning lokal xossalari butun tartibli hosilalar orqali ifodalanadi, biroq no-lokal xarakterga va xotira effektiga ega bo'lgan geometrik strukturalarni modellashtirishda bu yondashuv yetarli emas. Shu nuqtai nazardan, ishda kasr tartibli hosilalar (masalan, Kaputo ma'nosida) yordamida izotrop fazo uchun umumlashtirilgan geometrik usul shakllantirildi.

Tadqiqot doirasida sirtlarning kasr tartibli urunma vektorlari, birinchi va ikkinchi fundamental formalarining kasr analoglari aniqlandi hamda ular asosida izotrop fazodagi nisbiy (Gauss) va o'rtacha egriliklarning kasr tartibli invariantlari kiritildi.

Ishning asosiy natijasi sifatida izotrop fazoda "kasr tartibli Monj–Amper tenglamasining"analogi hosil qilindi.

Uch o'lchamli A_{n+1} affin fazoni qaraylik. A_{n+1} da Dekar koordinata sistemasi o'rnatilgan va $X\{x_1, x_2, ..., x_{n+1}\}$ va $Y\{y_1, y_2, ..., y_{n+1}\}$ vektorlar bo'lsin. X va Y vektorlarning skalyar ko'paytmasi quyidagicha:

$$\langle X, Y\rangle = \begin{cases} (X,Y)_1 = \sum\limits_{i=1}^{n} x_i y_i & (X,Y)_1 \neq 0 \\ (X,Y)_2 = x_{n+1}y_{n+1} & (X,Y)_1 = 0 \end{cases} \tag{1}$$

Ta'rif 1. A_{n+1} affin fazosida ikki vektorning skalyar ko'paytmasi (1) tenglik bilan aniqlansa, bu fazo $n+1$ o'lchovli izotrop fazo deyiladi va R^n_{n+1} bilan belgilanadi [1]

Kaputo ma'nosidagi kasr tartibli hosila quyidagicha aniqlanadi:

$${}^{C}D^{\alpha}f(x) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_a^x \frac{f(t)}{(x-t)^{\alpha}}\,dt \qquad \text{bu yerda} \quad 0<\alpha<1$$

Bizda 2 o'lchovli tekislikda $\mathbf{r}(s) = (x(s), y(s))$ silliq egri chiziq bo'lsin. Chiziqning egriligi ushbu $\kappa(s) = \langle \tau'(s), \mathbf{n}(s)\rangle$ tenglik bilan aniqlanadi.

Hosil o'rniga α tartibli Kaputo ${}^{C}D^{\alpha}\mathbf{r}(s)$ kasr tartibli hosilani qaraymiz. Bu vektor $\{\tau(s), \mathbf{n}(s)\}-$ urunma va normal vektorlar orqali chiziqli ifodalanadi:

$${}^{C}D^{\alpha}\mathbf{r}(s) = a_{\alpha}(s)\tau(s) + b_{\alpha}(s)\mathbf{n}(s)$$

Kasr tartibli egrilik quyidagicha aniqlaymiz:

$$\kappa_{\alpha}(s) = \left\langle \frac{d}{ds}\big({}^{C}D^{\alpha}\mathbf{r}(s)\big), \mathbf{n}(s)\right\rangle$$

yoki ekvivalent ravishda:$\kappa_{\alpha}(s) = a_{\alpha}(s)\kappa(s) + \frac{d}{ds}\big(b_{\alpha}(s)\big)$.

Agar $\alpha \to 1$ bo'lsa: $\kappa_{\alpha}(s) \to \kappa(s)$.

Bundan, kasr tartibli egrilik klassik egrilikning umumlashmasi bo'lib, u no-lokal (xotirali) geometrik xususiyatlarni ifodalaydi.

Ta'rif 2. Berilgan r gipersirt $u \in D$ nuqtada α-regulyar urunma fazosi deb[2]

$$T^{\alpha}_u r = D^{\alpha} r(u)(\mathbb{R}^n_{n+1}). \tag{2}$$

ga aytiladi.

Qaraliyotgan r giper sirtning izotrop fazoda α-tipdagi birinchi fundamental forma u nuqtada quydagicha aniqlanadi

$$\langle\cdot,\cdot\rangle : \mathbb{R}^{n-1}_n \times \mathbb{R}^{n-1}_n \to \mathbb{R}.$$

U quyidagicha belgilanadi:

$$I^{\alpha}(a,b) := \langle a, b\rangle, \tag{3}$$

bu yerda $a, b \in T^{\alpha}_u r$.

Agar $a = \frac{\partial^{\alpha} r(u)}{\partial u_i}, \quad b = \frac{\partial^{\alpha} r(u)}{\partial u_j}, \quad i,j \in \{1,\dots,n\}$, deb olsak, u holda α-birinchi fundamental forma quyidagi simmetrik va musbat aniqlangan matritsa orqali ifodalanadi:

$$\big(g^{\alpha}_{ij}(u)\big)^n_{i,j=1} = \left(\left\langle \frac{\partial^{\alpha} r(u)}{\partial u_i}, \frac{\partial^{\alpha} r(u)}{\partial r_j}\right\rangle\right)^n_{i,j=1}. \tag{4}$$

α-shakl operatorlari (4)-tenglamaga ko'ra har doim simmetrik bo'ladi. Har bir $u \in V$ uchun W^{α}_u operatorning haqiqiy xos qiymatlari $\kappa^{\alpha}_1(u), \dots, \kappa^{\alpha}_n(u)$ mavjud bo'ladi va agar $\kappa^{\alpha}_i(u) \neq \kappa^{\alpha}_j(u)$ bo'lsa, mos xos vektorlar ortogonal bo'ladi.

Ta'rif 3. Agar r α,-regulyar gipersirt bo'lsa, α-tipdagi ikkinchi fundamental forma

$$\mathrm{II}^{\alpha} : T_u^{\alpha} r \times T_u^{\alpha} r \to \mathbb{R}$$

quyidagicha aniqlanadi:

$$\mathrm{II}^{\alpha}(a,b) := I^{\alpha}\big(W_u^{\alpha}(a), b\big). \tag{5}$$

Ta'rif 4. Agar r α-regulyar gipersirt bo'lsa, $u \in V$ nuqtadagi W_u^{α} operatorning i-chi xos qiymati $\kappa_i^{\alpha}(u)$ $(i = 1, \ldots, n)$ ga α-tipdagi i-chi bosh egrilik deb aytiladi. Mos ravishda xossonga mos kelgan $k_i^{\alpha}(u)$ vektor esa i-chi bosh yo'nalish deyiladi.

Bosh egriliklardan foydalanib,

$$H^{(\alpha}(u) := \frac{1}{n} \sum_{i=1}^{n} \kappa_i^{\alpha}(u), \tag{6}$$

va

$$K_i^{\alpha}(u) := \frac{1}{\binom{n}{i}} \sum_{j_1 < \cdots < j_i} \kappa_{j_1}^{\alpha}(u) \cdots \kappa_{j_i}^{\alpha}(u), \tag{7}$$

mos ravishda α o'rtacha egrilik va i-chi α Gauss–Kroneker egriligi aniqlanadi.

Bundan $i = n$ bo'lganda α tartibli Gauss egriligi

$$K_n^{\alpha}(u) = \det\big(W_u^{\alpha}\big) = \frac{\det\big(\mathrm{II}^{\alpha}(u)\big)}{\det\big(I^{\alpha}(u)\big)}. \tag{8}$$

ga tengligi kelib chiqadi.

Agar ikki o'zgaruvchili $r(u,v) = \{u, v, f(u,v)\}$ sirt uchun isotrop fazoda $\det\big(I^{\alpha}(u,v)\big) = \big(\frac{d^{\alpha}u}{du}\frac{d^{\alpha}v}{dv}\big)^2$ bo'lishini hisobga olinsa, $\det\big(\mathrm{II}^{\alpha}(u,v)\big) = \frac{K_n^{\alpha}(u,v)}{\big(\Gamma(2-\alpha)\big)^4}(uv)^{2-2\alpha}$ ko'rinishga keladi.

Izotrop fazoda $K_n^{\alpha}(u,v)$ berilganda sirtni tiklash masalasi kasr tartibli Monje-Amper tenglamasiga keladi ya'ni:

$$\frac{\partial}{\partial u}\left(\frac{\partial^{\alpha} f(u,v)}{\partial u}\right)\frac{\partial}{\partial v}\left(\frac{\partial^{\alpha} f(u,v)}{\partial v}\right) - \frac{\partial}{\partial u}\left(\frac{\partial^{\alpha} f(u,v)}{\partial v}\right)\frac{\partial}{\partial v}\left(\frac{\partial^{\alpha} f(u,v)}{\partial u}\right) = \frac{K_n^{\alpha}(u,v)}{\big(\Gamma(2-\alpha)\big)^4}(uv)^{2-2\alpha}$$

Mazkur nochiziqli tenglama o'rganilayotgan sirt ko'chirish sirt (translation surface), ya'ni $z(x,y) = f(x) + g(y)$ ko'rinishida bo'lgan hol uchun quyidagi shartlarda to'liq integrallanib, aniq yechimlari topilgan:

Tenglamaning o'ng tomoni nolga teng bo'lganda ($K_{\alpha} = 0$, nol egrilikli sirtlar);

O'ng tomoni o'zgarmas songa teng bo'lganda ($K_{\alpha} = \mathrm{const}$, doimiy egrilikli sirtlar);

O'ng tomoni maxsus (o'zgaruvchilarga bog'liq xususiy) ko'rinishga ega bo'lgan holatda.

Olingan natijalar izotrop geometriyada kasr tartibli egrilik oqimlari va nochiziqli xususiy hosilali differensial tenglamalar nazariyasini rivojlantirishga xizmat qiladi.

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Local automorphisms of finite-dimensional totally graded filiform Lie algebras.

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The concept of local automorphisms was introduced by D. Larson and A. Sourour, who initiated the systematic study of local maps alongside R. Kadison's foundational work on local derivations. Based on these breakthroughs, local operators have been extensively studied across various operator and algebraic structures.

Following the systematic methodology developed by F. Arzikulov, I. Karimjanov, and S. Umrzaqov [2] for describing the local structures of algebraic systems, the present paper is mainly devoted to the study of local automorphisms on five exceptional one-parameter families of totally graded filiform Lie algebras, namely $\mathfrak{g}_{7,\alpha}$, $\mathfrak{g}_{8,\alpha}$, $\mathfrak{g}_{9,\alpha}$, $\mathfrak{g}_{10,\alpha}$, and $\mathfrak{g}_{11,\alpha}$, defined by the specific commutation relations presented in Table 1.

In this paper, automorphisms and local automorphisms of these five dimensional-restricted families are systematically investigated. Namely, general forms of the matrices of local automorphisms of these algebras are clarified. It turns out that due to rigid power constraints locked by internal commutation relations, the general form of the matrix of a local automorphism on these specific algebras strictly coincides with the global automorphism's matrix general form. Therefore, these exceptional filiform Lie algebras do not possess any pure local automorphisms that are not global automorphisms.

Definition 1. A vector space L over a field $\mathbb{F}$ equipped with a bilinear bracket $[\cdot,\cdot] : L\times L \to L$ is called a **Lie algebra** if for all $x, y, z \in L$ the following identities hold:

- Skew-symmetry: $[x, x] = 0$ (equivalently $[x, y] = -[y, x]$),
- Jacobi identity: $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$.

Definition 2. Let $\mathfrak{g}$ be an n-dimensional nilpotent Lie algebra over a field $\mathbb{F}$. The algebra $\mathfrak{g}$ is called a **totally graded filiform Lie algebra** if it satisfies:

1. Filiform Property: $\dim C^i\mathfrak{g} = n - i$ for $2 \le i \le n$.
2. Total Grading Property: $\mathfrak{g} = \mathfrak{g}_1 \oplus \cdots \oplus \mathfrak{g}_n$ with $[\mathfrak{g}_i, \mathfrak{g}_j] \subseteq \mathfrak{g}_{i+j}$ where $\dim \mathfrak{g}_i = 1$.

Definition 3. Let A be an associative algebra. Then a linear bijective map $\phi : A \to A$ is called an **automorphism**, if it satisfies

$$\phi(xy) = \phi(x)\phi(y) \tag{1}$$

for all $x, y \in A$.

Definition 4. A linear map $\Phi : A \to A$ is called a **local automorphism**, if for any element $x \in A$ there exists an automorphism $\phi_x : A \to A$ such that

$$\Phi(x) = \phi_x(x). \tag{2}$$

Theorem 1.[1] Let $\mathfrak{g}$ be a one parameter totally graded filiform Lie algebra. Then $\mathfrak{g}$ is isomorphic to a Lie algebra from thefollowing list:

Таблица 1: Five one-parameter families of totally graded filiform Lie algebras.

Algebra	**Commutation relations**
$\mathfrak{g}_{7,\alpha}$	$[e_1,e_j]=e_{j+1}$ $(2\le j\le 6)$, $[e_2,e_3]=(2+\alpha)e_5$, $[e_2,e_4]=(2+\alpha)e_6$, $[e_2,e_5]=(1+\alpha)e_7$, $[e_3,e_4]=e_7$
$\mathfrak{g}_{8,\alpha}$	relations of $\mathfrak{g}_{7,\alpha}$ and $[e_1,e_7]=e_8$, $[e_2,e_6]=\alpha e_8$, $[e_3,e_5]=e_8$
$\mathfrak{g}_{9,\alpha}$	relations of $\mathfrak{g}_{8,\alpha}$ and $(\alpha\neq-\frac{5}{2})$ $[e_1,e_8]=e_9$, $[e_2,e_7]=\frac{2\alpha^2+3\alpha-2}{2\alpha+5}e_9$, $[e_3,e_6]=\frac{2\alpha+2}{2\alpha+5}e_9$, $[e_4,e_5]=\frac{3}{2\alpha+5}e_9$
$\mathfrak{g}_{10,\alpha}$	relations of $\mathfrak{g}_{9,\alpha}$ and $(\alpha\neq-\frac{5}{2})$ $[e_1,e_9]=e_{10}$, $[e_2,e_8]=\frac{2\alpha^2+\alpha-1}{2\alpha+5}e_{10}$, $[e_3,e_7]=\frac{2\alpha-1}{2\alpha+5}e_{10}$, $[e_4,e_6]=\frac{3}{2\alpha+5}e_{10}$
$\mathfrak{g}_{11,\alpha}$	relations of $\mathfrak{g}_{10,\alpha}$ and $(\alpha\neq-\frac{5}{2},-1,-3)$ $[e_1,e_{10}]=e_{11}$, $[e_2,e_9]=\frac{2\alpha^3+2\alpha^2+3}{2(\alpha^2+4\alpha+3)}e_{11}$, $[e_3,e_8]=\frac{4\alpha^3+8\alpha^2-8\alpha-21}{2(\alpha^2+4\alpha+3)(2\alpha+5)}e_{11}$, $[e_4,e_7]=\frac{3(2\alpha^2+4\alpha+5)}{2(\alpha^2+4\alpha+3)(2\alpha+5)}e_{11}$, $[e_5,e_6]=\frac{3(4\alpha+1)}{2(\alpha^2+4\alpha+3)(2\alpha+5)}e_{11}$

Theorem 2. A linear map $\nabla: g_{k,\alpha}\to g_{k,\alpha}$, $k=7,8,9,10,11$, is an automorphism if and only if there exist complex numbers

$$a_i\ , b_i\in C \qquad such\quad that$$

matrix form

$$A_{g_{k,\alpha}}=\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0\\ 0 & a_1^2 & 0 & 0 & 0 & 0 & \ddots & 0 & 0\\ a_3 & b_3 & a_1^3 & 0 & 0 & 0 & \ddots & 0 & 0\\ a_4 & b_4 & a_1b_3 & a_1^4 & 0 & 0 & \ddots & 0 & 0\\ a_5 & b_5 & c_{5,3} & a_1^2b_3 & a_1^5 & 0 & \ddots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ a_{k-1} & b_{k-1} & c_{k-1,3} & c_{k-1,4} & c_{k-1,5} & c_{k-1,6} & \ddots & a_1^{k-1} & 0\\ a_k & b_k & c_{k,3} & c_{k,4} & c_{k,5} & c_{k,6} & \ddots & a_1^{k-3}b_3 & a_1^k \end{pmatrix}, \tag{3}$$

Theorem 3. A linear map ∇ on the one-parameter family of Lie algebras $\mathfrak{g}_{k,\alpha}$ ($k\in\{7,8,9,10,11\}$) is a local automorphism if and only if its matrix representation $M_\nabla=(b_{m,j})$ ($b_{m,j}\in\mathbb{C}$.) with respect to the homogeneous basis $\{e_1,e_2,\dots,e_k\}$ is lower triangular.

$$M_\nabla=\begin{pmatrix} b_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0\\ 0 & b_{1,1}^2 & 0 & 0 & \cdots & 0 & 0\\ b_{3,1} & b_{3,2} & b_{3,3} & 0 & \cdots & 0 & 0\\ b_{4,1} & b_{4,2} & b_{4,3} & b_{4,4} & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0\\ b_{n-1,1} & b_{n-1,2} & b_{n-1,3} & b_{n-1,4} & \cdots & b_{n-1,n-1} & 0\\ b_{n,1} & b_{n,2} & b_{n,3} & b_{n,4} & \cdots & b_{n,n-1} & b_{n,n} \end{pmatrix}, \tag{4}$$

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TRANSFER OF SECTION-PRESERVING KILLING FIELDS BETWEEN SURFACES ISOMETRIC BY SECTIONS

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Killing vector fields describe infinitesimal Riemannian symmetries. If X is a Killing vector field on a Riemannian manifold (M, g), then $\mathcal{L}_X g = 0$, and its local flow consists of local isometries. The geometry of Killing-field orbits and related invariant distributions has been studied in several settings; see, for example, [1,2].

The notion of surfaces *isometric by sections* compares surfaces through distinguished families of planar sections rather than through equality of their full first fundamental forms [3]. Let $F \subset \mathbb{R}^3$ be a regular surface, $e \in \mathbb{R}^3$ a unit vector, and

$$h_e(p) = \langle p, e\rangle, \qquad L_u = F \cap \{x \in \mathbb{R}^3 : \langle x, e\rangle = u\}.$$

We work on connected regular section domains where h_e is a submersion and the sets L_u are connected embedded curves. A smooth diffeomorphism $f : U_1 \to U_2$ is called a smooth isometry by sections if it maps sections onto sections, preserves the distances between the corresponding cutting planes, and preserves arc lengths on each section. Such a map need not be a Riemannian isometry of the surfaces.

Let $h_i(p) = \langle p, e_i\rangle$ be the corresponding height functions on $U_i \subset F_i$, $i = 1, 2$.

Proposition 1. If $f : U_1 \to U_2$ is a smooth isometry by sections, then there exist $\varepsilon \in \{1, -1\}$ and $c \in \mathbb{R}$ such that

$$h_2 \circ f = \varepsilon h_1 + c.$$

Consequently,

$$f(L^1_u) = L^2_{\varepsilon u + c},$$

and the restriction $f|_{L^1_u}$ is a one-dimensional Riemannian isometry between the corresponding sections.

Now let X be a Killing vector field on (U_1, g_1) tangent to the sections, that is, $X(h_1) = 0$, and put $\widetilde{X} = f_* X$. Since the restriction of f to each section is an isometry, $\widetilde{X}$ is automatically Killing along every corresponding target section. The question is whether it is Killing for the full target metric g_2.

Theorem 1. Let $f : U_1 \to U_2$ be a smooth isometry by sections and let X be a Killing vector field on (U_1, g_1) satisfying $X(h_1) = 0$. Then $\widetilde{X} = f_* X$ is Killing on (U_2, g_2) if and only if

$$\mathcal{L}_X\big(f^* g_2 - g_1\big) = 0. \tag{1}$$

Thus the tensor

$$D = f^* g_2 - g_1$$

measures exactly the metric discrepancy not detected by the sectionwise isometry. On vectors tangent to a section one has $D = 0$, whereas the obstruction to transfer is contained in the transverse components and their variation along X.

In section-adapted coordinates (u, v), where $u = h_1$ and the sections are $u = \text{const}$, write

$$g_1 = E\,du^2 + 2F\,du\,dv + G\,dv^2, \qquad f^*g_2 = \widetilde{E}\,du^2 + 2\widetilde{F}\,du\,dv + G\,dv^2.$$

If $X = \psi(u, v)\partial_v$, condition (1) is equivalent to

$$\psi(\widetilde{E} - E)_v + 2(\widetilde{F} - F)\psi_u = 0, \qquad \partial_v\Big[\psi(\widetilde{F} - F)\Big] = 0. \tag{2}$$

Hence only the transverse coefficients of the metric can obstruct the transferred Killing symmetry.

At a point where $X \neq 0$, one may choose local flow coordinates with $X = \partial_v$. Then

$$g_1 = E(u)\,du^2 + 2F(u)\,du\,dv + G(u)\,dv^2.$$

After a section-preserving change of the coordinate v, this becomes

$$g_1 = A(u)^2\,du^2 + B(u)^2\,dv^2. \tag{3}$$

For the transferred field to remain Killing, the coefficients of f^*g_2 must likewise be independent of the orbit coordinate v, while the section coefficient $G(u) = B(u)^2$ is fixed by sectionwise arc-length preservation.

Theorem 2. Assume that both X and $\widetilde{X} = f_*X$ are Killing and that $X \neq 0$ locally. In common section-adapted coordinates one can write

$$g_i = E_i(u)\,du^2 + 2F_i(u)\,du\,dv + G(u)\,dv^2, \qquad i = 1, 2,$$

where g_2 denotes f^*g_2. Put

$$B(u) = \sqrt{G(u)}, \qquad A_i(u) = \sqrt{E_i(u) - \frac{F_i(u)^2}{G(u)}}.$$

Then the signed geodesic curvatures of the corresponding sections satisfy

$$k_g^{(i)}(u) = -\frac{B'(u)}{A_i(u)B(u)}, \qquad i = 1, 2. \tag{4}$$

In particular,

$$L_u^1 \text{ is geodesic} \iff B'(u) = 0 \iff L_{\varepsilon u+c}^2 \text{ is geodesic.}$$

Therefore the numerical geodesic curvatures of corresponding sections need not coincide, but the property of being a geodesic section is preserved whenever the section-preserving Killing field transfers to a Killing field on the second surface. The criterion (1) separates the tangential information fixed by the section isometry from the transverse metric data and provides a natural basis for analogous transfer problems for higher-order Killing fields.

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Positivity Aspects of the Three-Particle Discrete Schrödinger Operator on a Lattice

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Let $\mathbb{T} = (-\pi; \pi]$ and $L^2(\mathbb{T}^3)$ be a Hilbert space of square-integrable functions defined on $\mathbb{T}^3$. Two particle discrete Schrödinger operator acts in $L^2(\mathbb{T}^3)$ by (see [1])

$$h_\mu(k) = h_0(k) - \mu\mathbf{v},$$

where $h_0(k)$ is operator multiplication by the function

$$\varepsilon(p) + \varepsilon(p+k)$$

and we assume that $\varepsilon(p)$ is continuous (periodic) real-valued functions on $\mathbb{T}^3$, with the single non-degenerate minimum point at the origin and

$$\lim_{|p|\to 0} \inf \frac{\varepsilon(p) - \varepsilon(0)}{|p|^2} > 0.$$

Here $\mathbf{v}$ is integral operator with the karnel $v(p-s)$, where $v(p) = \sum_{i=1}^{3} \cos p_i$.

Further, for simplicity, we assume $\varepsilon(\mathbf{0}) = 0$.

Since perturbation v is an integral operator, by the Weyl theorem the essential spectrum $\sigma_{ess}(h(k))$ of the operator $h(k), k \in \mathbb{T}^3$ coincides with the spectrum $\sigma(h_0(k))$ of $h_0(k)$. More specifically, $\sigma_{ess}(h(k))$ coincides with the range of the function $\varepsilon(\cdot) + \varepsilon(\cdot + k)$, i.e,

$$\sigma_{ess}(h(k)) = \sigma(h_0(k)) = [\varepsilon_{\min}(k); \varepsilon_{\max}(k)],$$

where

$$\varepsilon_{\min}(k) = \min_p \Big(\varepsilon(p) + \varepsilon(p+k)\Big), \quad \varepsilon_{\max}(k) = \max_p \Big(\varepsilon(p) + \varepsilon(p+k)\Big).$$

Let $K \in \mathbb{T}^3$ be a fixing and $L^2_K((\mathbb{T}^3)^2)$ be functions $f \in L^2((\mathbb{T}^3)^2)$ satisfying the equalities $f(p,q) = f(q,p) = f(K-p-q,q)$. A three identical particle Schrödinger operator $H_\mu(K)$ acts in the Hilbert space $L^2_K((\mathbb{T}^3)^2)$ (see [3]) as

$$H_\mu(K) = H_0(K) - \mu V_1 - \mu V_2 - \mu V_3,$$

where $H_0(K)$ is operator multiplication by the function

$$\mathcal{E}_K(p,q) = \varepsilon(p) + \varepsilon(q) + \varepsilon(K-p-q)$$

and

$$(V_1 f)(p,q) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{T}^3} v(p-s) f(s,q) ds,$$

$$(V_2 f)(p,q) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{T}^3} v(s-q) f(s,p) ds,$$

$$(V_3 f)(p,q) = \frac{1}{(2\pi)^{3/2}} \int_{\mathbb{T}^3} v(p-s) f(s,K-p-q) ds.$$

For the essintial spectrum $\sigma_{ess}(H_\mu(K))$ of the operator $H_\mu(K)$ the equality (see [2])

$$\sigma_{ess}(H_\mu(K)) = [m_K, M_K] \cup \bigcup_{p\in\mathbb{T}^3} \sigma_{disc}[(h_\mu(K-p)) + \varepsilon(p)]$$

holds, where

$$m_K = \min_{p,q} \mathcal{E}_K(p,q), \quad M_K = \max_{p,q} \mathcal{E}_K(p,q)$$

The main result of the paper is the following theorem

Theorem 1. If $h_{3\mu}(\mathbf{0}) \geq 0$, then $H_\mu(K) \geq 0$ for all $K \in \mathbb{T}^3$.

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Local and 2-Local δ-Derivations on Euclidean Lie Algebras

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The concept of a δ-derivation of a Lie algebra was introduced by Filippov in [1,2]. This notion includes several important classes of linear operators. Ordinary derivations correspond to $\delta = 1$, anti-derivations correspond to $\delta = -1$, while the case $\delta = \frac{1}{2}$ is closely related to the centroid. In particular, every operator belonging to the centroid is a $\frac{1}{2}$-derivation.

Filippov obtained fundamental restrictions on δ-derivations of prime Lie algebras. In particular, it was shown in [2] that, under the corresponding primeness assumptions, nonzero δ-derivations may occur only for the exceptional values $\delta \in \left\{0, 1, -1, \frac{1}{2}\right\}$.

Moreover, $\frac{1}{2}$-derivations of prime Lie algebras possessing a non-degenerate symmetric invariant bilinear form were investigated. In particular, for central simple Lie algebras satisfying the assumptions of [2], every $\frac{1}{2}$-derivation is a scalar multiple of the identity operator.

The study of δ-derivations was subsequently extended to other classes of non-associative algebras. Filippov investigated δ-derivations of prime alternative and non-Lie Malcev algebras in [3].

The notions of local derivations and 2-local derivations arose from the study of local operator behavior on associative and operator algebras. Local derivations were introduced independently by Kadison [4] and Larson and Sourour [5], while 2-local derivations were introduced by Šemrl [6]. These notions were later studied for many classes of Lie and non-associative algebras. In particular, Ayupov and Kudaybergenov proved that every local derivation on a finite-dimensional semisimple Lie algebra is a derivation [7], and Ayupov, Kudaybergenov, and Rakhimov obtained the corresponding result for 2-local derivations [8].

Definition 1. Let $\mathfrak{A}$ be an algebra and let $\delta \in \mathbb{C}$. A linear operator $D : \mathfrak{A} \to \mathfrak{A}$ is called a δ-derivation if

$$D([x, y]) = \delta\big([D(x), y] + [x, D(y)]\big)$$

for all $x, y \in \mathfrak{A}$. The vector space of all δ-derivations of $\mathfrak{A}$ is denoted by $\mathrm{Der}_\delta(\mathfrak{A})$.

Definition 2. A linear operator $\Delta : \mathfrak{A} \to \mathfrak{A}$ is called a local δ-derivation if, for every $x \in \mathfrak{A}$, there exists $D_x \in \mathrm{Der}_\delta(\mathfrak{A})$ such that

$$\Delta(x) = D_x(x).$$

The set of all local δ-derivations is denoted by $\mathrm{LocDer}_\delta(\mathfrak{A})$.

Definition 3. A mapping $\nabla : \mathfrak{A} \to \mathfrak{A}$, not necessarily linear, is called a 2-local δ-derivation if, for every $x, y \in \mathfrak{A}$, there exists $D_{x,y} \in \mathrm{Der}_\delta(\mathfrak{A})$ such that

$$\nabla(x) = D_{x,y}(x), \qquad \nabla(y) = D_{x,y}(y).$$

Let $n \geq 4$. The Euclidean Lie algebra $\mathfrak{e}(n)$ has a basis

$$\{E_{i,j}, H_k \mid 1 \leq i < j \leq n,\ 1 \leq k \leq n\},$$

where we use the convention $E_{j,i} = -E_{i,j}$. Its multiplication is

$$\begin{aligned}[E_{i,j}, E_{k,l}] &= \delta_{j,k}E_{i,l} - \delta_{i,k}E_{j,l} - \delta_{j,l}E_{i,k} + \delta_{i,l}E_{j,k},\\ [E_{i,j}, H_k] &= \delta_{j,k}H_i - \delta_{i,k}H_j, \quad [H_i, H_j] = 0,\end{aligned}$$

where $\delta_{p,q}$ is Kronecker delta. Thus $\mathfrak{e}(n) = \mathfrak{so}(n) \ltimes H(n)$, where

$$\mathfrak{so}(n) = \mathrm{span}\{E_{i,j} \mid 1 \leq i < j \leq n\} \quad \text{and} \quad H(n) = \mathrm{span}\{H_1, \ldots, H_n\}$$

is the natural $\mathfrak{so}(n)$-module.

Proposition 1. Let $n \geq 4$ and let $\delta \in \mathbb{C}$. Then

$$\mathrm{Der}_\delta(\mathfrak{e}(n)) = \begin{cases} \mathrm{ad}(\mathfrak{e}(n)) \oplus \mathbb{C}\tau, & \delta = 1,\\ \mathbb{C}\mathrm{Id}_{\mathfrak{e}(n)}, & \delta = \dfrac{1}{2},\\ 0, & \delta \notin \left\{1, \dfrac{1}{2}\right\}, \end{cases}$$

where $\tau : \mathfrak{e}(n) \to \mathfrak{e}(n)$ is the linear operator defined by

$$\tau\big|_{\mathfrak{so}(n)} = 0, \qquad \tau\big|_{H(n)} = \mathrm{Id}_{H(n)}\,.$$

Theorem 1. Every local δ-derivation and every 2-local δ-derivation on $\mathfrak{e}(n)$ is a δ-derivation.

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Parabolik Shteyn ko'pxilligining algebraik bo'lishi haqida

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Kompleks analiz, pluripotensial nazariya hamda algebraik geometriyaning o'zaro uyg'un rivojlanishi kompleks ko'pxilliklarda golomorf funksiyalarning o'sish xususiyatlarini rivojlantirdi. Mazkur yo'nalishning muhim masalalaridan biri kompleks ko'pxilliklarning algebraik xususiyatlarini ularda aniqlangan golomorf funksiyalarning ko'phadga xos o'sish sur'atlari orqali tavsiflashdan iborat. Xususan, umumiy Stein ko'pxilliklari sinfida affin algebraik ko'pxilliklarni ajratib ko'rsatishga imkon beruvchi ichki analitik hamda pluripotensial mezonlarni aniqlash zamonaviy kompleks analizning dolzarb muammolaridan biri hisoblanadi.

Algebraik struktura bilan pluripotensial nazariya o'rtasidagi uzviy bog'liqlik Sadullayev tomonidan olingan fundamental natijalarda o'zining chuqur ifodasini topgan. Xususan, u $\mathbb{C}^N$ fazodagi analitik to'plam uchun Siciak tipidagi ekstremal funksiyaning lokal chegaralangan bo'lishi mazkur to'plamning algebraik ekanligiga ekvivalentligini isbotlagan [1]. Mazkur kriteriy algebraiklik xususiyatini bevosita pluripotensial xossalar vositasida ifodalash imkonini beruvchi muhim natijalardan biridir. Bu yondashuv kompleks-analitik va algebraik geometriya o'rtasidagi ichki aloqalarni yanada ravshanlashtirib, keyingi ilmiy izlanishlar uchun nazariy asos yaratdi.

Paraboliklik konsepsiyasi yuqoridagi g‘oyalarni faqat affin algebraik ko‘pxilliklar doirasida emas, balki kompleks ko‘pxilliklarning kengroq sinflarida o‘rganish imkonini beradi. Riman sirtlari nazariyasida paraboliklik an’anaviy ravishda Liouville xossasi hamda Green funksiyasining mavjud emasligi bilan tavsiflanadi. Yuqori o‘lchamli kompleks ko‘pxilliklar uchun esa Stoll parabolik kompleks ko‘pxilliklarning nazariy asoslarini rivojlantirdi. Keyingi tadqiqotlarda Zeriahi parabolik Stein ko‘pxilliklarida cheksizlikda qutbga ega plurikompleks Green funksiyasi tushunchasini kiritib, unga mos Lelong sinflarining muhim xususiyatlarini o‘rgandi. Shuningdek, Bernstein–Walsh tipidagi teoremalar hamda ko‘phad tipidagi o‘sishga ega golomorf funksiyalar fazolari bo‘yicha olingan natijalar klassik pluripotensial nazariya usullarini parabolik ko‘pxilliklarga tatbiq etish imkoniyatlarini sezilarli darajada kengaytirdi.

1-Ta’rif. Stein ko‘pxilligi X da yuqoridan chegaralangan har qanday plurisubgarmonik funksiya aynan o‘zgarmas bo‘lsa, X *parabolik Stein ko‘pxilligi* deb ataladi. Boshqacha aytganda, X da yuqoridan chegaralangan o‘zgarmas bo‘lmagan plurisubgarmonik funksiya mavjud bo‘lmasa, ushbu ko‘pxillik parabolik deyiladi.

2-Ta’rif. $f \in \mathcal{O}(X)$ golomorf funksiya uchun shunday $c > 0$ va $d > 0$ o‘zgarmas sonlar mavjud bo‘lib, har bir $z \in X$ uchun $\ln|f(z)| \le d\,\rho^{+}(z) + c$, tengsizlik bajarilsa, f funksiya darajasi d dan oshmaydigan ρ-*ko‘phad* deb ataladi. Bu yerda $\rho^{+}(z) = \max\{0, \rho(z)\}$.

Yuqoridagi tengsizlikni qanoatlantiruvchi d sonlar orasidagi eng kichik mumkin bo‘lgan qiymat f funksiyaning *darajasi* deb yuritiladi. Umumiy holda ushbu minimal qiymat butun son bo‘lishi shart emas.

3-Ta’rif. Agar Stein ko‘pxilligi X biror $\mathbb{C}^N$ kompleks fazoda affin algebraik to‘plam ko‘rinishida realizatsiya qilinishi mumkin bo‘lsa, u holda X *algebraik Stein ko‘pxilligi* deb ataladi.

Ta’kidlash joizki, algebraiklikni tavsiflash uchun qo‘llaniladigan ayrim mezonlar butun funksiyalarning grafiklariga bevosita tatbiq etilmasligi mumkin. Shunga qaramay, bunday grafiklar algebraik ko‘rinishda realizatsiya qilinishi mumkin. Masalan,

$$X = \left\{ z \in \mathbb{C}^n : z_n = f(z') \right\},$$

bu yerda

$$z = (z', z_n) = (z_1, \ldots, z_{n-1}, z_n), \qquad f(z') \in \mathcal{O}\big(\mathbb{C}^{n-1}_{z'}\big),$$

ko‘rinishdagi istalgan butun funksiyaning grafigi mos almashtirish orqali

$$A = \{z_n = 0\}$$

algebraik to‘plam ko‘rinishida realizatsiya qilinadi. Bu holat kompleks-analitik realizatsiya bilan algebraik realizatsiya o‘rtasidagi munosabatning o‘ziga xos jihatini namoyon etadi.

1-Teorema. X S-parabolik Stein ko‘pxilligi bo‘lsin. U holda X ning algebraik bo‘lishi ρ-ko‘phadlar fazosi $\mathcal{P}_\rho(X)$ barcha golomorf funksiyalar fazosi $\mathcal{O}(X)$ ning chekli sondagi elementlar yordamida hosil qilinadigan va $\mathcal{O}(X)$ da hamma yerda zich bo‘lgan qism algebrasi bo‘lishi zarur va yetarlidir.

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METRIK FAZOLARDA QO‘ZG‘ALMAS NUQTA TEOREMALARI VA ULARNING TOPOLOGIK TALQINLARI

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Matematik analiz va funksional analizning zamonaviy taraqqiyoti metrik fazolar tushunchasi bilan uzviy bog‘liqdir. Qo‘zg‘almas nuqta nazariyasi esa chiziqli va chiziqli bo‘lmagan operatorli tenglamalarni yechishda asosiy metodologik baza hisoblanadi. $f(x) = x$ ko‘rinishidagi masalalar faqatgina sof matematik ob’ektlar bo‘lmay, balki iqtisodiy muvozanat modellarida, biologik populyatsiyalar dinamikasida va fizik jarayonlarning turg‘unlik holatlarini aniqlashda muhim o‘rin tutadi. Ushbu maqolaning maqsadi metrik fazolarda qo‘zg‘almas nuqta mavjudligini topologik nuqtai nazardan tahlil qilish va nazariyaning amaliy qo‘llanilish doirasini ko‘rsatishdan iborat.

Qo‘zg‘almas nuqta nazariyasining poydevori 1922-yilda Stefan Banax tomonidan qo‘yilgan. Uning "Siqib akslantirish prinsipi"(Banach Contraction Principle) metrik fazolarning to‘liqligi sharti ostida yagona yechimni kafolatlaydi. Keyinchalik, 1930-yillarda Yulius Shauder ushbu nazariyani cheksiz o‘lchamli Banax fazolari uchun umumlashtirdi. Topologik nuqtai nazardan, L. Brouwerning yopiq sharda uzluksiz akslantirish haqidagi teoremasi bu sohadagi eng muhim yutuqlardan biri bo‘ldi.

Zamonaviy tadqiqotchilardan R. Kannan (1968) va S. Reich (1971) kontraksiya shartini biroz yumshatgan holda yangi turdagi qo‘zg‘almas nuqta teoremalarini isbotladilar. Bugungi kunda bu nazariya "G-metrika"va "b-metrika"kabi umumlashtirilgan fazolarda faol o‘rganilmoqda.

Tadqiqotda metrik fazo (X, d) strukturasidan foydalaniladi.

Ta’rif. Agar $f : X \to X$ akslantirish uchun shunday $L \in [0, 1)$ mavjud bo‘lib, barcha $x, y \in X$ uchun

$$d(f(x), f(y)) \leq L \cdot d(x, y)$$

bo‘lsa, u **siqib akslantirish** deb ataladi.

Metrik fazoning topologiyasi undagi ochiq sharlar yordamida aniqlanadi. Qo‘zg‘almas nuqtaning mavjudligi ko‘pincha fazoning kompaktligi (har qanday ochiq yopqichdan chekli qism yopqich ajratish mumkinligi) bilan bog‘lanadi.

Ketma-ket yaqinlashish (Pikar iteratsiyasi) usuli qo‘llanildi.

(X, d) to‘la metrik fazo bo‘lsin. $f : X \to X$ siqib akslantirish bo‘lsa, uning yagona qo‘zg‘almas nuqtasi mavjudligini isbotlash uchun ixtiyoriy $x_0 \in X$ nuqtani olamiz va quyidagi ketma-ketlikni quramiz:

$$x_1 = f(x_0), \quad x_2 = f(x_1) = f^2(x_0), \quad \ldots, \quad x_n = f^n(x_0).$$

Ixtiyoriy n uchun qo‘shni hadlar orasidagi masofani baholaymiz:

$$d(x_{n+1}, x_n) = d(f(x_n), f(x_{n-1})) \leq L \cdot d(x_n, x_{n-1}) \leq \cdots \leq L^n \cdot d(x_1, x_0).$$

Koshi ketma-ketligi shartini tekshirish uchun $m > n$ bo‘lganda uchburchak tengsizligini qo‘llaymiz:

$$d(x_m, x_n) \le d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m)$$

$$d(x_m, x_n) \le (L^n + L^{n+1} + \cdots + L^{m-1}) \cdot d(x_1, x_0)$$

$$d(x_m, x_n) \le \frac{L^n}{1-L} \cdot d(x_1, x_0).$$

$L < 1$ bo‘lgani uchun, $n \to \infty$ da $L^n \to 0$, demak $\{x_n\}$ Koshi ketma-ketligidir. Fazoning to‘liqligidan $x_n \to x^*$ limit mavjud. f uzluksiz bo‘lgani uchun (kontraksiya shartidan kelib chiqadi), $f(x^*) = x^*$.

Agar fazo to‘la bo‘lsa-yu, lekin siqib akslantirish sharti bajarilmasa (masalan, $L = 1$), qo‘zg‘almas nuqta mavjud bo‘lmasligi mumkin. Biroq, agar fazo kompakt bo‘lsa, shartni

$d(f(x), f(y)) < d(x, y)$ ko‘rinishida yumshatish mumkin (Edelstein teoremasi).

Topologik nuqtai nazardan, bu holat fazoning "geometrik torayishi"sifatida ko‘riladi. Maqolada kompaktlikning roli quyidagi lemma orqali tushuntiriladi:

Kompakt metrik fazoda har qanday uzluksiz funksiya o‘zining quyi va yuqori chegaralariga erishadi.

Bu esa $g(x) = d(x, f(x))$ funksiyasining minimum nuqtasi qo‘zg‘almas nuqta ekanligini isbotlashga xizmat qiladi.

Metrik fazolardan Banax fazolariga o‘tishda Schauder teoremasi muhim o‘rin tutadi.

Teorema.[Schauder] Agar K to‘plam Banax fazosining qavariq, yopiq va cheklangan qism to‘plami bo‘lsa, u holda har qanday ixcham (kompakt) operator $A : K \to K$ kamida bitta qo‘zg‘almas nuqtaga ega.

Bu teorema differensial tenglamalar nazariyasida yechimning mavjudligini isbotlashda fundamental vositadir.Ushbu nazariya qanday qilib amaliyotga tatbiq etilishini ko‘rsatish uchun Koshi masalasini ko‘raylik:

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

Ushbu masala quyidagi integral tenglamaga ekvivalent:

$$y(x) = y_0 + \int_{x_0}^{x} f(t, y(t))\, dt.$$

Agar f funksiya Lipshits shartini qanoatlantirsa, u holda o‘ng tomondagi operator siqib akslantirish bo‘ladi. Banax teoremasiga ko‘ra, bu ketma-ketlik yagona yechimga yaqinlashadi. Bu usul Pikar iteratsiyasi deb ataladi va kompyuter modellashtirishi hamda hisoblash jarayonlarida keng qo‘llaniladi.

Tadqiqot natijalari shuni ko‘rsatadiki, metrik fazolarda qo‘zg‘almas nuqta nazariyasi faqatgina algebraik emas, balki chuqur topologik xarakterga ega.

- **To‘liqlik va Kompaktlik:** Bu ikki tushuncha qo‘zg‘almas nuqtaning mavjudligi uchun kafolat beruvchi asosiy ustunlardir.
- **Yagonalik:** Faqatgina qat’iy kontraksiya sharti ostidagina yagonalik kafolatlanadi, topologik teoremalarda (masalan, Brouwer teoremasida) yagonalik har doim ham mavjud bo‘lmasligi mumkin.
- **Algoritmik samaradorlik:** Banax prinsipi nafaqat yechim borligini aytadi, balki unga boradigan yo‘lni (iteratsiyani) ham aniq ko‘rsatib beradi.

Xulosa qilib aytganda, ushbu nazariya zamonaviy matematika, iqtisodiyot va muhandislikning kesishmasida turgan eng samarali vositalardan biri bo‘lib qolmoqda.

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Galiley fazosida sirtning bosh egriliklari bilan bog‘liq masalalar

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Galiley fazosida sirtlar nazariyasi A. Artikbaey, B.M. Sultanovlar tomonidan o‘rganilgan[1-3]. Yevklid fazosidagi sirtlar nazariyasi bilan galiley fazosidagi sirtlar nazariyasi orasidagi asosiy farq, sirtning koordinata chiziqlaridan birini maxsus yo‘nalishga mos keluvchi qilib tanlab olinishida. Bunday tanlangan sirtning $Oxyz$- koordinatalar sistemasidagi vektor tenglamasi

$$\vec{r}(u,v) = u\vec{i} + y(u,v)\,\vec{j} + z(u,v)\,\vec{k}$$

shaklida bo‘lib, $\left\{\vec{i},\vec{j},\vec{k}\right\}$ birlik bazis vektorlari, ulardan $\left\{\vec{j},\vec{k}\right\}$ fazo maxsus tekisligiga parallel, $u = const$ chiziqlar maxsus tekislikga tegishli.

Bunda maxsus yo‘nalishga mos chiziq egriligi bosh egriliklardan biri shaklida tanlab olinadi. Ikkinchi bosh egrilik esa, maxsus tekislikga parallel bo‘lmagan yo‘nalishlardagi normal egrilikning ekstrimal qiymati shaklida tanlanadi. Agar k_1, k_2- sirtning bosh egriliklari bo‘lsa, $K = k_1k_2$- sirtning Gauss egriligi, $H = \frac{k_1+k_2}{2}$- esa o‘rta egriligi sifatida aniqlanadi.

Bosh egriliklarga bog‘liq ushbu masalalar o‘rganildi

a) $K = k_1k_2 = const$. Bunday sirt $y = a\sin bx$- chiziqning bir arkasini Ox o‘qi atrofida aylantirishdan hosil bo‘lgan sirt bo‘lagi.

b) Agar $\frac{k_1}{k_2} = const$ bo‘lsa, $y = ax^2 + bx + c$-parabolani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan sirtga bir qiymatli proyeksiyalanuvchi sirt bo‘ladi.

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Construction of Leibniz dialgebras over 3-dimensional nilpotent Leibniz algebras

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Leibniz algebras represent a fundamental generalization of Lie algebras, obtained by relaxing the skew-symmetry condition while preserving the derivation property encoded in the Leibniz identity. The defining identity for a Leibniz algebra $(L, [\cdot,\cdot])$ is:

$$[[x,y],z] = [[x,z],y] + [x,[y,z]]$$

for all $x, y, z \in L$. When the bracket is skew-symmetric, this reduces to the Jacobi identity, recovering Lie algebras.

This class of algebras, first systematically studied by Loday in 1993, has emerged as a central object in non-associative algebra, mathematical physics, and operad theory.

Many classical theorems from Lie algebras theory have been extended to Leibniz algebras case and several classes of nilpotent, solvable, simple, semi-simple Leibniz algebras are classified, derivations, deformations and degenerations of these algebras are investigated.

In 2001 Loday introduced dialgebras to study natural connections between associative algebras and Leibniz algebras. It is well known that, on a given diassociative algebra, the bilinear operation defined by $[x,y] := x \vdash y - y \dashv x$ yields a Leibniz algebra structure [2]. Later, in 2008 Kolesnikov presented a general algorithm for obtaining the вЂЊVarвЂЌ dialgebra structures [3]. Accordingly, the definition of a Leibniz dialgebra is given as follows.

Definition 1.[2] A vector space A with a two multiplication operators $\dashv$ and $\vdash$ is called a *0-dialgebra* if

$$x \dashv (y \dashv z) = x \dashv (y \vdash z), \tag{1}$$

$$(x \vdash y) \vdash z = (x \dashv y) \vdash z, \tag{2}$$

for all $x, y, z \in A$.

Definition 2. A 0-dialgebra $(A, \vdash, \dashv)$ is called *(right) Leibniz dialgebra* if for any elements x, y, z of A

$$(x \vdash y) \dashv z - (x \dashv z) \vdash y - x \vdash (y \dashv z) = 0, \tag{3}$$

$$(x \dashv y) \vdash z - (x \vdash z) \dashv y - x \vdash (y \vdash z) = 0, \tag{4}$$

$$(x \dashv y) \dashv z - (x \dashv z) \dashv y - x \dashv (y \dashv z) = 0. \tag{5}$$

From Definition 2.1 we can easily derive that if $(A, \vdash, \dashv)$ is a (right) Leibniz dialgebra then $(A, \dashv)$ is a ordinary right Leibniz algebra.

In [1] provides a complete classification of 3-dimensional Leibniz algebras.

In the present work, we construct Leibniz dialgebra structures on the nilpotent Leibniz algebras appearing in that classification.Using this classification, we examine how the nilpotency property is related between the $\dashv$ and $\vdash$ products.

Theorem. Up to isomorphism, the Leibniz dialgebra structures over 3-dimensional nilpotent Leibniz algebras are precisely those listed below.

$L_1^1:$	$e_2 \dashv e_3 = e_1,$	$e_3 \dashv e_2 = -e_1$		$e_2 \vdash e_2 = e_1,$	$e_3 \vdash e_3 = e_1$
$L_1^2:$	$e_2 \dashv e_3 = e_1,$	$e_3 \dashv e_2 = -e_1$		$e_2 \vdash e_2 = e_1,$	$e_2 \vdash e_3 = \beta e_1$
$L_1^3:$	$e_2 \dashv e_3 = e_1,$	$e_3 \dashv e_2 = -e_1$		$e_2 \vdash e_3 = e_1,$	$e_3 \vdash e_2 = e_1$
$L_1^4:$	$e_2 \dashv e_3 = e_1,$	$e_3 \dashv e_2 = -e_1$		$e_2 \vdash e_3 = e_1$	
$L_1^5:$	$e_2 \dashv e_3 = e_1,$	$e_3 \dashv e_2 = -e_1$		$e_3 \vdash e_2 = e_1,$	$e_3 \vdash e_3 = \gamma e_1$
$L_1^6:$	$e_2 \dashv e_3 = e_1,$	$e_3 \dashv e_2 = -e_1$		$e_3 \vdash e_3 = e_1$	
$L_6^1:$	$e_2 \dashv e_2 = \gamma e_1,$	$e_3 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_2 = e_1,$	$e_3 \vdash e_2 = \beta e_1,$
				$e_3 \vdash e_3 = \gamma_1 e_1$	
$L_6^2:$	$e_2 \dashv e_2 = \gamma e_1,$	$e_3 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_2 = e_1,$	$e_2 \vdash e_3 = \beta e_1$
$L_6^3:$	$e_2 \dashv e_2 = \gamma e_1,$	$e_3 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_2 = e_1,$	$e_2 \vdash e_3 = \beta e_1,$
				$e_3 \vdash e_2 = \gamma e_1,$	$e_3 \vdash e_3 = \beta\gamma e_1$
$L_6^4:$	$e_2 \dashv e_2 = \gamma e_1,$	$e_3 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_3 = e_1,$	$e_3 \vdash e_2 = \beta e_1,$
				$e_3 \vdash e_3 = \gamma e_1$	
$L_6^5:$	$e_2 \dashv e_2 = \gamma e_1,$	$e_3 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_3 = e_1,$	$e_3 \vdash e_3 = \beta e_1$
$L_6^6:$	$e_2 \dashv e_2 = \gamma e_1,$	$e_3 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_3 = e_1$	
$L_7^1:$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_2 = e_1,$	$e_2 \vdash e_3 = \beta e_1,$	$e_3 \vdash e_3 = e_1$	
$L_7^2:$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_2 = e_1,$	$e_2 \vdash e_3 = \beta e_1$		
$L_7^3:$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_3 = e_1,$	$e_3 \vdash e_2 = e_1$		
$L_7^4:$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_3 = e_1$			
$L_7^5:$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_2 = e_1,$	$e_3 \vdash e_3 = \beta e_1$		
$L_7^6:$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_3 = e_1$			
$L_7^7:$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_3 = e_2$			
$L_8^1:$	$e_2 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_2 = e_1,$	$e_2 \vdash e_3 = \beta e_1,$	$e_3 \vdash e_3 = \gamma e_1$
$L_8^2:$	$e_2 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_2 \vdash e_3 = e_1,$	$e_3 \vdash e_2 = \beta e_1,$	$e_3 \vdash e_3 = \gamma e_1$
$L_8^3:$	$e_2 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_2 = e_1,$	$e_3 \vdash e_3 = \beta e_1$	
$L_8^4:$	$e_2 \dashv e_2 = e_1,$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_3 = e_1$		
$L_{15}^1:$	$e_1 \dashv e_3 = e_2,$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_1 = -e_2,$	$e_3 \vdash e_3 = -e_1$	
$L_{15}^2:$	$e_1 \dashv e_3 = e_2,$	$e_3 \dashv e_3 = e_1$	$e_1 \vdash e_3 = e_2,$	$e_3 \vdash e_3 = e_1 + \beta e_2$	
$L_{15}^3:$	$e_1 \dashv e_3 = e_2,$	$e_3 \dashv e_3 = e_1$	$e_3 \vdash e_3 = e_2$		

where γ, β are arbitrary complex numbers.

From the table above, we can directly see that the following result follows.

Remark. Let $(L, \dashv, \vdash)$be an arbitrary dialgebra constructed on a 3-dimensional nilpotent (right) Leibniz algebra $(L, \dashv)$. Then $(L, \vdash)$ is also nilpotent.

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YASASHGA DOIR BIR MASALANING ANALITIK YECHIMI

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Annotatsiya. Ushbu ishda klassik yasashga doir geometrik masala – berilgan AB kesmani o'zgarmas α burchak ostida ko'rish nuqtalarining geometrik o'rnini topish masalasi ko'rib chiqiladi. Masalaning sirkul va chizg'ich yordamida klassik yechimi keltiriladi, so'ngra bu yechim dekart koordinatalar sistemasida analitik usulda ifodalanadi. Natijada izlangan geometrik o'rin ikkita simmetrik aylanadan iborat ekanligi ko'rsatiladi va ularning tenglamalari aniq ko'rinishda olinadi. Natijalar geometriya o'qitish metodikasida foydalanish uchun tavsiya etiladi.

Kalit so'zlar: yasashga doir masalalar, nuqtalarning geometrik o'rni, analitik ifodalash, aylana tenglamasi, koordinatalar metodi.

1. Masalaning qo'yilishi

Uzunligi a bo'lgan AB kesma va α burchak berilgan bo'lsin. Shunday M nuqtalar to'plamini topish talab qilinadiki, ulardan AB kesma α burchak ostida ko'rinsin, ya'ni

$$\angle AMB = \alpha.$$

2. Klassik yechim

Ma'lumki, agar biror M nuqtadan AB kesma α burchak ostida ko'rinsa, u holda A, B, M nuqtalar orqali o'tuvchi aylana mavjud bo'lib, unda AB vatar bo'ladi. Aylanada bir xil vatarga tiralgan ichki burchaklar o'zaro teng bo'lgani uchun, AB kesmani α burchak ostida ko'radigan barcha nuqtalar shu aylananing yoyi ustida yotadi.

Yasash quyidagi bosqichlarda bajariladi:

1. AB kesmani chizamiz.

2. A nuqtadan AB bilan α burchak tashkil etuvchi nur o'tkazamiz.
3. Bu nurga perpendikulyar to'g'ri chiziq o'tkazamiz.
4. AB kesmaning o'rta perpendikulyarini chizamiz.
5. Bu ikki to'g'ri chiziqning kesishish nuqtasi – aylana markazi O.
6. O markazdan OA radiusda aylana chizamiz.

Izlangan geometrik o'rin shu aylananing ikkita simmetrik yoyidan iborat.

3. Analitik ifodalash

Dekart koordinatalar sistemasini shunday kiritamizki, AB kesmaning o'rtasi koordinata boshi bilan ustma-ust tushsin va kesma Ox o'qida yotsin:

$$A\left(-\frac{a}{2},0\right), \quad B\left(\frac{a}{2},0\right).$$

$M(x,y)$ – izlangan geometrik o'rindagi ixtiyoriy nuqta bo'lsin.

$\angle AMB = \alpha$ sharti A, B, M nuqtalar orqali o'tuvchi aylananing mavjudligini bildiradi. Bu aylananing markazi $O'(0,k)$ nuqtada joylashgan bo'lsin. U holda aylana tenglamasi:

$$x^2 + (y-k)^2 = R^2,$$

bu yerda R – aylana radiusi.

A va B nuqtalar aylanada yotgani uchun:

$$\left(\frac{a}{2}\right)^2 + k^2 = R^2.$$

Boshqa tomondan, AB vatariga tiralgan α ichki burchak radius R bilan quyidagicha bog'langan:

$$\sin\alpha = \frac{a}{2R},$$

bundan

$$R = \frac{a}{2\sin\alpha}.$$

Bu ifodani oldingi tenglikka qo'yib, topamiz:

$$k^2 = \frac{a^2}{4}\cot^2\alpha, \quad k = \pm\frac{a}{2}\cot\alpha.$$

Shunday qilib, izlangan geometrik o'rin analitik ravishda ikkita tenglama bilan ifodalanadi:

$$\boxed{x^2 + \left(y - \frac{a}{2}\cot\alpha\right)^2 = \frac{a^2}{4\sin^2\alpha}}$$

va

$$\boxed{x^2 + \left(y + \frac{a}{2}\cot\alpha\right)^2 = \frac{a^2}{4\sin^2\alpha}.}$$

Bu ikki aylana AB kesmaga nisbatan simmetrik joylashgan.

4. Xususiy hollar

- $\alpha = 90^\circ$ bo'lganda $k = 0$, ya'ni markazi AB o'rtasida bo'lgan bitta aylana hosil bo'ladi.
- $\alpha \to 0$ yoki $\alpha \to 180^\circ$ bo'lganda radius $R \to \infty$, aylana to'g'ri chiziqqa intiladi.

5. Xulosa

Ishda klassik yasashga doir masala to'liq analitik usulda ifodalandi. Olingan tenglamalar geometrik o'rinni nafaqat qat'iy asoslashga, balki uning xususiy va chegaraviy hollarini tadqiq qilishga ham imkon beradi. Bunday yondashuv geometriya va matematik analiz o'qitish metodikasida foydali bo'lishi mumkin.

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MSC 11L03, 11A07, 11N69

ENG KICHIK MUSBAT BOSHLANG'ICH ILDIZNI TRIGONOMETRIK YIG'INDILAR YORDAMIDA BAHOLASH

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Analitik sonlar nazariyasida tub sonlar moduli bo'yicha boshlang'ich ildizlarning taqsimoti, xususan, eng kichik musbat boshlang'ich ildiz g ning miqdoriy tartibini yuqoridan baholash fundamental nazariy va amaliy ahamiyatga ega. Ushbu muammo dastlab I. M. Vinogradov [1] hamda B. I. Segallarning [2] klassik ishlarida batafsil tadqiq etilgan bo'lib, ular yetarlicha katta p tub soni va ixtiyoriy $\varepsilon > 0$ uchun $g = O(p^{\frac{1}{2}+\varepsilon})$ asimptotik bahosini isbotlaganlar. Biroq mazkur bahoda noma'lum ko'paytuvchini o'z ichiga oluvchi asimptotik O-simvoli mavjudligi sababli undan aniq sonli tadqiqotlarda foydalanib bo'lmaydi. Ushbu ishning asosiy maqsadi — g uchun O-simvoliga bog'liq bo'lmagan, barcha koeffitsiyentlari effektiv hisoblanuvchi baholarni keltirib chiqarishdan iboratdir.

Faraz qilaylik, $p \geq 3$ tub son bo'lsin. $1, 2, \ldots, Q$ $(Q < p)$ sonlari orasida joylashgan p moduli bo'yicha boshlang'ich ildizlar sonini L bilan belgilaymiz. Ma'lumki, $a \equiv g^\gamma \pmod{p}$ soni boshlang'ich ildiz bo'lishi uchun $(\gamma, p-1) = 1$ bo'lishi zarur va yetarlidir. Har bir $a \in \{1, 2, \ldots, Q\}$ soniga mos $\gamma = \operatorname{ind} a$ va $\delta = (\gamma, p-1)$ soni uchun Myobius funksiyasi xossalariga ko'ra

$$L = \sum_{d|(p-1)} \mu(d) S_d \tag{1}$$

tenglik o'rinli bo'ladi. Bu yerda S_d oraliqdagi d-darajali chegirmalar soni bo'lib, trigonometrik yig'indilar yordamida $S_d = \frac{Q}{d} + \theta\sqrt{p}\ln p$ ko'rinishida baholanadi ($|\theta| \leq 1$). Multiplikativ

funksiyalarning asosiy ayniyatiga binoan quyidagi ifoda hosil qilinadi:

$$L = \frac{\varphi(p-1)}{p-1}Q + \theta\sigma\sqrt{p}\ln p, \tag{2}$$

bu yerda σ soni $p-1$ ning kvadratlardan xoli bo‘luvchilari soni, $\varphi(m)$ esa Eyler funksiyasidir.

$L > 0$ bo‘lishini ta’minlovchi Q chegarasini topish maqsadida bo‘luvchilar soni $\tau(k)$ ning bahosidan foydalaniladi. Ixtiyoriy $\varepsilon > 0$ uchun $\tau(k) \le c_1(\varepsilon)k^{\varepsilon}$ munosabat o‘rinli bo‘lib, bunda $c_1(\varepsilon)$ ko‘paytuvchisi uchun [4] ishda olingan aniq analitik ifoda jalb etiladi:

$$c_1(\varepsilon) = \prod_{p_i < e^{1/\varepsilon}} \max\left\{ \left[\frac{1}{\varepsilon \ln p_i}\right] p_i^{-\varepsilon\left[\frac{1}{\varepsilon \ln p_i}-1\right]}, \left(\left[\frac{1}{\varepsilon \ln p_i}\right]+1\right) p_i^{-\varepsilon\left[\frac{1}{\varepsilon \ln p_i}\right]} \right\}.$$

Bundan foydalanib $\varphi(k) \ge \frac{1}{c_1(\varepsilon)}k^{1-\varepsilon}$ quyi chegara olinadi. Natijada (2) tenglikdan quyidagi baho kelib chiqadi:

$$L \ge \frac{Q}{c_1(\varepsilon/4)}(p-1)^{-\varepsilon/4} - c_1(\varepsilon/4)(p-1)^{\varepsilon/4}\sqrt{p}\ln p.$$

Agar $Q = c_1^2(\varepsilon/4)(p-1)^{\varepsilon/2}\sqrt{p}\ln p$ deb tanlansa, $L > 0$ bo‘lishi kafolatlanadi va barcha $p \ge 3$ tub sonlar uchun quyidagi effektiv natijaga erishiladi:

$$g \le c_1^2(\varepsilon/4)(p-1)^{\varepsilon/2}\sqrt{p}\ln p.$$

Agar $p-1$ ning turli tub bo‘luvchilari soni k ma’lum bo‘lsa, $p-1 = p_1^{\alpha_1}\dots p_k^{\alpha_k}$ kanonik yoyilmasiga ko‘ra $\sigma = 2^k$ bo‘ladi. Shu bilan birga, $\varphi(p-1) > (p-1)2^{-k}$ ekanligini e’tiborga olsak, (2) tenglikdan quyidagi qat’iy quyi baho shakllanadi:

$$L > 2^{-k}Q - 2^k\sqrt{p}\ln p.$$

Teorema. *Faraz qilaylik, $p \ge 3$ ixtiyoriy tub son va k soni $p-1$ ning turli tub bo‘luvchilari soni bo‘lsin. U holda p moduli bo‘yicha eng kichik musbat boshlang‘ich ildiz g uchun quyidagi baho o‘rinlidir:*

$$g \le (4^k\sqrt{p}\ln p) + 1.$$

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Algebraik ko’pxillikda aniqlangan golomorf funksiyaning tartibi va tipining xossasi

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$A \subset C^N$ bo‘lsin. A kompleks o‘lchovi n ga teng bo‘lgan algebraik ko’pxillik deyiladi, agar har bir $a \in A$ nuqtasi uchun $U \subset C^N$ ochiq atrof va $N-n$ ta ko‘phad funksiyalar $f_1, f_2, \ldots, f_{N-n} \in C[z_1, \ldots, z_N]$ mavjud bo‘lib:

$$A \cap U = \{z \in U : f_1(z) = f_2(z) = \cdots = f_{N-n}(z) = 0\},$$

va shu bilan birga Yakobi matritsasi

$$\left[\frac{\partial f_i}{\partial z_j}\right]$$

$A \cap U$ dagi nuqtalarda to‘liq rangga ega bo‘lsa.

1-**Misol:** $A = \{(z_1, z_2) \in C^2 : z_1^2 + z_2^2 = 1\}$ to‘plam kompleks o‘lchovi 1 bo‘lgan algebraik ko’pxillik hisoblanadi.

Ko‘pxillikdada aniqlangan f funksiya lokal koordinatalarda analitik bo‘lsa, $f \in \mathcal{O}(A)$ deyiladi. Xuddi shuningdek, u funksiya lokal koordinatalarda plurisubgarmonik bo‘lsa, $u \in psh(A)$ deyiladi.

1-**Ta’rif:** A algebraik ko’pxillik regulyar [2,3] deyiladi, agar ρ–ko‘phadlar fazosi $\mathcal{P}_\rho(A)$ $\mathcal{O}(A)$ analitik funksiyalar fazosida zich bo‘lsa.

A da aniqlangan golomorf funksiyalar butun funksiyalar deyiladi. Klassik nazariyadagi kabi , regulyar parabolik ko‘pmanifollarda butun funksiyalarning o‘sish tartibi kiritiladi. Bu holatda ko‘phadlar asosiy solishtirish ob’ekti sifatida xizmat qiladi (ya’ni, butun funksiyalar ichida eng kichik o‘sish ko‘rsatkichiga ega).

Shu maqsadda psevdoshar aniqlanadi:

$$B_R = \{z \in \partial A : \rho(z) < \ln R\}, \quad R > 0.$$

Butun $f(z) \in \mathcal{O}(A)$ funksiya uchun:

$$M(R, f) = \sup\{|f(z)| : z \in \partial \overline{B}_R\}.$$

2-**Ta’rif .** Quyidagi kattalik

$$\tau(f) = \limsup_{R \to \infty} \frac{\ln\ln M(R, f)}{\ln R}$$

f butun funksiyaning o‘sish tartibi deb ataladi.

Aniqki, $0 \le \tau(f) \le +\infty$. Agar $\tau(f) < +\infty$ bo‘lsa, quyidagi qiymat aniqlanadi:

$$\sigma(f) = \limsup_{R \to \infty} \frac{\ln M(R, f)}{R^\tau}.$$

$\sigma(f)$ butun funksiyaning turi deyiladi:

$K \subset A$ bo‘lsin — regulyar kompakt to‘plam. Ekstremal funksiyaning darajali to‘plamini qaraymiz:

$$D_R = \{z \in A : \Phi_A(z, K) < R\}, \quad R > 1,$$

bu yerda

$$\Phi_A(z, K) = \sup\{|f(z)|^{1/\deg f} : f \in \mathcal{P}_\rho(A), \quad \|f\|_K \le 1\}.$$

Shuningdek,

$$M_K(R, f) = \sup\{|f(z)| : z \in \partial D_R\}, \quad \tau_K(f) = \limsup_{R \to \infty} \frac{\ln\ln M_K(R, f)}{\ln R}.$$

1-**Teorema** Agar (A, ρ) algebraik ko‘pxillik bo‘lsa, u holda har qanday regulyar kompakt $K \subset A$ uchun $\tau_K(f) = \tau(f)$ tenglik o‘rinli.

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Algebraik ko'pxillikda aniqlangan golomorf funksiyani davom qildirish haqidagi teorema

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Bizga A algebraik ko'pxillik berilgan bo'lsin. Bilamizki algebraik ko'pxilliklar bu parabolik ko'pxilliklarning bir qismi hisoblanadi. Klassik nazariyadagi kabi , regulyar parabolik ko'pxilliklarda ham butun funksiyalarning o'sish tartibi kiritiladi. Bu holatda ko'phadlar asosiy solishtirish ob'ekti sifatida xizmat qiladi (ya'ni, butun funksiyalar ichida eng kichik o'sish ko'rsatkichiga ega).

Shu maqsadda psevdoshar aniqlanadi:

$$\mathrm{B_R}=\{\mathrm{z}\in \mathrm{A}:\rho(\mathrm{z})<\mathrm{lnR}\},\qquad \mathrm{R}>0.$$

Butun $\mathrm{f(z)}\in\mathcal{O}(\mathrm{A})$ funksiya uchun:

$$\mathrm{M(R,f)}=\sup\{|\mathrm{f(z)}|:\mathrm{z}\in\partial\overline{\mathrm{B}}_{\mathrm{R}}\}.$$

1-**Ta'rif .** Quyidagi kattalik

$$\tau(\mathrm{f})=\limsup_{\mathrm{R}\to\infty}\frac{\mathrm{lnlnM(R,f)}}{\mathrm{lnR}}$$

f butun funksiyaning o'sish tartibi deb ataladi.

Aniqki, $0\le\tau(\mathrm{f})\le+\infty$. Agar $\tau(\mathrm{f})<+\infty$ bo'lsa, quyidagi qiymat aniqlanadi:

$$\sigma(\mathrm{f})=\limsup_{\mathrm{R}\to\infty}\frac{\mathrm{lnM(R,f)}}{\mathrm{R}^{\tau}}.$$

σ(f) butun funksiyaning turi deyiladi:

2-Ta'rif: $f(z)\in\mathcal{O}(A)$ funksiya uchun $\forall z\in A$ uchun $|f(z)|\le Ce^{\sigma z^{\tau}}$, $C=const$ tengsizlik bajarilsa $f(z)$ golomorf funksiya A algebraik ko'pxillikda σ tip va τ tartibga ega deyiladi.

1-Teorema: Agar $K\subset A$ dagi regulyar kompaktda aniqlangan $f(z)\in\mathcal{O}(A)$ funksiya quyidagi $\overline{\lim_{d\to\infty}}d^{\frac{1}{\tau}}e_d^{\frac{1}{d}}(f,K)\le(e\tau\sigma)^{\frac{1}{\tau}}$ tengsizlikni qanoatlantirsa, u holda $f(z)$funksiya A ga tartibi $\le\tau$ va tipi $\le\sigma$ bo'lgan butun funksiya bo'lib davom qiladi.

Izoh: Bu yerda $e_d(f,K)$ minimal chetlashishdir.

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LOCAL DERIVATIONS ON PARA-HURWITZ ALGEBRAS

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Local derivations are linear mappings which, although not required to be derivations globally, agree at every individual element with some derivation depending on that element. This notion has been extensively studied for associative, Lie, Jordan and other nonassociative algebras. In this paper we consider local derivations on para-Hurwitz algebras, with particular attention to the eight-dimensional real case. We describe the relation between ordinary derivations and local derivations and indicate how the multiplication and norm structure of para-Hurwitz algebras constrain local operators.

Keywords: para-Hurwitz algebra, symmetric composition algebra, derivation, local derivation, octonion algebra, nonassociative algebra.

1. Introduction

Composition algebras are finite-dimensional algebras equipped with a nondegenerate quadratic form satisfying a multiplicativity condition. The classical examples are the real numbers, complex numbers, quaternions and octonions. A symmetric composition algebra is a composition algebra whose polar form is associative with respect to the multiplication. Over an algebraically closed field of characteristic different from 2 and 3, the main examples are para-Hurwitz and Okubo algebras.

The study of derivations is one of the basic ways to understand the internal symmetries of an algebra. A linear map $D : A \to A$ is called a derivation if

$$D(x * y) = D(x) * y + x * D(y), \qquad x, y \in A.$$

A weaker concept is obtained by requiring this identity only after choosing, for every fixed element, a suitable derivation. A linear map $\Delta : A \to A$ is a *local derivation* if for every $x \in A$ there exists a derivation D_x of A such that

$$\Delta(x) = D_x(x).$$

The central question is whether every local derivation must necessarily be an ordinary derivation.

2. Para-Hurwitz algebras

Let (C, n) be a Hurwitz algebra with canonical involution $x \mapsto \bar{x}$. The corresponding para-Hurwitz algebra C^{p} is obtained by changing the multiplication to

$$x * y = \bar{x}\,\bar{y}.$$

Its norm is still multiplicative:

$$n(x * y) = n(x)n(y).$$

Thus the para-Hurwitz construction produces a symmetric composition algebra. In dimensions $1, 2, 4$ and 8 the corresponding classical models are obtained from the real, complex, quaternionic and octonionic Hurwitz algebras.

For the eight-dimensional real case, the algebra is the para-octonion algebra. Its underlying vector space has a basis

$$e_0, e_1, \ldots, e_7,$$

with multiplication determined by the octonion multiplication and the para-Hurwitz rule. The norm form is nondegenerate, and the multiplication satisfies the composition identity. These properties impose strong restrictions on derivations.

The derivation algebra of the octonions is the exceptional Lie algebra $\mathfrak{g}_2$. For the para-octonion algebra the same exceptional symmetry appears through the corresponding derivation structure. Consequently, local derivations can be studied by expressing a general linear operator in the chosen basis and comparing it with the finite-dimensional space of derivations.

3. Local derivations

Suppose A is a finite-dimensional para-Hurwitz algebra and $\Delta : A \to A$ is linear. Choose a basis and write

$$\Delta(e_j) = \sum_i a_{ij} e_i.$$

If Δ is a local derivation, then for every vector x there is a derivation D_x whose value at x equals $\Delta(x)$. In particular, this condition can be tested on basis vectors and on carefully selected linear combinations of basis vectors.

The key observation is that the local condition is stronger than it first appears because derivations preserve the algebraic identities determined by the composition structure. For a derivation D, differentiation of the norm identity gives

$$\langle D(x), x\rangle = 0$$

for the polar form associated with the norm. Hence every local derivation satisfies the corresponding pointwise orthogonality condition

$$\langle \Delta(x), x\rangle = 0, \qquad x \in A.$$

By polarization, this produces a system of linear equations in the coefficients a_{ij}. Combining these equations with the multiplication relations yields the same coefficient restrictions as those defining the derivation algebra.

In the eight-dimensional real para-Hurwitz case, the computation may be organized by considering vectors of the form

$$x = e_i + te_j,$$

where $t \in \mathbb{R}$. For each such vector, the local property supplies a derivation D_x. Comparing the coefficients of different powers of t gives polynomial identities. Since the identities hold for all real t, every coefficient must vanish separately. Repeating this for a suitable collection of pairs (i, j) forces the matrix of Δ to satisfy the derivation equations.

4. Main conclusion

The coefficient comparison method shows that, for the eight-dimensional real para-Hurwitz algebra considered here, the local condition determines the operator sufficiently strongly to

recover the ordinary derivation identities. Therefore, under the stated finite-dimensional real setting, a local derivation is forced to belong to the derivation algebra.

This conclusion is significant because it identifies local derivations not merely as pointwise approximations to derivations, but as operators governed by the global symmetry of the para-Hurwitz multiplication. It also connects the problem with the exceptional Lie algebra $\mathfrak{g}_2$ and with the broader theory of local and 2-local mappings on nonassociative algebras.

The methods can be extended to other symmetric composition algebras. In particular, comparing the para-Hurwitz case with Okubo algebras may clarify which properties of the multiplication table are responsible for the coincidence of local and ordinary derivations. Such comparisons also provide a route toward studying local automorphisms and 2-local derivations.

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On the Infinitude of Eigenvalues of the Weyl Function Associated with a Second-Order Operator Matrix

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Denote by $\mathbb{T}$ the one dimensional torus. Let $\mathcal{H}_1 := L_2(\mathbb{T})$ be the Hilbert space of square integrable (complex) functions defined on $\mathbb{T}$ and $\mathcal{H}_2 := L_2^{\mathrm{sym}}((\mathbb{T})^2)$ be the Hilbert space of square integrable symmetric (complex) functions defined on $(\mathbb{T})^2$. The spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ are called one- and two-particle subspaces of the bosonic Fock space $\mathcal{F}_{\mathrm{s}}(L_2(\mathbb{T}))$ over $L_2(\mathbb{T})$, respectively.

In the Hilbert space $\mathcal{H} := \mathcal{H}_1 \oplus \mathcal{H}_2$ we consider the operator $\mathcal{A}_\mu$ that admit an 2×2 tridiagonal block operator matrix representation

$$\mathcal{A}_\mu := \begin{pmatrix} A_{11} & \mu A_{12} \\ \mu A_{12}^* & A_{22} \end{pmatrix}$$

with the entries $A_{ij} : \mathcal{H}_j \to \mathcal{H}_i$, $i \le j$, $i, j = 1, 2$ defined by

$$(A_{11}f_1)(p) = af_1(p), \quad (A_{12}f_2)(p) = \int_{\mathbb{T}} f_2(p,t)dt,$$
$$(A_{22}f_2)(p,q) = w(p-q)f_2(p,q), \quad f_i \in \mathcal{H}_i, \quad i = 1, 2.$$

Here A_{12}^* denotes the adjoint operator to A_{12}, $a \in \mathbb{R}\backslash\{0\}, \mu > 0$ and $w(\cdot)$ is an even continuous function on $\mathbb{T}$. Under these assumptions the operator $\mathcal{A}_\mu$ is bounded and self-adjoint.

The spectrum, the essential spectrum and the discrete spectrum of a bounded self-adjoint operator will be denoted by $\sigma(\cdot)$, $\sigma_{\text{ess}}(\cdot)$ and $\sigma_{\text{disc}}(\cdot)$, respectively.

We define an analytic function $\Delta_\mu(\cdot)$ in $\mathbb{C} \setminus \text{Ran}(w)$ by

$$\Delta_\mu(z) := z - a + \frac{\mu^2}{2} \int_{\mathbb{T}} \frac{dt}{w(t) - z}.$$

For completeness, we recall the definition of the Weyl function [1,2].

Definition 1. A triple $\Pi = \{\mathcal{H}, \Gamma_0, \Gamma_1\}$ consisting of a separable Hilbert space $\mathcal{H}$ and linear mapping $\Gamma_j : \text{dom}(A^*) \to \mathcal{H}$, $j \in \{0, 1\}$, is called a boundary triple for the adjoint relation A^* of A if:

(i) the following Green's identity takes place

$$(f', g) - (f, g') = (\Gamma_1 \widehat{f}, \Gamma_0 \widehat{g})_{\mathcal{H}} - (\Gamma_0 \widehat{f}, \Gamma_1 \widehat{g})_{\mathcal{H}}, \quad \widehat{f} := \{f, f'\},\ \widehat{g} := \{g, g'\} \in \text{dom}(A^*),$$

(ii) the mapping $\Gamma := \{\Gamma_0, \Gamma_1\} : \text{dom}(A^*) \to \mathcal{H} \oplus \mathcal{H}$ is surjective.

Definition 2. Let $\Pi = \{\mathcal{H}, \Gamma_0, \Gamma_1\}$ be a boundary triple for A^* and $A_0 = A^* \restriction \ker(\Gamma_0)$. The operator valued function $M(\cdot) :\ \rho(A_0) \to [\mathcal{H}]$ defined by

$$M(z) := \Gamma_1 \widehat{\gamma}(z), \quad z \in \rho(A_0),$$

is called the Weyl function corresponding to the boundary triple Π.

Using the definition after direct calculations, we conclude the corresponding Weyl function $M_\mu(\cdot)$ for operator $\mathcal{A}_\mu$ is given by:

$$(M_\mu(z)f)(p) = \Delta_\mu(z)f(p) + \frac{\mu^2}{2} \int_{\mathbb{T}} \frac{f(t)dt}{w(p-t) - z}, \quad z \in \mathbb{R} \setminus \text{Ran}(w), \quad f \in L_2(\mathbb{T}).$$

Since for any fixed $z_0 \in \mathbb{R} \setminus \text{Ran}(w)$ the function

$$\frac{1}{w(p-q) - z_0}$$

is a continuous as function of (p, q) on the compact set $(\mathbb{T})^2$ we have

$$\sigma_{\text{ess}}(M_\mu(z_0)) = \{\Delta_\mu(z_0)\}).$$

For a bounded self-adjoint operator A acting in the Hilbert space $\mathcal{H}$ denote by $\mathcal{H}_A(\lambda)$, a subspace such that $(Af, f) < \lambda\|f\|$ for any $f \in \mathcal{H}_A(\lambda)$ and set

$$N_{(-\infty,\lambda)}(A) := \sup_{\mathcal{H}_A(\lambda)} \dim \mathcal{H}_A(\lambda).$$

Certainly, for an operator A not having any essential spectrum on the left from the point λ, the value $N_{(-\infty,\lambda)}(A)$ coincides with the number of an eigenvalues of A smaller than λ. Therefore, $N_{(-\infty,\lambda)}(A) = N_{(-\lambda,\infty)}(-A)$.

Theorem 1. For any $\varepsilon > 0$ the operator $M_\mu(z)$ has infinitely many eigenvalues in $(\Delta_\mu(z) - \varepsilon, \Delta_\mu(z)) \cup (\Delta_\mu(z), \Delta_\mu(z) + \varepsilon)$, that is,

$$N_{(\Delta_\mu(z)-\varepsilon, \Delta_\mu(z))}(M_\mu(z)) + N_{(\Delta_\mu(z), \Delta_\mu(z)+\varepsilon)}(M_\mu(z)) = \infty.$$

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On the Geometry of Orbits of Vector Fields

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Let M be a Riemannian (smooth) manifold of dimension n.

Definition 1. A subset L of M is said to be a $k-leaf$ of M if there exists a differentiable structure σ on L such that

(i) (L,σ) is a connected k-dimensional immersed submanifold of M,

(ii) if N is an arbitrary locally connected topological space, and $f : N \to M$ is a continuous function such that $f(N) \subset L$, then $f : N \to (L,\sigma)$ is continuous.

It follows from the properties of immersions that if $f : N \to M$ is a differentiable mapping of manifolds such that $f(N) \subset L$, then $f : N \to (L,\sigma)$ is also differentiable. In particular, σ is the unique differentiable structure on L which makes L into an immersed k-dimensional submanifold of M. Since M is paracompact,every connected immersed submanifold of M is separable,and so the dimensional k of a leaf L is uniquely determined.

Definition 2. We say that $\mathbf{F}$ is a C^q-foliation of M with singularities if $\mathbf{F}$ is partition of M into C^q-leaves of M such that, for every $x\epsilon M$, there exists a local C^q-chart ψ of M with the following properties:

(a) The domain of ψ is of the from $U \times W$, where U is an open neighbourhood of 0 in R^k, W is an open neighbourhood of 0 in R^{n-k}, and k is the dimension of the leaf through x.

(b) $\psi(0,0) = x$.

(c) If L is a leaf of $\mathbf{F}$, then $L \cap \psi(U \times W) = \psi(U \times l)$, where $l = \{w\epsilon W : \psi(0,w)\epsilon L\}$. If the dimension of L is maximal, it is called regular, otherwise, L is called singular. It is known that orbits of vector fields generate singular foliation [1],[2].

Let us consider a set of vector fields $D \subset V(M)$of the Lie algebra of all smooth (class C^∞) vector fields $V(M)$ and the smallest Lie subalgebra containing D by $A(D)$. Let $t \to X^t(x)$ be an integral curve of the vector field X with the initial point x for $t = 0$, which is defined in some region $I(x)$ of real line.

Definition 3. The orbit $L(x)$ of a system D of vector fields through a point x is the set of points y in M such that there exist $t_1, t_2, ..., t_k \in R$ and vector fields $X_{i_1}, X_{i_2}, ..., X_{i_k} \in D$ such that

$$y = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_1}^{t_1})...)),$$

where k is an arbitrary positive integer[3].

let us recall some characteristics of two-dimensional surface F in four-dimensional Euclidean space E^4. Consider on the surface F at the point x some nonzero vector ξ. A hyperplane $E^3(x,\xi N)$ in E^4, defined by the vector ξ and the normal plane N at the point x, intersects the surface F along some curve γ. By its construction, the curve γ is a three-dimensional curve. Curvature $k_N(x,\xi)$ and torsion $\chi_N(x,\xi)$ of the curve γ at the point x are called, respectively, the normal curvature and the normal torsion of the surface at the point x in the direction ξ.

Geometry of two-dimensional surfaces in E^4 is an essential part of differential geometry and studied by many authors [4]. It is known that two-dimensional hyperplane surfaces are zero normal torsion surfaces. However, two-dimensional torus $S^1 \times S^1$ on the hypersphere S^3 in E^4 is zero normal torsion surface, although it is not hyperplane surface. The geometry of hyperplane surfaces is given in.

The Gaussian torsion χ_G is an invariant of the extrinsic geometry of the surface. If a and b are the semiaxes of an ellipse of normal curvature, then $\chi_G = \pm 2ab$, where the sign is taken to be plus in the case when under a rotation of the tangent vector in the positive direction the corresponding point on the ellipse moves in the positive direction in accordance with the orientation in the normal plane, and minus if this point moves in the negative direction [4].

Let us consider a family of $D = \{X_1, X_2\}$ vector fields on four-dimensional Euclidean space E^4 with Cartesian coordinates t, x_1, x_2, u, where

$$X_1 = \exp(t)\frac{\partial}{\partial t} + \exp(t)u\frac{\partial}{\partial u}, X_2 = (x_1^2 - x_2^2)\frac{\partial}{\partial x_1} + 2x_1x_2\frac{\partial}{\partial x_2} - 4x_1u\frac{\partial}{\partial u}. \quad (1)$$

Theorem 1. The family of orbits of the vector fields (1) generates a singular foliation whose regular leaf is not hyperplane surface (a surface with nonzero normal torsion).

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ON (A, φ)–STATISTICAL CONVERGENCE IN GENERALIZED SEQUENCE SPACES DEFINED BY MODULUS FUNCTION

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Abstract. Let w denote the set of all real and complex sequences $x = (x_k)$. By ℓ_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\|x\| = \sup_n |x_n|$, respectively. A linear functional L on ℓ_∞ is said to be a Banach limit if it has the following properties:

1. $L(x) \geq 0$ if $n \geq 0$ (i.e. $x_n \geq 0$ for all n),
2. $L(e) = 1$ where $e = (1, 1, \ldots)$,
3. $L(Dx) = L(x)$, where the shift operator D is defined by $D(x_n) = \{x_{n+1}\}$.

Let B be the set of all Banach limits on ℓ_∞. A sequence $x \in \ell_\infty$ is said to be almost convergent if all Banach limits of x coincide. Let $\hat{c}$ denote the space of almost convergent sequences. Lorentz [2] has shown that

$$\hat{c} = \left\{ x \in \ell_\infty : \lim_m t_{m,n}(x) \text{ exists uniformly in } n \right\},$$

where

$$t_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

By a lacunary $\theta = (k_r)$, $r = 0, 1, 2, \ldots$, where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and $h_r = k_r - k_{r-1}$. The ratio k_r/k_{r-1} will be denoted by q_r.

A modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that

(i) $f(x) = 0$ if and only if $x = 0$,

(ii) $f(x + y) \leq f(x) + f(y)$ for all $x, y \geq 0$,

(iii) f is increasing,

(iv) f is continuous from the right at zero.

In the present paper, we introduce and study some properties of the following sequence space that is defined using the φ-function and generalized three parametric real matrix.

Let φ and f be given φ-function and modulus function, respectively, and $p = (p_k)$ be a sequence of positive real numbers. Moreover, let $A = (A_i)$ be the generalized three parametric real matrix with $A_i = (a_{n,k}(i))$ and a lacunary sequence θ be given. Then we define the following sequence space,

$$N_\theta^0(A, \varphi, f, p) = \left\{ x = (x_k) : \lim_{r \to \infty} \frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{n,k}(i) \varphi(|x_k|) \right| \right)^{p_k} = 0, \text{ uniformly in } i \right\}.$$

If $x \in N_\theta^0(A, \varphi, f)$, the sequence x is said to be lacunary strong (A, φ)-convergent to zero with respect to a modulus f.

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Unital 3-dimensional structurable algebras

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A class of unital algebras with an involution, satisfying one exotic quasi-identity depending on the fixed element unit and the involution, was introduced by Allison in. If this involution is the identity mapping, the definition of structurable algebras reduces to the definition of Jordan algebras. Allison gave some important examples of simple structurable algebras. Since then, it has been under active investigation.

This paper is devoted to the classification and studying properties of complex unital 3-dimensional structurable algebras. We provide a complete list of non-isomorphic classes, identifying five algebras for type $(2,1)$ and two algebras for type $(1,2)$. For each obtained algebra, we describe the derivation algebra, the lattice of subalgebras.

Definition 1. Let $\mathcal{A}$ be an algebra. A map $^- : \mathcal{A} \to \mathcal{A}$ is called an involution if, for all $x, y \in \mathcal{A}$, the following conditions hold: $\overline{\overline{x}} = x,\ \overline{xy} = \overline{y}\ \overline{x}$.

For each complex n-dimensional unital algebra $\mathcal{A}$ with involution $^-$, we have $\mathcal{A} = \mathcal{H} \oplus \mathcal{S}$, where $\mathcal{H} = \{a \in \mathcal{A} \mid \overline{a} = a\}$ and $\mathcal{S} = \{a \in \mathcal{A} \mid \overline{a} = -a\}$.

If $\dim(\mathcal{H}) = \mathrm{k}$, then we call $\mathcal{A}$ an algebra of type $(k, n-k)$. For $x, y \in \mathcal{A}$, define $V_{x,y} \in \mathrm{End}(\mathcal{A})$ and $T_x = V_{x,e_1}$ as $V_{x,y}(z) = (x\overline{y})z + (z\overline{y})x - (z\overline{x})y,\ \ T_x(z) = xz + zx - z\overline{x}$. In particular, if $a \in \mathcal{H}$, then $T_a = L_a$.

Definition 2.[1] A unital algebra $\mathcal{A}$ with an involution $^-$ is structurable if it satisfies the following special identity

$$[T_z, V_{x,y}] \ = \ V_{T_z(x),y} - V_{x,T_{\overline{z}}(y)}. \tag{1}$$

Theorem 1. Let $\mathcal{A}$ be a complex unital structurable algebra $\mathcal{A}$ of type $(2,1)$, then it is isomorphic to the universal unital algebra A_1 of type $(2,1)$, or it is isomorphic to an algebra with an involution $^-$, given by $\overline{e_1} = e_1$, $\overline{e_2} = e_2$, $\overline{e_3} = -e_3$, and the multiplication table given by one of the following:

$\mathrm{A}_2 : e_3e_3 = e_2$

$\mathrm{A}_3 : e_2e_2 = e_2$

$\mathrm{A}_4 : e_2e_2 = e_2,\ \ e_3e_3 = -e_1 + e_2$

$\mathrm{A}_5 : e_2e_3 = e_2,\ \ e_3e_2 = -e_2,\ \ e_3e_3 = e_1$

Theorem 2. Let $\mathcal{A}$ be a complex unital structurable algebra $\mathcal{A}$ of type $(1,2)$, then it is isomorphic to the universal unital algebra S_1 of type $(1,2)$, or it is isomorphic to an algebra with an involution $^-$, given by $\overline{e_1} = e_1$, $\overline{e_2} = -e_2$, $\overline{e_3} = -e_3$, and the multiplication table given by one of the following:

$\mathrm{S}_2 : e_2e_3 = e_2\ e_3e_2 = -e_2\ e_3e_3 = e_1$

Definition 3.[2] Let $\mathcal{A}$ be a structurable algebra. A linear map $D : \mathcal{A} \to \mathcal{A}$ is called a derivation if $D(xy) = D(x)y + xD(y)$. If a derivation of $\mathcal{A}$ satisfies $D(x) = \overline{D(\overline{x})}$, then it is called a derivation commuting with the involution $\big($or $\overline{derivation}\big)$. The set of all derivations of $\mathcal{A}$ and the set of all $\overline{derivations}$ are denoted by $\mathfrak{Der}(\mathcal{A})$ and $\overline{\mathfrak{Der}}(\mathcal{A})$.

Proposition 1. Derivations and $\overline{derivations}$ of complex unital 3-dimensional structurable algebras are given below:

$\mathfrak{Der}(\mathrm{A}_1)=\langle d_1, d_2, d_3, d_4 \mid d_1(e_2) = e_2;\ d_2(e_3) = e_3;\ d_3(e_2) = e_3;\ d_4(e_3) = e_2\rangle$ $\overline{\mathfrak{Der}}(\mathrm{A}_1)=\langle d_1, d_2 \mid d_1(e_2) = e_2;\ d_2(e_3) = e_3\rangle$
$\mathfrak{Der}(\mathrm{A}_2)=\langle d_1, d_2 \mid d_1(e_2) = 2e_2,\ d_1(e_3) = e_3;\ d_2(e_3) = e_2\rangle$ $\overline{\mathfrak{Der}}(\mathrm{A}_2)=\langle d_1 \mid d_1(e_2) = 2e_2,\ d_1(e_3) = e_3\rangle$
$\mathfrak{Der}(\mathrm{A}_3)=\langle d_1 \mid d_1(e_3) = e_3\rangle$ $\overline{\mathfrak{Der}}(\mathrm{A}_3)=\langle d_1 \mid d_1(e_3) = e_3\rangle$
$\mathfrak{Der}(\mathrm{A}_4)=\langle 0\rangle$ $\overline{\mathfrak{Der}}(\mathrm{A}_4)=\langle 0\rangle$

$\mathfrak{Der}(\mathrm{A}_5)=\langle d_1, d_2 \mid d_1(e_2)=e_2;\ d_2(e_3)=e_2\rangle$
$\overline{\mathfrak{Der}}(\mathrm{A}_5)=\langle d_1 \mid d_1(e_2)=e_2\rangle$
$\mathfrak{Der}(\mathrm{S}_1)=\langle d_1, d_2, d_3, d_4 \mid d_1(e_2)=e_2;\ d_2(e_2)=e_3;\ d_3(e_3)=e_2;\ d_4(e_3)=e_3\rangle$
$\overline{\mathfrak{Der}}(\mathrm{S}_1)=\langle d_1, d_2, d_3, d_4 \mid d_1(e_2)=e_2;\ d_2(e_2)=e_3;\ d_3(e_3)=e_2;\ d_4(e_3)=e_3\rangle$
$\mathfrak{Der}(\mathrm{S}_2)=\langle d_1, d_2 \mid d_1(e_2)=e_2;\ d_2(e_3)=e_2\rangle$
$\overline{\mathfrak{Der}}(\mathrm{S}_2)=\langle d_1, d_2 \mid d_1(e_2)=e_2;\ d_2(e_3)=e_2\rangle$

Theorem 3. Let $\mathfrak{s}$ be a one-dimensional subalgebra of A. Up to isomorphisms, all one-dimensional subalgebras of A are equivalent to

A Table of one - dimensional subalgebras
$\mathcal{A}_1 : \langle e_1\rangle,\ \langle e_2\rangle.$
$\mathcal{A}_2 : \langle e_1\rangle,\ \langle e_2\rangle.$
$\mathcal{A}_3 : \langle e_1\rangle,\ \langle e_2\rangle,\ \langle e_3\rangle,\ \langle e_1-e_2\rangle.$
$\mathcal{A}_4 : \langle e_1\rangle,\ \langle e_2\rangle,\ \langle e_1-e_2\rangle,\ \langle e_1\pm e_2+\mathbf{i}e_3\rangle.$
$\mathcal{A}_5 : \langle e_1\rangle,\ \langle e_2\rangle,\ \langle e_1\pm e_3\rangle.$
$\mathcal{S}_1 : \langle e_1\rangle,\ \langle e_2\rangle.$
$\mathcal{S}_2 : \langle e_1\rangle,\ \langle e_2\rangle,\ \langle e_1\pm e_3\rangle.$

Theorem 4. Let $\mathfrak{s}$ be a two-dimensional subalgebra of A. Up to isomorphisms, all two-dimensional subalgebras of A are equivalent to

A Table of two - dimensional subalgebras
$\mathcal{A}_1 : \langle e_1, e_2\rangle,\ \langle e_2, e_3\rangle.$
$\mathcal{A}_2 : \langle e_1, e_2\rangle,\ \langle e_2, e_3\rangle.$
$\mathcal{A}_3 : \langle e_1, e_3\rangle,\ \langle e_1-e_2, e_3\rangle,\ \langle e_1, e_2\rangle,\ \langle e_2, e_3\rangle.$
$\mathcal{A}_4 : \langle e_1, e_2\rangle,\ \langle e_1+ie_3, e_2\rangle,\ \langle e_1-e_2, e_3\rangle,\ \langle e_1, ie_2+e_3\rangle.$
$\mathcal{A}_5 : \langle e_1, e_2\rangle,\ \langle e_1\pm e_3, e_2\rangle,\ \langle e_1, e_3\rangle.$
$\mathcal{S}_1 : \langle e_1, e_2\rangle,\ \langle e_2, e_3\rangle.$
$\mathcal{S}_2 : \langle e_1, e_2\rangle,\ \langle e_1\pm e_3, e_2\rangle,\ \langle e_1, e_3\rangle.$

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CHEBYSHEV FUNCTIONS AND EXPLICIT FORMULAS FOR THEM

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Introduction. In number theory, the study of the distribution of prime numbers among the natural numbers essentially relies on the investigation of the prime-counting function $\pi(x)$, which denotes the number of primes not exceeding a given real number x. However, since performing direct analytic operations on $\pi(x)$ is technically challenging, P. L. Chebyshev introduced the

first and second Chebyshev functions into the field. These functions serve as a foundation for investigating asymptotic laws using complex analysis methods.

The objective of this paper is to analyze von Mangoldt's explicit formula for the second Chebyshev function, which links it directly to the complex zeros of the Riemann zeta function, and to evaluate the remainder term under the assumption of the Riemann Hypothesis.

Basic Definitions and Properties. The first and second Chebyshev functions, $\theta(x)$ and $\psi(x)$, are defined as follows [1]:

$$\theta(x) = \sum_{p \le x} \ln p, \quad \psi(x) = \sum_{p^m \le x} \ln p = \sum_{n \le x} \Lambda(n),$$

where p denotes prime numbers and $\Lambda(n)$ represents the von Mangoldt function. It is well known that $\Lambda(n) = \ln p$ if $n = p^m$ $(m \ge 1)$, and vanishes otherwise $(\Lambda(n) = 0)$.

In analytic number theory, examining the properties of $\psi(x)$ is of crucial importance because its logarithmic derivative is directly connected to the Riemann zeta function through the Dirichlet series:

$$-\frac{\zeta'(s)}{\zeta(s)} = \sum_{n=1}^{\infty} \frac{\Lambda(n)}{n^s}, \quad \mathrm{Re}(s) > 1.$$

Analytic Analysis and von Mangoldt's Formula. By applying Perron's formula to this Dirichlet series, the function $\psi(x)$ can be expressed in terms of a contour integral [1]:

$$\psi_0(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left(-\frac{\zeta'(s)}{\zeta(s)}\right) \frac{x^s}{s}\, ds, \quad c > 1.$$

Applying Cauchy's residue theorem over the integration contour leads to von Mangoldt's explicit formula for $\psi(x)$.

Theorem 1. If $x > 1$ is not an integer power of a prime, then the following explicit formula holds:

$$\psi(x) = x - \sum_{\rho} \frac{x^\rho}{\rho} - \ln(2\pi) - \frac{1}{2}\ln(1 - x^{-2}), \tag{1}$$

where $\rho = \beta + i\gamma$ runs over all non-trivial zeros of the Riemann zeta function $\zeta(s)$ located in the critical strip $(0 < \mathrm{Re}(s) < 1)$, and the sum is evaluated in increasing order of the imaginary parts $|\gamma|$.

The leading term x in formula (1) corresponds to the residue of the integrand at the simple pole $s = 1$. The logarithmic term reflects the sum of residues originating from the trivial zeros of $\zeta(s)$ at $s = -2n$.

The Riemann Hypothesis and Remainder Estimation. The explicit formula (1) demonstrates the deepest connection between prime number theory and complex analysis. It reveals that the rate of convergence of $\psi(x)$ to x depends directly on the real parts β of the non-trivial zeros [2].

Theorem 2. The validity of the Riemann Hypothesis, namely that all non-trivial zeros of $\zeta(s)$ satisfy $\mathrm{Re}(\rho) = 1/2$ (the critical line), is equivalent to the following asymptotic bound:

$$\psi(x) = x + O(x^{1/2} \ln^2 x). \tag{2}$$

Conclusion. The explicit formulas obtained for Chebyshev functions allow the arithmetic properties of prime numbers to be translated into the geometric language of the zeros of the zeta function, providing an essential analytical framework for establishing asymptotic distribution laws of primes and investigating binary and ternary additive problems in prime number theory [3, 4].

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MSC 11D04, 11P32, 11D75

OLTITA TUB NOMA'LUMLI TO'RTTA CHIZIQLI TENGLAMALAR SISTEMASINING YECHIMGA EGA BO'LISHLIK SHARTLARI HAQIDA

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Natural sonlarni bir vaqtda tub o'zgaruvchili chiziqli formalar orqali ifodalash masalasi analitik sonlar nazariyasining klassik muammolaridan biri bo'lib, umumiy holda Xua Lo-Ken tomonidan qo'yilgan [1]. Butun koeffitsiyentli s ta tenglama va m ta tub o'zgaruvchili

$$b_i = \sum_{j=1}^{m} a_{ij} p_j, \quad i = 1, \ldots, s \tag{1}$$

chiziqli tenglamalar sistemasi uchun Wu Fang [2] $m \geq 2s + 1$ bo'lganda yechimlar soni uchun asimptotik formula oldi. Keyinchalik M. C. Liu va K. M. Tsang [3] sistemaning $s = 2, m = 3$ xususiy holi uchun istisno to'plami $E_{2,3}(X)$ ning o'lchovi uchun darajali yuqoridan baho oldilar. Ushbu natija I. Allakov tomonidan yaxshilandi va $s = n, m = n + 1$ holiga umumlashtirildi [4, 5]. Shuningdek, $s = 2, m = 4$ holati I. Allakov va B. X. Abrayev ishlarida [6, 7], $s = 3, m = 5$ holi esa B. X. Erdonov tomonidan [8] atroflicha o'rganildi.

Ushbu ishda biz to'rtta tenglama va oltita tub noma'lumli ($s = 4, m = 6$) quyidagi chiziqli tenglamalar sistemasini qaraymiz:

$$b_i = a_{i1}p_1 + a_{i2}p_2 + a_{i3}p_3 + a_{i4}p_4 + a_{i5}p_5 + a_{i6}p_6, \quad i = 1, 2, 3, 4. \tag{2}$$

Yechimga ega bo'lish shartlari va asosiy tushunchalar. (2) sistemaning tub sonlarda yechimga ega bo'lishi quyidagi ikki zaruriy shartning bajarilishiga bog'liq:

a) Kongruent yechimga ega bo'lishlik sharti: Ixtiyoriy p tub son uchun

$$\sum_{j=1}^{6} a_{ij} l_j \equiv b_i \pmod{p}, \quad i = 1, 2, 3, 4$$

taqqoslamalar sistemasini qanoatlantiruvchi $1 \leq l_j \leq p-1$ butun sonlar majmuasi mavjud;

b) Musbat yechimga ega bo'lishlik sharti: Shunday musbat haqiqiy $y_1, \ldots, y_6 > 0$ sonlari mavjudki,

$$\sum_{j=1}^{6} a_{ij} y_j = b_i, \quad i = 1, 2, 3, 4.$$

Ushbu a) va b) shartlarni qanoatlantiruvchi $\vec{b} = (b_1, b_2, b_3, b_4)$, $1 \le b_i \le X$ vektorlar to‘plamini $W_{4,6}(X)$ orqali belgilaymiz. $B = \max\{4|a_{ij}|\}$ $(i = 1, \dots, 4;\ j = 1, \dots, 6)$ bo‘lsin. $A = (a_{ij})_{4\times 6}$ matritsaning ixtiyoriy to‘rtta ustunidan tuzilgan 4×4 tartibli minorlarini $\Delta_{j_1j_2j_3j_4} = \det(a_{i,j_k})_{i,k=1}^{4}$ deb belgilaymiz $(1 \le j_1 < j_2 < j_3 < j_4 \le 6)$. Bunday minorlar soni $\binom{6}{4} = 15$ tani tashkil etadi. Trivial bo‘lmagan holni ta’minlash uchun quyidagi shartlar bajarilgan deb hisoblaymiz:

$$\prod_{1\le j_1<j_2<j_3<j_4\le 6} \Delta_{j_1j_2j_3j_4} \neq 0, \quad \text{EKUB}\left(\{\Delta_{j_1j_2j_3j_4}\}\right) = 1. \tag{3}$$

Maqolaning asosiy natijasi $W_{4,6}(X)$ to‘plamining quvvati uchun quyidan effektiv baho olishdan iborat.

Teorema. *Agar (3) shart bajarilsa va biror haqiqiy musbat $y_1, \dots, y_6 > 0$ sonlari uchun $\sum_{j=1}^{6} a_{ij}y_j > 0$ $(i = 1, 2, 3, 4)$ tengsizliklar o‘rinli bo‘lsa, u holda yetarlicha katta X lar uchun quyidagi baho o‘rinli:*

$$\operatorname{card} W_{4,6}(X) \gg X^4 B^{-c} (\ln\ln B)^{-1}, \tag{4}$$

bu yerda $c > 0$ — (2) sistemaning koeffitsiyentlar matritsasi tuzilishiga bog‘liq bo‘lgan effektiv hisoblanuvchi o‘zgarmas son.

Isbot sxemasi. Ixtiyoriy butun $q \ge 1$ soni uchun $\sum_{j=1}^{6} a_{ij}l_j \equiv b_i \pmod q$, $(l_j, q) = 1$ taqqoslamalar sistemasining yechimlari sonini $N(q)$ bilan belgilaymiz.

1-lemma. *$p > 5$ tub son bo‘lsin. $N(p) = 0$ bo‘lishi uchun shunday bir $k \in \{1, \dots, 6\}$ indeksi mavjud bo‘lib, $\{j_1, j_2, j_3, j_4\} \subset \{1, 2, 3, 4, 5, 6, b\} \setminus \{k\}$ to‘rtliklar uchun $\Delta_{j_1j_2j_3j_4} \equiv 0 \pmod p$ munosabatning bajarilishi zarur va yetarlidir (bu yerda b indeksi mos ustun ozod hadlar $\vec{b}$ ustuniga almashtirilganini bildiradi).*

Ko‘p o‘lchovli Z_p maydoni ustida yotuvchi ko‘phadlarning nolga aylanishi tahlil qilinib, $N(p) = 0$ bo‘ladigan "yomon"tub sonlar soni chegaralanganligi ko‘rsatiladi. Kramer qoidasi yordamida $y_1, \dots, y_4$ o‘zgaruvchilarni ajratib, to‘rt o‘lchovli musbat konus ichidagi butun nuqtalar soni $z_i \equiv b_i \pmod m$ $(m \ll B^{60})$ arifmetik progressiyalar bo‘yicha yig‘iladi va bu $N_1 \gg X^4 m^{-4} B^{-c_1}$ miqdorni beradi. Buni taqqoslamalar yechimlarining zichligi bilan birlashtirib, (4) bahoga ega bo‘lamiz.

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The max-plus representation of the Shilkret integral

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Let X be a compact Hausdorff space and $\mathfrak{B}(X)$ the family of Borel subsets of X. We denote $\overline{\mathbb{R}}_+ = [0, +\infty) \cup \{+\infty\} = [0, +\infty]$. The symbol $\mathfrak{A}$ denotes a directed set.

Definition 1. [1] A set function $\mu\colon \mathfrak{B}(X) \to \overline{\mathbb{R}}_+$ is said to be an idempotent measure on X if the following conditions hold

1) $\mu(\varnothing) = 0$;

2) $\mu(A \cup B) = \max\{\mu(A), \mu(B)\}$ for any $A, B \in \mathfrak{B}(X)$;

3) $\mu\left(\bigcup\limits_{\alpha\in\mathfrak{A}} A_\alpha\right) = \sup\limits_{\alpha\in\mathfrak{A}}\{\mu(A_\alpha)\}$ for every increasing net $\{A_\alpha\colon \alpha \in \mathfrak{A}\} \subset \mathfrak{B}(X)$ such that $\bigcup\limits_{\alpha\in\mathfrak{A}} A_\alpha \in \mathfrak{B}(X)$.

The set of all idempotent measure on X we denote by $IM(X)$.

If $\mu(X) = 1$, the idempotent measure μ is called an idempotent probability measure on X. We denote $I(X) = I_{\mathfrak{B}}(X) = \{\mu \in IM(X)\colon \mu(X) = 1\}$.

Let (X, μ) be an idempotent measure space such that $\mu(X) < \infty$. We adopt the convention that $\infty \cdot 0 = 0$.

Definition 2. [2], [3] For a function $f\colon X \to \overline{\mathbb{R}}_+$ we define the idempotent integral of f with respect to μ by

$$I_\mu(f) = \int\limits_X^{\oplus} f d\mu = \sup_{t\in\overline{\mathbb{R}}_+}\{t \cdot \mu\{x \in X\colon f(x) \geq t\}\}.$$

Furthermore, define the set function $\nu\colon \mathfrak{B}(X) \to \overline{\mathbb{R}}_{\max}$ by

$$\nu(A) = \ln\big(\mu(A)\big), \qquad A \in \mathfrak{B}(X).$$

Lemma 1. If μ is an idempotent measure on X, then ν is also an idempotent measure on X.

Theorem 1. Let $g\colon X \to \mathbb{R}_{\max}$ and let $f\colon X \to \overline{\mathbb{R}}_+$ be defined by

$$f(x) = e^{g(x)}.$$

Then we have the following equality

$$\ln I_\mu(f) = I_\nu(g).$$

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MAX-PLUS ALGEBRADA TROPIK KVADRAT KO‘PHADLARNI KO‘PAYTUVCHILARGA AJRATISH VA ILDIZLARI TUZILISHI

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Ushbu ishda tropik algebra sharoitida aniqlangan tropik ko‘phadlarning asosiy xossalari, ildiz tushunchasi va ko‘paytuvchchilarga ajratish masalasi o‘rganilgan. Tropik yarim halqa ustida Algebraning fundamental teoremasining analogi keltirilib, tropik kvadrat ko‘phadlarning ikki hol bo‘yicha ko‘paytuvchilarga ajralishi isbotlangan. Shuningdek, ayrim misollar yordamida tropik ko‘phadlarning algebraik xususiyatlari tahlil qilingan. Tropik algebra - zamonaviy matematikaning istiqbolli yo‘nalishlaridan biri bo‘lib, u max-plus yoki min-plus algebraik amallariga asoslanadi. Tropik algebra va tropik geometriya optimallashtirish masalalari, diskret matematika, nazorat nazariyasi hamda matematik fizikaning turli sohalarida keng qo‘llaniladi. Max-plus algebra amallar quyidagi ko‘rinishda aniqlanadi:

$$a \oplus b = \max(a, b), \qquad a \odot b = a + b$$

Bu yerda neytral elementlar mos ravishda $-\infty$ va 0 dan iborat bo‘ladi. Shu algebraik strukturada aniqlangan ko‘phadlar tropik ko‘phadlar deyiladi. Tropik ko‘phadlar klassik algebraik ko‘phadlardan farqli ravishda bo‘lakli chiziqli va qavariq funksiyalarni ifodalaydi.

Lemma. Aytaylik, P- $T = \mathbb{R} \setminus \{-\infty\}$ ustida aniqlangan n o‘zgaruvchili tropik ko‘phad bo‘lsin. U holda P uzluksiz, chekli sondagi bo‘laklardan iborat bo‘lgan bo‘lakli chiziqli va qavariq funksiyadir.

Biz tropik ildiz tushunchasini quyidagicha aniqlaymiz.$T = \mathbb{R} \setminus \{-\infty\}$ element P tropik ko‘phadning ildizi deyiladi, agar shunday tropik ko‘phad Q mavjud bo‘lsaki, barcha $x \in T$ uchun:

$$P(x) = (x \oplus \lambda) \odot Q(x)$$

tenglik bajariladi.

Klassik algebraik geometriyada ildizlar ko‘phadning nolga teng bo‘ladigan qiymatlari sifatida aniqlanadi. Biroq tropik holatda bu ta‘rif to‘g‘ridan-to‘g‘ri mos kelmaydi, chunki bu strukturada oddiy ayirish amali mavjud emas. Shuning uchun ildizlar faktorizatsiya orqali aniqlanadi, bu esa tropik algebra ichida tabiiy va foydali tushunchadir. Tropik ko‘phadlar va ularning ildizlari tushunchasini kiritgach, tropik kvadrat uchhad uchun algebraning fundamental teoremasini kiritamiz.

Teorema. Umumiy ko‘rinishdagi ushbu:

$$P(x) = c_2 \odot x^2 \oplus c_1 \odot x \oplus c_0, \quad c_2 \neq \infty$$

tropik kvadrat ko‘phad uchun quyidagi holatlar o‘rinli bo‘ladi.

1. Agar $c_1 - c_2 > c_0 - c_1$ bo‘lsa, u holda

$$P(x) = c_2 \odot (x \oplus (c_1 - c_2)) \odot (x \oplus (c_0 - c_1))$$

tenglik o‘rinli bo‘ladi.

2. Agar $c_1 - c_2 \leq c_0 - c_1$ bo‘lsa, u holda

$$P(x) = c_2 \odot \left(x \oplus \frac{c_0 - c_2}{2}\right)^2$$

tenglik o‘rinli bo‘ladi.

Isbot. 1- holatni isbotlash uchun avval ichki ko‘paytmani hisoblaymiz:

$$(x \oplus (c_1 - c_2)) \odot (x \oplus (c_0 - c_1)) =$$
$$= (x \odot x \oplus (c_1 - c_2) \odot x \oplus x \odot (c_0 - c_1) \oplus (c_1 - c_2) \odot (c_0 - c_1)) =$$
$$= x^2 \oplus x \odot (c_1 - c_2) \oplus (c_0 - c_2)$$

Hosil bo‘lgan ifodani c_2 ga ko‘paytiramiz va natijada:

$$c_2 \odot \big(x^2 \oplus x \odot (c_1 - c_2) \oplus (c_0 - c_2)\big) = c_2 \odot x^2 \oplus c_1 \odot x \oplus c_0 = P(x)$$

1-holatni isbotladik.

Endi 2-holatni, ya’ni $c_1 - c_2 \leq c_0 - c_1$ bo‘lsa, quyidagini isbotlaymiz:

$$P(x) = c_2 \odot \left(x \oplus \frac{c_0 - c_2}{2}\right)^2$$

Quyidagi bir o‘zgaruvchili tropik kvadrat ko‘phadlarni ko‘paytuvchilarga ajratishni qaraylik:

1) $P(x) = 3 \odot x^{\odot 2} \oplus 1 \odot x \oplus 1$ 2) $P(x) = x^{\odot 2} \oplus 4 \odot x \oplus 4$

1) $P(x) = 3 \odot x^{\odot 2} \oplus 1 \odot x \oplus 1$ ushbu tropik kvadrat ko‘phadning koeffitsiyentlari uchun $1 - 3 < 1 - 1$ munosabat o‘rinli. Demak, 2-holatga ko‘ra bu kvadrat ko‘phad uchun

$$P(x) = c_2 \odot \left(x \oplus \frac{c_0 - c_2}{2}\right)^{\odot 2}$$

formula o‘rinli bo‘lib, quyidagi ko‘rinishga keladi:

$$P(x) = 3 \odot x^{\odot 2} \oplus 1 \odot x \oplus 1 = 3 \odot \left(x \oplus \frac{-2}{2}\right)^{\odot 2} = 3 \odot (x \oplus -1)^{\odot 2}$$

2) $P(x) = x^{\odot 2} \oplus 4 \odot x \oplus 4$, $c_1 - c_2 = 3$, $c_0 - c_1 = 0$ ekanligidan 1-holatga ko‘ra ko‘phadni quyidagicha ko‘paytuvchiga ajratish mumkin:

$$P(x) = x^{\odot 2} \oplus 4 \odot x \oplus 4 = 1 \odot (x \oplus 3) \odot (x \oplus 0)$$

E’tiborga loyiq jihat shundaki, yuqoridagi ko‘paytuvchiga ajratish natijalari koeffitsiyentlar cheksizlikka teng bo‘lgan chegara holatlarda ham o‘z kuchini saqlab qoladi. Umuman olganda, har qanday tropik ko‘phadni shunday ko‘rinishga keltirish mumkinki, uning ildizlari koeffitsiyentlar orasidagi farqlarning umumlashgan shakli orqali aniqlanadi. Bu hodisa klassik algebraik ko‘phadlardan muhim farq qiladi, chunki tropik algebra sharoitida turli koeffitsiyentlarga ega bo‘lgan ko‘phadlar bir xil funksiyani ifodalashi mumkin.

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CONFIGURATION-SPACE MODELS FOR EXACT-SUPPORT STRATA OF IDEMPOTENT PROBABILITY MEASURES

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Idempotent mathematics is based on the max-plus semiring

$$\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}, \qquad a \oplus b = \max\{a, b\}, \qquad a \odot b = a + b.$$

Let K be a compact Hausdorff space. An idempotent probability measure on K is a functional $\mu : C(K) \to \mathbb{R}$ satisfying

$$\mu(c_K) = c, \qquad \mu(c \odot \varphi) = c + \mu(\varphi), \qquad \mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi).$$

The space $I(K)$ of all idempotent probability measures is endowed with the topology of pointwise convergence. The support $\operatorname{supp}\mu$ is the smallest closed subset of K carrying μ [3].

Let X be a nonempty Tychonoff space and let βX be its Stone–Čech compactification. Put

$$I_\beta(X) = \{\mu \in I(\beta X) : \operatorname{supp}\mu \subset X\}$$

and, for $n \geq 1$,

$$I_\beta^{(n)}(X) = \{\mu \in I_\beta(X) : |\operatorname{supp}\mu| = n\}.$$

We describe the exact-support stratum $I_\beta^{(n)}(X)$ in terms of configuration spaces.

Define

$$\operatorname{Conf}_n(X) = \{(x_1, \ldots, x_n) \in X^n : x_i \neq x_j \text{ whenever } i \neq j\},$$

and

$$B_n(X) = \operatorname{Conf}_n(X)/S_n.$$

The symmetric group S_n acts freely on $\operatorname{Conf}_n(X)$ by permutations of coordinates, and the quotient map

$$\operatorname{Conf}_n(X) \longrightarrow B_n(X)$$

is a principal S_n-bundle [1].

Let

$$V_n = \mathbb{R}^n/\mathbb{R}\mathbf{1}, \qquad \mathbf{1} = (1, \ldots, 1),$$

with the natural action of S_n. The normalization

$$[a_1, \dots, a_n] \longmapsto \left(a_1 - \max_j a_j, \dots, a_n - \max_j a_j \right)$$

identifies V_n equivariantly with the space of finite coefficient vectors $(\lambda_1, \dots, \lambda_n)$ satisfying $\max_i \lambda_i = 0$.

Theorem 1. The map

$$\Theta_{n,X} : \mathrm{Conf}_n(X) \times_{S_n} V_n \longrightarrow I_\beta^{(n)}(X),$$

defined by

$$\Theta_{n,X}\big([(x_1, \dots, x_n), [a_1, \dots, a_n]]\big) = \bigoplus_{i=1}^{n} \left(a_i - \max_{1 \le j \le n} a_j \right) \odot \delta_{x_i}, \tag{1}$$

is a homeomorphism. Hence $I_\beta^{(n)}(X)$ is the total space of a real vector bundle of rank $n-1$ over $B_n(X)$.

Define the equal-weight subspace

$$E_n(X) = \{\delta_{x_1} \oplus \cdots \oplus \delta_{x_n} : (x_1, \dots, x_n) \in \mathrm{Conf}_n(X)\}.$$

It is canonically homeomorphic to $B_n(X)$ and, under Theorem 1, corresponds to the zero section of the associated vector bundle.

Theorem 2. The subspace $E_n(X)$ is a strong deformation retract of $I_\beta^{(n)}(X)$. In particular,

$$I_\beta^{(n)}(X) \simeq B_n(X).$$

Consequently, the coefficient directions do not change the homotopy type of an exact-support stratum. In particular,

$$I_\beta^{(2)}(X) \cong \mathrm{Conf}_2(X) \times_{\mathbb{Z}_2} \mathbb{R},$$

where the involution acts by

$$(x, y, s) \longmapsto (y, x, -s).$$

Thus the two-point stratum is the real line bundle associated with the double covering

$$\mathrm{Conf}_2(X) \longrightarrow B_2(X).$$

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THE FOUR-STATE CHUI-WEEKS MODEL ON A CAYLEY TREE OF ORDER THREE: TRANSLATION-INVARIANT GROUND STATES

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The Chui-Weeks model on the Cayley tree (see [2]) is defined by the following Hamiltonian

$$H(\sigma) = J \sum_{\langle x,y\rangle \in L} \mid \sigma(x) - \sigma(y) \mid + \alpha \sum_{x \in V} \delta_{\sigma(x),0}, \tag{1}$$

where $J, \alpha \in \mathbb{R}$, α is an external field, and $\sigma \in \Phi^V$, $\Phi = \{0, 1, 2, 3\}$, as well as $\delta_{i,j}$ denotes the Kronecker delta, that is, $\delta_{i,j} = 1$ if $i = j$ and 0 otherwise.

Recall that model (1) coincides with the SOS model under the condition $\alpha = 0$ (see, e.g., [1]).

Let M be the set of all unit balls with vertices in V and let $S_1(x)$ be the set of all nearest-neighbor vertices of $x \in V$. We denote the restriction of a configuration σ to the ball $b \in M$ by σ_b. The energy of configuration σ_b on b is defined by the formula

$$U(\sigma_b) = \frac{J}{2} \sum_{x \in S_1(c_b)} \mid \sigma(x) - \sigma(c_b) \mid + \frac{\alpha}{k+2} \sum_{x \in b} \delta_{\sigma(x),0}.$$

where $J = (J, \alpha) \in \mathbb{R}^2$ and c_b is the center of the unit ball b.

The Hamiltonian (1) can be written as

$$H(\sigma) = \sum_{b \in M} U(\sigma_b).$$

We proved the following lemma.

Lemma. *Let $k = 3$. Then for each configuration φ_b, we have the following*

$$U(\varphi_b) \in \{U_i : i = 1, 2, ..., 46\},$$

where

$$U_1 = 0;\ U_2 = \alpha;\ U_3 = \frac{J}{2};\ U_4 = \frac{J}{2} + \frac{\alpha}{5};\ U_5 = \frac{J}{2} + \frac{4\alpha}{5};\ U_6 = J;\ U_7 = J + \frac{\alpha}{5};$$

$$U_8 = J + \frac{2\alpha}{5};\ U_9 = J + \frac{3\alpha}{5};\ U_{10} = J + \frac{4\alpha}{5};\ U_{11} = \frac{3J}{2};\ U_{12} = \frac{3J}{2} + \frac{\alpha}{5};\ U_{13} = \frac{3J}{2} + \frac{2\alpha}{5};$$

$$U_{14} = \frac{3J}{2} + \frac{3\alpha}{5};\ U_{15} = \frac{3J}{2} + \frac{4\alpha}{5};\ U_{16} = 2J;\ U_{17} = 2J + \frac{\alpha}{5};\ U_{18} = 2J + \frac{2\alpha}{5};\ U_{19} = 2J + \frac{3\alpha}{5};$$

$$U_{20} = 2J + \frac{4\alpha}{5};\ U_{21} = \frac{5J}{2};\ U_{22} = \frac{5J}{2} + \frac{\alpha}{5};\ U_{23} = \frac{5J}{2} + \frac{2\alpha}{5};\ U_{24} = \frac{5J}{2} + \frac{3\alpha}{5};$$

$$U_{25} = 3J;\ U_{26} = 3J + \frac{\alpha}{5};\ U_{27} = 3J + \frac{2\alpha}{5};\ U_{28} = 3J + \frac{3\alpha}{5};\ U_{29} = \frac{7J}{2};\ U_{30} = \frac{7J}{2} + \frac{\alpha}{5};$$

$$U_{31} = \frac{7J}{2} + \frac{2\alpha}{5};\ U_{32} = \frac{7J}{2} + \frac{3\alpha}{5};\ U_{33} = 4J;\ U_{34} = 4J + \frac{\alpha}{5};\ U_{35} = 4J + \frac{2\alpha}{5};$$

$$U_{36} = 4J + \frac{4\alpha}{5};\ U_{37} = \frac{9J}{2} + \frac{\alpha}{5};\ U_{38} = \frac{9J}{2} + \frac{2\alpha}{5};\ U_{39} = \frac{9J}{2} + \frac{3\alpha}{5};\ U_{40} = 5J + \frac{\alpha}{5};$$

$$U_{41} = 5J + \frac{2\alpha}{5};\ U_{42} = 5J + \frac{3\alpha}{5};\ U_{43} = \frac{11J}{2} + \frac{\alpha}{5};\ U_{44} = \frac{11J}{2} + \frac{3\alpha}{5};\ U_{45} = 6J + \frac{\alpha}{5};\ U_{46} = 6J + \frac{4\alpha}{5}.$$

Definition. A configuration φ is called a ground state for the Hamiltonian (1), if

$$U(\varphi_b) = \min\{U_i : i = 1, 2, ..., 46\}$$

for any $b \in M$.

We denote $A_\xi = \{(J,\alpha) \in \mathbb{R}^2 : U_\xi = \min\{U_i : i = 1, 2, ..., 46\}\}$.

Calculations show that:

$$A_1 = \{(J,\alpha) \in \mathbb{R}^2 : J \geq 0,\ \alpha \geq 0\};\ A_2 = \{(J,\alpha) \in \mathbb{R}^2 : 30J \geq \alpha,\ \alpha \leq 0\};$$

$$A_3 = A_6 = A_{11} = A_{16} = A_{21} = A_{25} = A_{29} = \{(J,\alpha) \in \mathbb{R}^2 : J = 0, \alpha \geq 0\};\ A_4 = A_5 = A_7 = A_8 = A_9$$

$$= A_{10} = A_{12} = A_{13} = A_{14} = A_{15} = A_{17} = A_{18} = A_{19} = A_{20} = A_{22} = A_{23} = A_{24} = A_{26} = A_{27} = A_{28}$$

$$= A_{30} = A_{31} = A_{32} = A_{34} = A_{35} = A_{36} = ... = A_{43} = A_{44} = \{(J,\alpha) \in \mathbb{R}^2 : J = 0,\ \alpha = 0\};$$

$$A_{33} = \{(J,\alpha) \in \mathbb{R}^2 : -\alpha \leq 10J \leq 0\};\ A_{45} = \{(J,\alpha) \in \mathbb{R}^2 : 0 \leq \alpha \leq -10J\};$$

$$A_{46} = \{(J,\alpha) \in \mathbb{R}^2 : 30J \leq \alpha \leq 0\}$$

and $\bigcup_{i=1}^{46} A_i = \mathbb{R}^2$.

We study all translation-invariant ground states for the Chui-Weeks model on the Cayley tree of order three. The following theorem describes all translation-invariant ground states for the four-state Chui-Weeks model.

Theorem. *Let $k = 3$. Then, for the four-state Chui-Weeks model, the following assertions hold:*
(i) The configurations $\sigma(x) = i$, $i \in \{1,2,3\}$, for all $x \in V$ are translation-invariant ground states iff $(J,\alpha) \in A_1$;
(ii) The configuration $\sigma(x) = 0$ for all $x \in V$ is the translation-invariant ground state iff $(J,\alpha) \in A_2$;
(iii) If $(J,\alpha) \in \mathbb{R}^2 \setminus (A_1 \cup A_2)$, then there is no translation-invariant ground state.

Proof. Let $k = 3$.

(i) We consider the configuration $\sigma(x) = i$, $i \in \{1,2,3\}$, for all $x \in V$. For any $b \in M$ by Lemma we have $U(\sigma_b) = U_1 = 0$. Thus the configuration $\sigma(x) = i, i \in \{1,2,3\}$, for all $x \in V$ is ground state iff $(J,\alpha) \in A_1$;

(ii) We consider the configuration $\sigma(x) = 0$ for all $x \in V$. For any $b \in M$ by Lemma we have $U(\sigma_b) = U_2 = \alpha$. Thus the configuration $\sigma(x) = 0$ for all $x \in V$ is ground state iff $(J,\alpha) \in A_2$;

(iii) It is obvious. Theorem is proved.

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Surface recovery using the operator Ω

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Let there be given an affine space A_3 with the coordinate system $Oxyz$. Consider $\overrightarrow{X}\{x_1, y_1, z_1\}$ and $\overrightarrow{Y}\{x_2, y_2, z_2\}$ vectors in the A_3.

Definition 1. If the scalar product of the vectors $\overrightarrow{X}$ and $\overrightarrow{Y}$ is determined by the following:

$$\begin{cases} (\overrightarrow{X}, \overrightarrow{Y})_1 = x_1x_2 + y_1y_2, \ if \ \ x_1x_2 + y_1y_2 \neq 0 \\ (\overrightarrow{X}, \overrightarrow{Y})_2 = z_1z_2, \ \ if, \ x_1x_2 + y_1y_2 = 0 \end{cases} \tag{1}$$

then A_3 is called an isotropic space R_3^2 [1]. There be given a regular surface F by the explicit form:

$$z = f(x, y) \tag{2}$$

The total curvature of this surface is [2]:

$$K = f_{xx}f_{yy} - f_{xy}^2$$

Let us consider the following operator [3]:

$$\Omega = f_{xxx}f_{yyy} - f_{xxy}f_{xyy}$$

Here, above expressions are the third-order partial derivatives of the $z = f(x, y)$.

The operator presented above is a third-order nonlinear differential operator that plays a significant role in mathematical physics, the geometry of surfaces, and dispersionless integrable systems.

The geometric interpretation of the expression Ω is associated with the spatial variation (gradient) of curvature. For the surface $z = f(x, y)$, the second-order partial derivatives fxx and f_{yy} are proportional to the normal curvatures of the surface along the corresponding coordinate directions. Therefore, the operator Ω characterizes the anisotropic interaction of the rates of curvature variation (curvature gradients) in the principal curvature directions, in other words, it measures the intensity of third-order twisting and deformation of the surface.

The translation surface is given by the following vector equation:

$$\overrightarrow{r}(x, y) = x\overrightarrow{i} + y\overrightarrow{j} + (f(x) + g(y))\overrightarrow{k}, \tag{3}$$

here, $f(x), g(y)$ are smooth functions.

Let there be given the main theorem:

Theorem 1. Let the translation surface is parameterized by the (3), then three cases are valid: If $\Omega = 0$, then the surface equation is:

$$\overrightarrow{r}(x, y) = (x, y, \frac{c_1}{2}x^2 + \frac{d_1}{2}y^2 + c_2x + c_3 + d_2y + d_3)$$

$\Omega = c$,

$$\overrightarrow{r}(x, y) = (x, y, \frac{\tau x^3}{6} + \frac{cy^3}{6\tau} + \frac{c_1}{2}x^2 + \frac{d_1}{2}y^2 + c_2x + c_3 + d_2y + d_3)$$

$\Omega = \varphi(x)\psi(y)$

$$\overrightarrow{r}(x, y) = (x, y, \tau \int [\int [\int \varphi(x)dx]dx]dx + \frac{1}{\tau}\int [\int [\int \psi(y)dy]dy]dy + \frac{c_1}{2}x^2 + \frac{d_1}{2}y^2 + c_2x + c_3 + d_2y + d_3)$$

Here, $\tau, \ c_i, \ d_i - const.$

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Fermionli Fok fazodagi uchinchi tartibli operatorli matritsa muhim spektrining komponentalari

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Dastlab, $\mathbb{T}^{\mathrm{d}} := (-\pi;\pi]^{\mathrm{d}}$ orqali d o‘lchamli torni, $\mathcal{H}_0 := \mathbb{C}$ orqali bir o‘lchamli kompleks sonlar fazosini, $\mathcal{H}_1 := L_2(\mathbb{T}^{\mathrm{d}})$ orqali $\mathbb{T}^{\mathrm{d}}$ to‘plamda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini, $\mathcal{H}_2 := L_2^{\mathrm{as}}((\mathbb{T}^{\mathrm{d}})^2)$ orqali $(\mathbb{T}^{\mathrm{d}})^2$ to‘plamda aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) antisimmetrik funksiyalarning Hilbert fazosini belgilaymiz.

Umumiy holda $n \geq 3$ uchun $(\mathbb{T}^{\mathrm{d}})^n$ to‘plamda aniqlangan kvadrati bilan integrallanuvchi hamda ixtiyoriy ikkita argumentiga nisbatan antisimmetrik bo‘lgan funksiyalarning Hilbert fazosini $\mathcal{H}_n := L_2^{\mathrm{as}}((\mathbb{T}^{\mathrm{d}})^n)$ orqali belgilaymiz.

Ushbu fazolarning chekli hamda cheksiz to‘g‘ri yig‘indilari uchun quyidagi belgilashlarni kiritamiz:

$$\mathcal{F}_{\mathrm{as}}\left(L_2(\mathbb{T}^{\mathrm{d}})\right) := \bigoplus_{i=0}^{+\infty}\mathcal{H}_i; \quad \mathcal{F}_{\mathrm{as}}^{(1)}\left(L_2(\mathbb{T}^{\mathrm{d}})\right) := \mathcal{H}_0 \oplus \mathcal{H}_1;$$

$$\mathcal{F}_{\mathrm{as}}^{(2)}\left(L_2(\mathbb{T}^{\mathrm{d}})\right) := \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2; \quad ...; \quad \mathcal{F}_{\mathrm{as}}^{(\mathrm{m})}\left(L_2(\mathbb{T}^{\mathrm{d}})\right) := \bigoplus_{i=0}^{\mathrm{m}}\mathcal{H}_i.$$

Odatda, $\mathcal{F}_{\mathrm{as}}\left(L_2(\mathbb{T}^{\mathrm{d}})\right)$ Hilbert fazosiga $L_2(\mathbb{T}^{\mathrm{d}})$ Hilbert fazosi yordamida qurilgan fermionli Fok fazo, $\mathcal{F}_{\mathrm{as}}^{(\mathrm{m})}\left(L_2(\mathbb{T}^{\mathrm{d}})\right), (\mathrm{m} = 1, 2..)$ fazoga esa $\mathcal{F}_{\mathrm{as}}\left(L_2(\mathbb{T}^{\mathrm{d}})\right)$ fazoning $(\mathrm{m}+1)$ zarrachali qirqilgan qism fazosi deyiladi.

$\mathcal{F}_{\mathrm{as}}^{(2)}\left(L_2(\mathbb{T}^{\mathrm{d}})\right)$ Hilbert fazosining elementlari $f = (f_0, f_1, f_2)$ ko‘rinishidagi vektorlardan iborat bo‘lib, bu yerda $f_\alpha \in \mathcal{H}_\alpha$, $\alpha = 0, 1, 2$.

$\mathbb{C}^2 \otimes \mathcal{F}_{\mathrm{as}}^{(2)}(L_2(\mathbb{T}^{\mathrm{d}}))$ fazoning elementlari esa $f = \{(f_0^{(\mathrm{s})}, f_1^{(\mathrm{s})}, f_2^{(\mathrm{s})}),\ s = \pm\}$ ko‘rinishga ega. Ushbu fazoning ixtiyoriy ikkita

$$f = \{(f_0^{(\mathrm{s})}, f_1^{(\mathrm{s})}, f_2^{(\mathrm{s})}),\ \mathrm{s} = \pm\} \quad \text{va} \quad g = \{(g_0^{(\mathrm{s})}, g_1^{(\mathrm{s})}, g_2^{(\mathrm{s})}),\ \mathrm{s} = \pm\},$$

elementlarining skalyar ko‘paytmasi uchun quyidagi tenglik o‘rinli:

$$(f,g) := \sum_{\mathrm{s}=\pm}\left(f_0^{(\mathrm{s})}\overline{g_0^{(\mathrm{s})}} + \int_{\mathbb{T}^{\mathrm{d}}} f_1^{(\mathrm{s})}(k_1)\overline{g_1^{(\mathrm{s})}(k_1)}\,dk_1 + \frac{1}{2}\int_{(\mathbb{T}^{\mathrm{d}})^2} f_2^{(\mathrm{s})}(k_1,k_2)\overline{g_2^{(\mathrm{s})}(k_1,k_2)}\,dk_1dk_2\right)$$

Mazkur ishda $\mathbb{C}^2 \otimes \mathcal{F}_{\mathrm{as}}^{(2)}(L_2(\mathbb{T}^{\mathrm{d}}))$ Hilbert fazoda aniqlangan

$$\mathcal{A} := \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{01}^* & A_{11} & A_{12} \\ 0 & A_{12}^* & A_{22} \end{pmatrix}$$

uch diagonalli uchinchi tartibli operatorli matritsani qaraymiz. $\mathcal{A}$ operatorli matritsaning A_{ij} elementlari quyidagicha ta'sir formulasiga ega:

$$A_{00}f_0^{(\mathrm{s})} = s\varepsilon f_0^{(\mathrm{s})}, \quad A_{01}f_1^{(\mathrm{s})} = \alpha\int_{\mathbb{T}^{\mathrm{d}}} v(t)f_1^{(-\mathrm{s})}(t)dt,$$
$$(A_{11}f_1^{(\mathrm{s})})(k_1) = (\mathrm{s}\varepsilon + w(k_1))f_1^{(\mathrm{s})}(k_1), \quad (A_{12}f_2^{(\mathrm{s})})(k_1) = \alpha\int_{\mathbb{T}^{\mathrm{d}}} v(t)f_2^{(-\mathrm{s})}(k_1,t)dt,$$
$$(A_{22}f_2^{(\mathrm{s})})(k_1,k_2) = (\mathrm{s}\varepsilon + w(k_1) + w(k_2))f_2^{(\mathrm{s})}(k_1,k_2),$$

bu yerda, $f = \{(f_0^{(\mathrm{s})}, f_1^{(\mathrm{s})}, f_2^{(\mathrm{s})}),\ \mathrm{s} = \pm\}$, ε tayinlangan haqiqiy musbat son, $w(\cdot)$ va $v(\cdot)$ funksiyalar $\mathbb{T}^{\mathrm{d}}$ torda aniqlangan haqiqiy qiymatli uzluksiz funksiyalar hamda $\alpha > 0$ ta'sirlashish parametri.

Parametrlarga qo'yilgan yuqoridagi shartlarda $\mathcal{A}$ operatorli matritsa $\mathbb{C}^2 \otimes \mathcal{F}_{\mathrm{as}}^{(2)}(L_2(\mathbb{T}^{\mathrm{d}}))$ fazoda chiziqli, chegaralangan va o'z-o'ziga qo'shma operatordir.

Qulaylik uchun $\sigma(\cdot)$, $\sigma_{\mathrm{ess}}(\cdot)$ va $\sigma_{\mathrm{disc}}(\cdot)$ orqali mos ravishda Hilbert fazosidagi chiziqli, chegaralangan va o'z-o'ziga qo'shma operatorning spektri, muhim spektri va diskret spektrini belgilaymiz.

$\mathcal{A}$ operatorli matritsaning spektral xossalarini tadqiq qilish maqsadida $\mathbb{C}^2 \otimes \mathcal{F}_{\mathrm{as}}^{(1)}(L_2(\mathbb{T}^{\mathrm{d}}))$ fazoda

$$\mathcal{A}_1 := \begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & A_{11} \end{pmatrix},$$

operatorli matritsani va $\mathcal{F}_{\mathrm{as}}^{(1)}(L_2(\mathbb{T}^{\mathrm{d}}))$ fazoda quyidagi diskret parametrga ega bo'lgan ikkinchi tartibli chiziqli, chegaralangan va o'z-o'ziga qo'shma operatorli matritsani qaraymiz:

$$\mathcal{A}_1^{(\mathrm{s})} := \begin{pmatrix} \widehat{A}_{00}^{(\mathrm{s})} & \widehat{A}_{01} \\ \widehat{A}_{01}^* & \widehat{A}_{11}^{(\mathrm{s})} \end{pmatrix}, \qquad \mathrm{s} = \pm.$$

$\mathcal{A}_1^{(\mathrm{s})}, \mathrm{s} = \pm$ operatorli matritsaning elementlari quyidagi tengliklar orqali aniqlanadi:

$$\widehat{A}_{00}^{(\mathrm{s})} f_0 = s\varepsilon f_0, \qquad \widehat{A}_{01}f_1 = \alpha\int_{\mathbb{T}^d} v(t)f_1(t)\,dt,$$

$$\left(\widehat{A}_{11}^{(\mathrm{s})} f_1\right)(k_1) = (-\mathrm{s}\varepsilon + w(k_1))\,f_1(k_1), \qquad (f_0, f_1) \in \mathcal{F}_{\mathrm{as}}^{(1)}(L_2(\mathbb{T}^d)).$$

Sodda mulohazalardan

$$\left(\widehat{A}_{01}^* f_0\right)(k_1) = \alpha v(k_1) f_0.$$

tenglikning o'rinli ekanligini ko'rsatish oson.

Klassik Weyl teoremasiga ko'ra [2], $\mathcal{A}_1^{(\mathrm{s})}$ operatorli matritsaning muhim spektri uchun

$$\sigma_{\mathrm{ess}}\left(\mathcal{A}_1^{(\mathrm{s})}\right) = [-\mathrm{s}\varepsilon + m;\ -\mathrm{s}\varepsilon + M]$$

tenglik o'rinli bo'lib, bu yerda

$$m := \min_{k_1 \in \mathbb{T}^d} w(k_1), \qquad M := \max_{k_1 \in \mathbb{T}^d} w(k_1).$$

Mazkur ishda

$$\int_{\mathbb{T}^{\mathrm{d}}} \frac{v^2(t)dt}{w(t) - m} = +\infty \quad \text{va} \quad \int_{\mathbb{T}^{\mathrm{d}}} \frac{v^2(t)dt}{M - w(t)} = +\infty \tag{1}$$

bo‘lgan hollarni tahlil qilamiz. Dastlab, (1) ifodaning har ikkala shartlarini qanoatlantiradigan parametrlar va funksiyalarga misol quramiz.

Agar d = 1, $v(k_1) = 1$ va $w(k_1) = 1 - \cos k_1$ bo‘lsa, u holda

$$m = \min_{k_1 \in \mathbb{T}} (1 - \cos k_1) = 0 \quad \text{va} \quad M = \max_{k_1 \in \mathbb{T}} \{1 - \cos k_1\} = 2$$

tengliklar o‘rinli bo‘ladi. Ushbu tanlangan shartlarda esa integrallar uchun

$$\int_{\mathbb{T}} \frac{dt}{1 - \cos t} = \int_{\mathbb{T}} \frac{dt}{2\sin^2(t/2)} = \frac{1}{2} \int_{\mathbb{T}} \frac{dt}{\sin^2(t/2)} = +\infty.$$

$$\int_{\mathbb{T}} \frac{dt}{2 - (1 - \cos t)} = \int_{\mathbb{T}} \frac{dt}{2\cos^2(t/2)} = +\infty$$

munosabatlar o‘rinli bo‘ladi.

Tadqiqot ishining asosiy natijasini bayon qilish uchun zarur belgilashlarni kiritamiz:

$$\sigma^{(\mathrm{s})} := \bigcup_{p \in \mathbb{T}^d} \left\{ w(p) + \sigma_{\mathrm{disc}} \left(\mathcal{A}_1^{(\mathrm{s})} \right) \right\},$$

$$\sigma_{\mathrm{ess}}(\mathcal{A}) := \bigcup_{s=\pm} \left(\sigma^{(\mathrm{s})} + [-s\varepsilon + 2m;\ -\mathrm{s}\varepsilon + 2M] \right),$$

$$\sigma_1^{(\mathrm{s})} := \sigma^{(\mathrm{s})} \cap (-\infty; -\varepsilon + 2m], \quad \sigma_2^{(\mathrm{s})} := \sigma^{(\mathrm{s})} \cap [\varepsilon + 2M; \infty),$$

$$E_{\min}^{(k,\mathrm{s})} := \min(\sigma_k^{(\mathrm{s})}), \quad E_{\max}^{(k,\mathrm{s})} := \min(\sigma_k^{(\mathrm{s})}), \quad k = 1, 2,$$

$$E_{\min} := \min\{E_{\min}^{(1,+)}, E_{\min}^{(1,-)}\}.$$

Asosiy natijalarni bayon qilamiz.

1-teorema. Faraz qilaylik,(1) ifodaning har ikkala shartlari ham o‘rinli bo‘lsin. U holda $\mathcal{A}_1$ operatorli matritsa $\alpha > 0$ ta’sirlashish parametrining barcha qiymatlarida sonlari uchun $-\varepsilon + m$ nuqtadan chapda hamda $\varepsilon + M$ nuqtadan o‘ngda bittadan xos qiymatga ega.

(1) ifodadagi integrallar yaqinlashuvchi bo‘lgan hol [1] maqolada batafsil tahlil qilingan.

2-teorema. Shunday $\mathrm{s} \in \{+, -\}$ topilib, $\mathcal{A}$ operatorli matritsaning muhim spektri quyidagi komponentalardan tashkil topadi:

$$\sigma_{\mathrm{ess}}(\mathcal{A}) = [E_{\min}^{(1,\mathrm{s})}; E_{\max}^{(1,\mathrm{s})}] \cup [\varepsilon + 2m; \varepsilon + 2M] \cup [\varepsilon + 2m; \varepsilon + 2M] \cup [E_{\min}^{(2,\mathrm{s})}; E_{\max}^{(2,\mathrm{s})}],$$

bu yerda $E_{\min}^{(1,\mathrm{s})} < -\varepsilon + 2m,\ E_{\max}^{(2,\mathrm{s})} > \varepsilon + 2M,\ \mathrm{s} = \pm.$

Shuni ta’kidlash joizki, (1) ifodada qatnashuvchi integrallarning yaqinlashuvchanligi yoki uzoqlashuvchanligiga bog‘liq holda, $\mathcal{A}$ operatorli matritsaning muhim spektri 1 tadan 6 tagacha komponentadan iborat bo‘lishini ta’minlovchi $\alpha > 0$ ta’sirlashish parametrining qiymatlarini aniqlash mumkin.

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On the existence of the eigenvalues of a family of Friedrichs models with two-dimensional perturbation

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As is well known, some current problems, in particular those in quantum mechanics, statistical mechanics, and hydrodynamics, can be reduced to studying the spectrum of the Friedrichs model with an n-dimensional perturbation. This note considers the case $n = 2$.

First, let's focus on the concepts used in the work. Let $\mathbb{T}^3$ be the three-dimensional torus and $L_2(\mathbb{T}^3)$ the Hilbert space of square-integrable (complex-valued) functions defined on $\mathbb{T}^3$. Consider the family of Friedrichs models $H_\mu(k)$, acting in the Hilbert space $L_2(\mathbb{T}^3)$ as

$$H_\mu(k) = H_0(k) - \mu_1 V_1 - \mu_2 V_2, \quad \mu = (\mu_1, \mu_2), \qquad \mu_1, \mu_2 > 0, \qquad k \in \mathbb{T}^3,$$

operators $H_0(k)$, $k \in \mathbb{T}^3$, and V_α, $\alpha = 1, 2$, are defined by

$$(H_0(k)f)(p) = \big(\varepsilon(p) + \varepsilon(k-p)\big) f(p),$$

$$\varepsilon(p) := \sum_{i=1}^{3} \left(1 - \cos(3p_i)\right), \qquad p = (p_1, p_2, p_3) \in \mathbb{T}^3,$$

and

$$(V_\alpha f)(p) = v_\alpha(p) \int_{\mathbb{T}^3} v_\alpha(t) f(t)\, dt, \qquad \alpha = 1, 2.$$

Here the functions $v_\alpha(\cdot)$, $\alpha = 1, 2$, are real-valued continuous function on $\mathbb{T}^3$.

It should be noted that the family of Friedrichs models $H_\mu(k)$ is a linear, bounded and self-adjoint operator in the Hilbert space $L_2(\mathbb{T}^3)$.

We set

$$m(k) := 2\sum_{i=1}^{3} \left(1 - \left|\cos\frac{3k_i}{2}\right|\right), \qquad M(k) := 2\sum_{i=1}^{3} \left(1 + \left|\cos\frac{3k_i}{2}\right|\right).$$

Using Weyl's theorem, it can be shown that the essential spectrum of the family of Friedrichs models $H_\mu(k)$ does not depend on the parameter μ, and the equality

$$\sigma_{\text{ess}}\big(H_\mu(k)\big) = [m(k), M(k)]$$

holds.

For each $k \in \mathbb{T}^3$ and for the numbers $\mu_1, \mu_2 > 0$, consider the function

$$\Delta_\mu(k; z) := \Delta_{\mu_1}^{(1)}(k; z)\Delta_{\mu_2}^{(2)}(k; z) - \mu_1\mu_2 I_{12}(k; z),$$

which is regular in the domain $\mathbb{C} \setminus [m(k), M(k)]$, where the functions $\Delta_{\mu_1}^{(1)}(\cdot)$, $\Delta_{\mu_2}^{(2)}(\cdot)$ and $I_{\alpha\beta}(\cdot)$ with $\alpha, \beta = 1, 2$ are defined in the domain $\mathbb{C} \setminus [m(k), M(k)]$ by

$$\Delta_{\mu_1}^{(1)}(k; z) := 1 - \mu_1 I_{11}(k; z), \qquad \Delta_{\mu_2}^{(2)}(k; z) := 1 - \mu_2 I_{22}(k; z),$$

and

$$I_{\alpha\beta}(k;z) := \int_{\mathbb{T}^3} \frac{v_\alpha(t)v_\beta(t)}{\varepsilon(t)+\varepsilon(k-t)-z}\,dt, \qquad \alpha,\beta = 1,2.$$

Usually, the function $\Delta_\mu(k;\cdot)$ is referred to as the Fredholm determinant corresponding to the family of Friedrichs models $H_\mu(k)$. This function plays an important role [1,2] in the study of the discrete spectrum of the Friedrichs model $H_\mu(k)$. Below, we present a result describing the relationship between the eigenvalues of the family of Friedrichs models $H_\mu(k)$ and the zeros of the Fredholm determinant $\Delta_\mu(k;\cdot)$.

Lemma 1. A number $z \in \mathbb{C}\setminus[m(k),M(k)]$ is an eigenvalue of the family of Friedrichs models $H_\mu(k)$ if and only if $\Delta_\mu(k;z)=0$.

The principal result of this work is formulated in the following theorem.

Teorema 1. If for some $\mu=(\mu_1,\mu_2)$ and $k\in\mathbb{T}^3$ the condition $\Delta_\mu(k;m(k))<0$ holds, then the Friedrichs model $H_\mu(k)$ has a unique eigenvalue $E_\mu(k)$ and $E_\mu(k)<m(k)$.

In conclusion, the results serve as an important basis for studying the discrete spectrum of Friedrichs models with two-dimensional discontinuities and can be applied to relevant problems in quantum mechanics and statistical mechanics.

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Hyperspace of the weak Π-complete spaces

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In the present paper under space we mean a topological T_1-space.

A finite sequence of subsets $M_0, \ldots, M_s$ of a set X is a chain connecting sets M_0 and M_s, if $M_{i-1}\cap M_i \neq \varnothing$ for $i=1,\ldots,s$. A collection ω of subsets of a set X is said to be connected if for any pair of sets M, $M^{'}\subset X$ there exists a chain in ω connecting the sets M and $M^{'}$. The maximal connected subcollections of ω are called components of ω [1].

A star-finite open cover of a spaces X is said to be a finite-component cover if the number of elements of each component is finite.

For a collection $\omega=\{O_\alpha : \alpha\in A\}$ of subsets of a space X we put

$$[\omega]=[\omega]_X=\{[O_\alpha]_X : \alpha\in A\}.$$

For a space X, its some subspace W and a point $x\in X\setminus W$ they say that an open cover λ of the space W pricks out the point x in X if $x\notin \cup[\lambda]_X$.

Definition 1. A Tychonoff space X is Π-complete if for every point $x\in bX\setminus X$ of an arbitrary perfect compactification bX there exists a finite component open cover ω of X with $Kp\,\omega=1$ and, pricking out x in bX (i. e. $x\notin\cup[\omega]_{bX}$) [3].

Definition 2. A space X is said to be weak Π-complete if for every point $x \in \beta X \setminus X$ there exists an open-closed cover ω of X which pricks out the point x in βX [3].

A closed subset of a weak Π-complete space is weak Π-complete.

Every Π-complete space is weak Π-complete. But the inverse is not true.

Example 1. The space $T(\omega_1) = \{\alpha : \alpha < \omega_1\}$ of all ordinal numbers less than the first uncountable ordinal ω_1 is weak Π-complete, but not Π-complete.

By $\exp X$ we denote a set of all nonempty closed subsets of a topological space X. A family of sets of the view

$$O\langle U_1, \ldots, U_n\rangle = \left\{ F \in \exp X \colon F \subset \bigcup_{i=1}^{n} U_n,\, F \cap U_1 \neq \varnothing,\, \ldots,\, F \cap U_n \neq \varnothing \right\}$$

forms a base of a topology in $\exp X$, where $U_1, \ldots, U_n$ are open nonempty sets in X. This topology is called the Vietoris topology. A space $\exp X$ equipped with the Vietoris topology is called hyperspace of X. For a compact Hausdorff space X its hyperspace $\exp X$ is also a compact Hausdorff space.

Let $f \colon X \to Y$ be continuous map of compact Hausdorff spaces, $F \in \exp X$. Putting

$$(\exp f)(F) = f(F)$$

we define a map $\exp f \colon \exp X \to \exp Y$. Since each closed subset of a compact Hausdorff space is compact set, and a continuous image of a compact set is also a compact set, for a continuous map f the map $\exp f$ is well defined. Otherwise, for a continuous map f the map $\exp f$ is also continuous. Really, it follows from the equality

$$(\exp f)^{-1} O\langle U_1, ..., U_m\rangle = O\langle f^{-1}(U_1), ..., f^{-1}(U_m)\rangle$$

what one can check directly. Note that if $f \colon X \to Y$ is an epimorphism, then $\exp f$ is also an epimorphism.

For a Tychonoff space X let βX become the Stone-Čech compact extension of X. Then we put

$$\exp_\beta X = \{F \in \exp \beta X : F \subset X\}.$$

It is clear, that $\exp_\beta X \subset \exp \beta X$. Consider the set $\exp_\beta X$ as a subspace of the space $\exp \beta X$. For a Tychonoff spaces X the space $\exp_\beta X$ is also a Tychonoff space with respect to the induced topology. Obtained topological space we call as a hyperspace of X.

For a continuous map $f \colon X \to Y$ of Tychonoff spaces we put

$$\exp_\beta f = (\exp \beta f)|_{\exp_\beta X},$$

where $\beta f \colon \beta X \to \beta Y$ is the Stone-Čech compact extension of f (it is unique) [2].

Theorem 1. For a Tychonoff space X its hyperspace $\exp_\beta X$ is weak Π-complete if and only if X is weak Π-complete.

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$\mathcal{J}-$ o‘z-o‘ziga qo‘shma umumlashgan Fridrixs modeli kvadratik sonli tasvirining bog‘langan komponentalari

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Kreyn tipidagi fazolarda aniqlangan operatorlarning spektral xossalarini tadqiq etish zamonaviy funksional analiz va operatorlar nazariyasining muhim yo‘nalishlaridan birihisoblanadi. Bunday fazolarda ichki ko‘paytmaning ishorasi aniqlanmaganligi sababli operatorlarning spektral tuzilishi Hilbert fazosidagi holatdan sezilarli darajada farq qiladi. Xususan, $\mathcal{J}-$ o‘z-o‘ziga qo‘shma operator larning spektri umumiy holda haqiqiy bo‘lishi shart emas. Shu sababli bunday operatorlarning spektral xossalarini o‘rganishda kvadratik sonli tasvir muhim vositalardan biri hisoblanadi.

Umumlashgan Fridrixs modeli kvant mexanikasidagi ko‘plab masalalarni, xususan, bog‘langan holatlar va sochilish jarayonlarini tavsiflashda qo‘llaniladigan operatorli modellar sinfiga kiradi. Mazkur modelning Kreyn tipidagi fazoda qaralishi uning spektral xossalarini yangi sharoitlarda tadqiq etish imkonini beradi [1].

$\mathbb{T} := (-\pi, \pi]$ - bir o‘lchamli tor, $\mathbb{C}$ - bir o‘lchamli kompleks sonlar to‘plami, $L_2(\mathbb{T})$ - $\mathbb{T}$ torda aniqlangan kvadrati bilan integrallanuvchi (kompleks qiymatli) funksiyalarning Hilbert fazosi bo‘lsin. Bu fazolarni $\mathcal{H}_0 := \mathbb{C}, \mathcal{H}_1 := L_2(\mathbb{T})$ kabi belgilab olamiz.

$\mathcal{H}_0, \mathcal{H}_1$ fazolarning to‘g‘ri yig‘indisidan tashkil topgan $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1$ chiziqli fazoni qaraymiz. $(\cdot,\cdot)_0$ va $(\cdot,\cdot)_1$ orqali mos ravishda $\mathcal{H}_0, \mathcal{H}_1$ fazolarning skalyar ko’paytmalarini belgilaymiz, ya’ni $f, g \in \mathcal{H}$ elementlari uchun

$$(f_0, g_0)_0 = f_0\overline{g_0}, \qquad (f_1, g_1)_1 = \int_T f_1(t)\overline{g_1(t)}\, dt.$$

$\mathcal{H}$ chiziqli fazo

$$(f, g) = (f_0, g_0)_0 + (f_1, g_1)_1$$

skalyar ko‘paytmaga nisbatan Hilbert fazoni tashkil qiladi va Fok fazosining qirqilgan ikki zarrachali qism fazosi deyiladi.

$\mathcal{H}$ fazoning ixtiyoriy $f, g \in \mathcal{H}$ elementlari uchun ularning ichki ko‘paytmasini quyidagicha kiritamiz:

$$[f, g] = (f_0, g_0)_0 - (f_1, g_1)_1.$$

$\mathcal{H}$ chiziqli fazoda

$$\mathcal{J} := \begin{pmatrix} I_0 & 0 \\ 0 & -I_1 \end{pmatrix}$$

operatorni qaraymiz, bu yerda $I_k, k = 0, 1$ orqali $\mathcal{H}_k$ fazodagi birlik operator belgilangan. Bundan $(\mathcal{H}, [\cdot,\cdot])$ fazoda ishorasi aniqlanmagan ichki ko‘paytma uchun quyidagi munosabat o‘rinli ekanini ko‘rish mumkin:

$$[f, g] = (\mathcal{J}f, g).$$

$(\mathcal{H}, [\cdot,\cdot])$ Hilbert fazosida $\mathcal{A}$ operatorli matritsani qaraymiz:

$$\mathcal{A} := \begin{pmatrix} A_{00} & A_{01} \\ -A_{01}^* & A_{11} \end{pmatrix}. \tag{1}$$

Odatda, (1) ko‘rinishida aniqlangan operatorli matritsaga umumlashgan Fridrixs modeli deyiladi. $\mathcal{A}$ operatorli matritsaning

$$A_{ij} : H_j \to H_i, i, j = 0, 1$$

elementlari quyidagi ta’sir formulalariga ega:

$$A_{00}f_0 = af_0, \quad A_{01}f_1 = \int_{\mathbb{T}} v(t)f_1(t)dt,$$
$$(A_{01}^*f_0)(x) = v(x)f_0, \quad (A_{11}f_1)(x) = u(x)f_1(x);$$

Bu yerda $a-$ fiksrlangan biror haqiqiy son, $v(\cdot), u(\cdot)$ funksiyalar $\mathbb{T}$ torda aniqlangan haqiqiy qiymatli uzluksiz funksiyalardir.

Parametrlarga qo‘yilgan bunday shartlarda $\mathcal{A}$ umumlashgan Fridrixs modeli $(\mathcal{H}, [\cdot,\cdot])$ Krein tipidagi Fok fazosida chiziqli, chegaralangan va $\mathcal{J}$- o‘z-o‘ziga qo‘shma operator bo‘ladi. Qulaylik uchun $\mathcal{A}$ operatorli matritsaning spektri va nuqtali spektrini mos ravishda $\sigma(\mathcal{A})$, $\sigma_p(\mathcal{A})$ kabi belgilaymiz

Endi

$$S^2 := \{f = (f_0, f_1) \in (\mathcal{H}, [\cdot,\cdot]) : \|f_0\| = \|f_1\| = 1\}$$

sifatida birlik sferalarning Dekart ko‘paytmasini belgilaymiz.

Har bir $f \in S^2$ element uchun

$$\mathcal{A}_f := \begin{pmatrix} (A_{00}f_0, f_1)_0 & (A_{01}f_1, f_0)_0 \\ (-A_{01}^*f_0, f_1)_1 & (A_{11}f_1, f_1)_1 \end{pmatrix}$$

sonli matritsani qaraylik. Kvadratik sonli tasvir ta’rifiga ko‘ra [1] $\mathcal{A}$ operatorli matritsaning kvadratik sonli tasviri

$$W^2(\mathcal{A}) = \bigcup_{f \in S^2} \sigma_p(\mathcal{A}_f)$$

tenglik yordamida aniqlanadi.

Ishning asosiy teoremasini bayon qilamiz.

Teorema 1. $\mathcal{A}$ operatorli matritsaning kvadratik sonli tasviri uchun quyidagi tasdiqlar o‘rinli:
(i) Agar $\overline{W^2(\mathcal{A})} \backslash \mathcal{R} \neq \emptyset$ bo‘lsa,u holda $W^2(\mathcal{A})$ bog‘langandir.
(ii) Agar $\overline{W^2(\mathcal{A})}$ ikkita komponentdan iborat bo‘lsa, u holda $\sigma(\mathcal{A}) \subset \mathcal{R}$ munosabat o‘rinlidir.

Umumlashgan Fridrixs modelining kvadratik sonli tasviri [2] ishda batafsil o‘rganilgan.

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Birman-Schwinger principle for a Generalized Friedrichs model with rank seven perturbations

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To establish the necessary framework for the subsequent spectral analysis, we first introduce several basic definitions and notations. Let $\mathbb{C}$ denote the one-dimensional complex space and $L_2(\mathbb{T}^2)$ the Hilbert space of square-integrable functions defined on the two-dimensional torus $\mathbb{T}^2$. We consider the Hilbert space $\mathcal{H}$ as the direct sum of two Hilbert spaces, where $\mathcal{H}_0 := \mathbb{C}$ and $\mathcal{H}_1 := L_2(\mathbb{T}^2)$; that is,

$$\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1.$$

Within the Fock space framework, the subspaces $\mathcal{H}_0$ and $\mathcal{H}_1$ correspond to the zero-particle and one-particle subspaces, respectively.

We introduce a second-order operator matrix acting in the Hilbert space $\mathcal{H}$ as

$$H_{\mu,\lambda}(k) := \begin{pmatrix} H_{00}(k) & \mu H_{01} \\ \mu H_{01}^* & H_{11}(k) - V_\lambda \end{pmatrix}.$$

The entries of this operator matrix are specified as follows:

$$H_{00}(k)f_0 = a(k)f_0, \qquad H_{01}f_1 = \int_{\mathbb{T}^2} v(t)f_1(t)\,dt,$$

$$(H_{11}(k)f)(p) = E_k(p)f(p), \qquad (V_\lambda f)(p) = \int_{\mathbb{T}^2} v_\lambda(p-t)f(t)\,dt.$$

Here

$$E_k(p) = \varepsilon_1(p) + \varepsilon_2(k-p), \qquad v_\lambda(p) = \lambda_0 + \lambda_1\cos p_1 + \lambda_2 \cos p_2, \qquad p = (p_1, p_2) \in \mathbb{T}^2,$$

and $f \in \mathcal{H}_i$, $i = 0, 1$, μ, λ_0, λ_1, λ_2 are real positive parameters, and we denote $\lambda = (\lambda_0, \lambda_1, \lambda_2)$. The functions $a(\cdot)$, $v(\cdot)$, $\varepsilon_1(\cdot)$, and $\varepsilon_2(\cdot)$ are real-valued and continuous functions on the torus $\mathbb{T}^2$.

Using fundamental results from functional analysis, particularly the theory of adjoint operators in Hilbert spaces, it follows that the adjoint operator H_{01}^* is defined as

$$(H_{01}^* f_0)(p) = v(p)f_0.$$

The operator H_{01} can be interpreted as the annihilation operator of a single particle, whereas its adjoint H_{01}^* represents the corresponding creation operator. Consequently, the operator matrix $H_{\mu,\lambda}(k)$ is commonly viewed as a family of generalized Friedrichs models. Under the assumptions imposed on its coefficients, $H_{\mu,\lambda}(k)$ is a linear, bounded, and self-adjoint operator acting in the Hilbert space $\mathcal{H}$. Setting the parameters $\mu = \lambda_0 = \lambda_1 = \lambda_2 = 0$, we obtain the unperturbed operator matrix denoted by $H_{0,0}(k)$:

$$H_{0,0}(k) = \begin{pmatrix} H_{00}(k) & 0 \\ 0 & H_{11}(k) \end{pmatrix}.$$

A straightforward computation shows that the perturbation operator $H_{\mu,\lambda}(k) - H_{0,0}(k)$ is a finite-rank operator of rank at most seven. Indeed, if $v(\cdot) \notin \{1, \sin p_1, \cos p_1, \sin p_2, \cos p_2\}$, then

$$\dim \operatorname{Im}\big(H_{\mu,\lambda}(k) - H_{0,0}(k)\big) = 7.$$

Otherwise,

$$\dim \operatorname{Im}\big(H_{\mu,\lambda}(k) - H_{0,0}(k)\big) < 7.$$

By means of spectral analysis, the essential spectrum of the operator $H_{0,0}(k)$ can be characterized as follows:

$$\sigma_{\text{ess}}(H_{0,0}(k)) = [m(k), M(k)], \qquad \sigma_{\text{pp}}(H_{0,0}(k)) = \{a(k)\},$$

where

$$m(k) = \min_{p\in\mathbb{T}^2} E_k(p), \qquad M(k) = \max_{p\in\mathbb{T}^2} E_k(p).$$

If $a(k) \in [m(k), M(k)]$, then

$$\sigma(H_{0,0}(k)) = [m(k), M(k)], \qquad \sigma_{\text{ess}}(H_{0,0}(k)) = [m(k), M(k)].$$

For the case $a(k) \notin [m(k), M(k)]$, we have

$$\sigma_{\text{disc}}(H_{0,0}(k)) = \{a(k)\}.$$

Here $a(k)$ is a simple eigenvalue, and the corresponding eigenvector-function has the form $f = (c, 0)$, $c = \mathbb{C} \setminus \{0\}$.

According to Weyl's theorem, which states that the essential spectrum remains invariant under finite-rank perturbations (in our case, an at most seven-rank perturbation), the essential spectrum of the generalized Friedrichs model $H_{\mu,\lambda}(k)$ coincides with the essential spectrum of $H_{0,0}(k)$. Therefore, the essential spectrum of the model $H_{\mu,\lambda}(k)$ is independent of the spectral parameters $\mu > 0$, $\lambda = (\lambda_0, \lambda_1, \lambda_2)$, $\lambda_0, \lambda_1, \lambda_2 > 0$, and satisfies

$$\sigma_{\text{ess}}(H_{\mu,\lambda}(k)) = [m(k), M(k)].$$

Let $C(\mathbb{T}^2)$ denote the Banach space of continuous functions on $\mathbb{T}^2$. Furthermore, let $G(k, z, p, q)$ denote the integral Birman–Schwinger operator with kernel given by

$$G(k, z, p, q) = \frac{v(p-q)}{E_k(q) - z} - \frac{\mu^2}{(z - a(k))(E_k(q) - z)}.$$

Theorem. A number $z \in \mathbb{C} \setminus [m(k), M(k)]$ is an eigenvalue of the operator $H_{\mu,\lambda}(k)$ if and only if 1 is an eigenvalue of the operator $G(k, z, p, q)$.

The eigenvalues of such type generalized Friedrichs model was investigated in [1].

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THE ARCHIMEDEAN SPIRAL AND ITS MANIFESTATIONS IN THE WORLD AROUND US

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Abstract. Spiral form combines symmetry and the golden ratio, with visual perception, it causes a feeling of harmony. The basis of many forms of beauty created by nature, is a spiral. This is article discusses types of coils – from the simplest to the most complex, are found in the world around us.

Keywords: Archimedean spiral, descriptive geometry, engineering graphics, symbol, shape, golden section.

Introduction. A spiral is a planar curve that repeatedly winds around a point on the plane, which is called the pole of the spiral. "Eadem mutate resurgo" – "though changed, I rise again" – signifies that the spiral possesses the remarkable property of restoring its form after various transformations. The spiral shape combines symmetry and the golden ratio; when perceived visually, it evokes a sense of harmony and beauty. The shells of marine animals can grow in only one direction. To avoid stretching out too much, they have to curl up, with each subsequent coil being similar to the previous one. Such growth can only occur along a logarithmic spiral or its analogues [1].

Spirals. The Meaning of the Symbol. The spiral is a very complex symbol that has been used since the Paleolithic era. This symbol became widespread at the dawn of humanity. It should also be noted that the spiral sign (and therefore the snake) is also a sign of a hurricane or storm. According to Tibetan myths about the creation of the world, a storm arose in the original void, filling the original ocean with water. The wind drove the water into the ground like milk mixed into butter. So, the spiral is a way of life. Like a circle, spiral is associated with the image of infinity and embodies the ideas of development, continuity, cosmicrhythms. In the mythopoetic system of correspondences, it stands alongside objects and phenomena such as the moon, snake, lightning, water, fertility, birth, and the afterlife. Life, and is often used in ornamental and pictorial traditions. The spiral represents a schematic representation of the evolution of the universe. This figure expresses the dynamic aspect of being, movement toward or away from the center, representing evolution and involution respectively; it serves as a model for various physical processes (spiral galaxies, a whirlpool, a hurricane). In the Egyptian system of hieroglyphs, the spiral is associated with the idea of the dynamics of cosmic forms and the connection between the one and the many.

Development occurs in a spiral pattern. Each turn is the end of one cycle and, at the same time, the beginning of another. Each cycle is the flowering of those that preceded it and the preparation of more favorable conditions for those following. In addition to material objects, the spiral also characterizes the spheres of the psyche and spirit, even the very word "spiral" close to the word "spirit" – spirit. It also symbolizes our inner self, travel, because if we do not know ourselves, we cannot hope to know the world around us of the world [2].

The spiral also becomes one of the images of the maze. Spiral labyrinths were depicted on coins and seals in Crete. But it is such a labyrinth that, as you walk through it, you will inevitably find yourself in the center of the labyrinth, as well as in the center of the 6-pointed star, where, as the floor gali cretans, the 5th element – the quintessence – is placed. Therefore, in this case, the center symbolizes the seed of spiritual life or sema, that is, meaning. In other words, we

are talking about initiation, rebirth. In addition, the spiral represents the forces of the Sun and Moon, air, water, thunder, and lightning in various ways. It is also a whirlwind, a great creative force, emanation. As a divergent and convergent, it can mean the increase and decrease of the sun, the stay and decrease of the moon, and by analogy, growth and spread, death and contraction, twisting and untwisting, birth and death. The spiral and the vortex have one and the same symbolism, especially when they act as symbols of energy in nature. The time factor, combined with rotation and directional motion, forms the shape of a spiral. The spirals present in the structure of artworks are related to time.

An Archimedean spiral is a set of positions of a point participating simultaneously in two uniform motions, one of which is performed along a line and the other along a circle. The Archimedean spiral was discovered by Archimedes. This happened in the 3rd century BC, when he experimented with a compass. He pulled the compass needle at a constant speed, turning the compass itself clockwise. The resulting curve was a spiral, which shifted by the same amount as the compass rotated, and the same distance was maintained between the turns of the spiral. The spiral associated with the symbolism of the center has long been the beginning of beginnings, from which evolution, development, and the movement of life originate. At one time, Archimedes paid attention to its shape. An ancient Greek scientist from Syracuse studied the shape of a spirally twisted shell and derived the equation of a spiral. The coil he drew according to this equation is named after him – the Archimedean spiral [3]. Its construction is carried out as follows: a circle with a radius equal to the spiral's pitch is drawn from center O, and the pitch and circle are divided into several equal parts, with the division points numbered (Fig. 1). In descriptive geometry, the Archimedean spiral belongs to the regular curves. It is constructed using patterns connecting points on a circle.

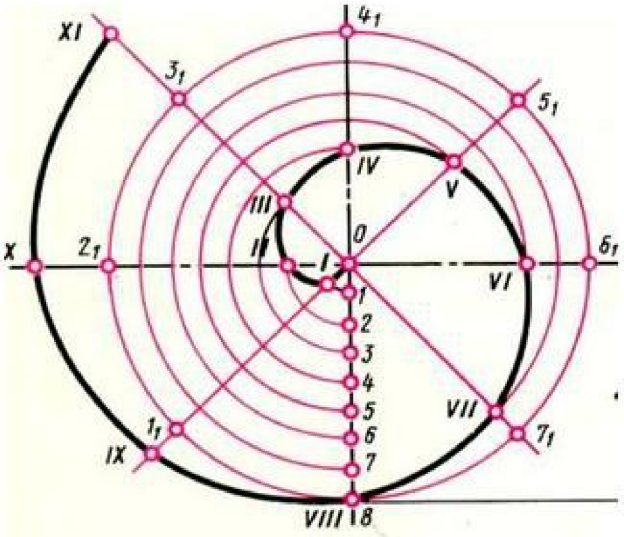


Рис. 1: Constructing an Archimedean spiral

In the 3rd century BC, Archimedes invented a screw based on his spiral, which was successfully used to transfer water from water bodies to irrigation canals. Archimedean screw became the prototype of a screw ("slug") – a device widely used in various machines for mixing liquid, bulk, and dough-like materials. Its most common variety is the screw rotor in a conventional meat grinder. An example of application in archimedean spiral technique is also a self-centering chuck. This mechanism is used in sewing machines to wind the threads evenly.

Archimedes' spiral and Fibonacci sequence. The Archimedes spiral is closely related to the Fibonacci sequence of numbers. What do these, at first glance, completely different concepts have in common? The Fibonacci series is a sequence of numbers, in where each subsequent number is equal to the sum of the two preceding ones. The Fibonacci sequence looks as follows: 1, 1, 2, 5, 8, 13, 21, 34, 55, 89 ... And the ratio of each subsequent number to the previous one in this series of numbers is 1,618... This number is called the Fibonacci number [4].

Archimedes' spiral in nature. In nature, the spiral manifests in three main forms: frozen (like a snail shell), expanding (like images of spiral galaxies), or contracting (like a whirlpool). Spiral forms range from evolutionary depths (DNA molecules) to the laws of dialectics. The DNA molecule is twisted into a double helix. Goethe called the spiral the "curve of life". The spiral is close to the circle – the most perfect shape of all that nature has created (Fig. 2). Indeed, spontaneous and natural elements you, which have a spiral shape, are very common in nature. These include spiral nebulae, galaxies, whirlpools, vortices, tornadoes, and plant structures. Nature loves repetition, in her works utilize the same principles.



Рис. 2: Archimedes' spiral in nature

In nature, the Archimedean spiral is encountered at every turn. Even spiders spin their webs in a spiral, twisting the threads in a spiral around the center. The sunflower head consists of Archimedes' spirals, some of which are rotated clockwise, while others are counterclockwise: the medium-sized head has 34 spirals in one direction and 55 in another. Pine cones and cactus thorns also have spirals oriented clockwise, or counterclockwise. Moreover, the number of these spirals will always be equal to the adjacent numbers of the Fibonacci series. For example, pine cones have 5 and 8 spirals, while pineapples have 8 and 13 [5].

Conclusion. The spiral in science, art, and nature has long been a subject of study. The Archimedean spiral is closely related to the Fibonacci sequence, which derived a chamber value that increases proportionally to the ratio 1,618 while the shape of the spirals remains unchanged. This law of mathematics describes the principle of the Archimedean spiral and the golden section. Their close connection can be observed in many natural phenomena and elements – in the structure of mollusk shells, sunflower inflorescences and succulent plants, fractal cabbage and pine cones, humans and entire galaxies.

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Fridrixs modelidagi operatorning muhim spektrining yuqori chegarasining yuqorisida yotuvchi musbat chekli xos qiymatlari

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Faraz qilaylik, $u(x)$- funksiya $\Omega = [0,1]$ to'plamda haqiqiy qiymatli uzluksiz funksiya bo'lsin. K- operator $L_2(\Omega)$ Gilbert fazosida $k(x,s) = L_2(\Omega^2)$ simmetrik yadroli kompakt integral operator bo'lsin. Kvant mexanikasi va statistik fizikaning [1] bir qancha masalalari $L_2(\Omega)$ Gilbert fazosida quyidagi

$$H = H_0 + K \tag{1}$$

(1) formula orqali aniqlangan H operatorning spektrini o'rganishga keltiriladi, bu yerda $(H_0 f)(x) = u(x)f(x), (Kf)(x) = \int_\Omega k(x,s)f(s)d\mu(s)$.

Bunda K- kompakt integral operator Lebeg ma'nosida integral va $\mu(.)$ - ifoda R to'plamda Lebeg o'lchovi hisoblanadi.

Qulaylik uchun $\sigma(A)$, $\sigma_{ess}(A)$ va$\sigma_{disc}(A)$ bilan operatorning mos ravishda spektr, muhim spektr va diskret spektrlarni belgilaymiz.

Kompakt qo'zg'alish haqidagi Veyl teoremasidan H operatorning $\sigma_{ess}(H)$ muhim spektri $u(x)$ funsiyaning qiymatlar to'plamidan iborat bo'lishi kelib chiqadi, ya'ni $\sigma_{ess}(H) = [u_{min}, u_{max}]$ tenglik o'rinli, bu yerda $u_{min} = \min\limits_{x\in\Omega} u(x), u_{max} = \max\limits_{x\in\Omega} u(x)$ va $u_{max} > 0$ deb olamiz. (1) ko'rinishdagi operator Fridrixs modelidagi operator deyiladi. Fridrixs modelidagi operatorlar [2-3] ishlarda o'rganilgan. Minimaks va maksimin prinsiplaridan [3] foydalanib, agar K integral operatorning yadrosi ajralgan bo'lsa, u holda (1) Fridrixs modelining diskret spektri chekli bo'lishi isbotlangan.

Faraz qilaylik, $\mathcal{H}$ - separabel Gilbert fazo va $A : \mathcal{H} \to \mathcal{H}$ chiziqli chegaralangan o'z-o'ziga qo'shma operator berilgan bo'lsin. Shuningdek quyidagi belgilashlarni kiritamiz

$$E_{min}(A) = \inf\{\lambda : \lambda \in \sigma_{ess}(A)\}, \quad E_{max}(A) = \sup\{\lambda : \lambda \in \sigma_{ess}(A)\}$$

Bu yerda $E_{min}(A)$ ($E_{max}(A)$ son) son A operator muhim spektrining quyi (yuqori) chegarasi deb ataladi. $\eta_k(A)_{k\in 1,2,...,n}$ bilan o'z-o'ziga qo'shma A operator uchun maksimina prinsipi [3] yordamida qurilgan chegaralangan o'suvchi (ya'ni $\eta_1(A) \geq \eta_2(A) \geq ... \geq \eta_n(A)$) haqiqiy sonlar ketma-ketligini belgilaymiz. U holda har bir $\eta_k(A), k \in 1, 2, ..., n$ son A operatorning xos qiymati bo'ladi va $\eta_n(A) = E_{max(A)}$ tenglik o'rinli.

Lemma 1.[3] $A, B : \mathcal{H} \to \mathcal{H}$ - chiziqli chegaralangan o'z-o'ziga qo'shma operatorlar va $A \leq B$ tengsizlik o'rinli bo'lsin. U holda $\eta_n(A) \leq \eta_n(B), n \in \mathbb{N}$ tengsizlik o'rinli bo'ladi, bu yerda

$$\eta_k(A) = \inf_{L\subset H, dimL=k-1} \sup_{\|x\|=1, x\perp L} (Ax, x), k \in \mathbb{N}$$

Lemma 1 dan quyidagi tasdiq kelib chiqadi.

Tasdiq 1. $\mathcal{H}$ Gilbert fazosida A va B chiziqli chegaralangan o'z-o'ziga qo'shma operatorlar uchun $A \leq B$ tengsizlik va $E_{max}(A) = E_{max}(B)$ tenglik o'rinli bo'lsin. Agar B operator muhim spektri yuqori chegarasining yuqorisida yotuvchi cheklita xos qiymatga ega bo'lsa, u holda A operatorning ham muhim spektri yuqori chegarasining yuqorisida yotuvchi xos qiymatlari soni ham cheklita bo'ladi. Aytaylik, (1) Fridrixs modelidagi K integral operator musbat va K integral operatorning yadrosi ajralgan ko'rinishda bo'lsin, ya'ni

$$k(x, s) = \sum_{k=1}^{n} a_k \varphi_k(x)\overline{\varphi_k(s)},$$

bu yerda $\{a_k\}_{k\in\{1,2,...n\}} \in l_2, a_k > 0, k \in 1, 2, ...n$ har bir a_k son K operatorning xos qiymati hamda $\varphi_k(x)_{k\in 1,2,...,n}$ - sistema K operatorning $L_2(\Omega)$ Gilbert fazosida a_k xos qiymatlariga mos ortonormal xos funksiyalar sistemasi bo'ladi.

H (1) operatorning o'z-o'ziga qo'shma ekanligidan $\sigma(H) \subset \mathbb{R}$ munosabat kelib chiqadi. Shuningdek, K operatorning musbatligidan esa $\sigma_{disc}(H) \cap (-\infty, u_{min}) = \emptyset$ tenglikning o'rinli bo'lishi kelib chiqadi, shuning uchun H operatorning diskret spektri $(u_{max}, +\infty)$ oraliqda yotadi.

Tasdiq 2. H (1) Fridrixs modelidagi operator chekli sondagi musbat xos qiymatlarga ega bo'lsin. Agar uzluksiz $\nu(x), \nu_{max} > 0$ (bu yerda $\nu_{max} = \max_{x\in\Omega} \nu(x)$) funksiyaga ko'paytirish operatori bo'lgan V_0 operator uchun $V_0 \leq H_0$ shart bajarilsa, u holda

$$H_1 = V_0 + K$$

operator ham chekli sondagi musbat xos qiymatlarga ega bo'ladi.

Tasdiq 3. H (1) Fridrixs modelidagi operator chekli sondagi musbat xos qiymatlarga ega bo'lsin. Agar $q(x, s) = \sum_{k=1}^{m} b_k \psi_k(x)\overline{\psi_k(x)}, b_k > 0, k \in 1, 2, ..., m$ ajralgan yadroli Q musbat kompakt integral operator uchun $K \geq Q$ shart o'rinli bo'lsa, u holda

$$H_2 = H_0 + Q$$

operator ham chekli sondagi mubat xos qiymatlarga ega bo'ladi.

Fridrixs modelidagi operatorni quyidagicha qaraymiz

$$H_3 = V_0 + Q$$

bu yerda V_0 - uzluksiz $\nu(x), \nu_{max} > 0$ (bu yerda $\nu_{max} = \max\limits_{x\in\Omega} \nu(x)$) funksiyaga ko'paytirish operatori, Q - ajralgan $q(x,s) = \sum_{k=1}^{m} b_k \psi_k(x)\overline{\psi_k(x)}, b_k > 0, k \in 1,2,...,m$ yadroli musbat kompakt integral operator.

Teorema 1. H (1) Fridrixs modelidagi operator chekli sondagi musbat xos qiymatlarga ega bo'lsin. Agar $V_0 \le H_0$ va $Q \le K$ shartlar bajarilsa, u holda $H_3 = V_0 + Q$ operator ham chekli sondagi musbat xos qiymatlarga ega bo'ladi.

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Birinchi tur klassik sohalarning dekart kopaytmasi orqali aniqlangan matrissaviy poliedrda Bishop integral formulasi

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Faraz qilaylik,

$$Z = (Z_1, Z_2, \dots, Z_n)$$

-elementlari kompleks sonlar maydoni $\mathbb{C}$ da aniqlangan $[m \times m]$-tartibli kvadrat Z_j $(1 \le j \le n)$ matritsalardan iborat vektor bo'lsin. Z ni

$$\mathbb{C}^n_{[m\times m]} \cong \mathbb{C}^{nm^2}$$

fazosining elementi deb qarash mumkin.

Ushbu

$$T = T_n = \{Z = (Z_1, Z_2, \dots, Z_n) : I - Z_1 Z_1^* > 0,\ I - Z_2 Z_2^* > 0, \dots,\ I - Z_n Z_n^* > 0\}$$

soha birlik matritsaviy polikrug deb ataladi. Bunda I- m-tartibli birlik matritsa, Z_j^* matritsa esa Z_j matritsaning transponerlanganining kompleks qo'shma matritsasi hisoblanadi (bu yerda $H > 0$ Ermit matritsasining musbat aniqlanganligini bildiradi, ya'ni barcha xos sonlari musbat) ([1–2]).

Birlik matritsaviy polikrugning chegarasi va ostovi mos ravishda quyidagicha aniqlanadi [2]:

$$\partial T_n = \left\{ Z \in \mathbb{C}^n_{[m\times m]} : \det(I - Z_1 Z_1^*) = 0,\ I - Z_1 Z_1^* \ge 0, \dots, \det(I - Z_n Z_n^*) = 0,\ I - Z_n Z_n^* \ge 0 \right\},$$

$$\Gamma = \left\{ Z \in \mathbb{C}^n_{[m\times m]} : Z_1 Z_1^* = I, \dots, Z_n Z_n^* = I \right\}.$$

Bu T_n sohada golomorf va uning yopig‘ida uzluksiz har qanday $h(Z)$ funksiya uchun ([2], 32-saxifa)

$$h(Z) = c_{m,n} \int_{\Gamma} \frac{h(X)\, dX}{\prod\limits_{k=1}^{n} \det^{m}(X - Z)}$$

Xua Lo-ken integral formulasi o‘rinli bo‘ladi. Bunda

$$dX = \bigwedge_{k=1}^{n} \left(\bigwedge_{i,j=1}^{m} dx_{ij}^{k} \right),$$

ko‘rinishda aniqlanib, $c_{m,n}$ o‘zgarmas esa

$$c_{m,n} \int_{\Gamma} \frac{dX}{\prod\limits_{k=1}^{n} \det^{m}(X)} = 1,$$

ko‘rinishda aniqlanadi.

Biror

$$G \subset \mathbb{C}^{n}_{[m\times m]}$$

sohada golomorf bo‘lgan

$$f = (f_1, \ldots, f_n) : G \longrightarrow \mathbb{C}^{n}_{[m\times m]}$$

akslantirishni qaraymiz.

Yuqoridagi

$$f = (f_1, \ldots, f_n) : G \longrightarrow \mathbb{C}^{n}_{[m\times m]}$$

akslantirishni quyidagicha matritsa ko‘rinishida tushunish kerak:

$$f(Z) = \big(f_1(Z), \ldots, f_n(Z)\big) = \left(\begin{pmatrix} f_{11}^{1}(Z) & \cdots & f_{1m}^{1}(Z) \\ \vdots & \ddots & \vdots \\ f_{m1}^{1}(Z) & \cdots & f_{mm}^{1}(Z) \end{pmatrix}, \ldots, \begin{pmatrix} f_{11}^{n}(Z) & \cdots & f_{1m}^{n}(Z) \\ \vdots & \ddots & \vdots \\ f_{m1}^{n}(Z) & \cdots & f_{mm}^{n}(Z) \end{pmatrix} \right).$$

Endi matritsaviy poliedr tushunchasini keltiramiz.

1-ta’rif. Agar

$$f : G \longrightarrow \mathbb{C}^{n}_{[m\times m]}$$

golomorf akslantirish yordamida aniqlangan

$$f^{-1}(T_{n,r}) = \big\{ Z \in G : r^2 I - f^1(Z) f^1(Z)^* > 0, \ldots, r^2 I - f^n(Z) f^n(Z)^* > 0,\ r > 0 \big\}$$

to‘plam G sohada kompakt yotsa, ya‘ni,

$$f^{-1}(T_{n,r}) \Subset G,$$

u holda $f^{-1}(T_{n,r})$ to‘plamga ***matritsaviy poliedrik to‘plam*** deyiladi.

Shuni alohida ta‘kidlash lozimki, bu to‘plam har doim ham bog‘lamli bo‘lavermaydi.

2-ta’rif. Matritsaviy poliedrik to‘plamning bog‘lamli komponentasini matritsaviy poliedrik soha yoki qisqacha ***matritsaviy poliedr*** deb ataymiz va uni $\Pi_{f,r}$ kabi belgilaymiz.

Matritsaviy poliedrning ostovi

$$\Gamma_{f,r} = \left\{ Z \in G : r^2 I = f^1(Z) f^1(Z)^*, \ldots, r^2 I = f^n(Z) f^n(Z)^*,\ r > 0 \right\},$$

ko‘rinishda aniqlanadi.

Aytaylik,

$$f = (f_1, f_2, \ldots, f_n) : D \longrightarrow G$$

akslantirish

$$D \subset \mathbb{C}^n_{[m\times m]}$$

sohani

$$G \subset \mathbb{C}^n_{[m\times m]}$$

sohaga akslantiruvchi golomorf akslantirish bo‘lsin. Agar har bir $W \in G$ nuqtalar uchun

$$f(Z) = W$$

tenglama D sohada karrali ildizlarining tartibi bilan qo‘shib hisoblaganda doimo bir xil sondagi yechimlarga ega bo‘lsa, u holda $f(Z)$ akslantirish ***chekli turdagi akslantirish*** deb ataladi (bu yerda nollar soni karrasi bilan olingan).

Aytaylik,

$$f : D \longrightarrow G$$

akslantirish

$$D \subset \mathbb{C}^n_{[m\times m]}$$

sohani

$$G \subset \mathbb{C}^n_{[m\times m]}$$

sohaga akslantiruvchi chekli turdagi golomorf akslantirish bo‘lsin va

$$H(Z) = \frac{\varphi(Z)}{\psi(Z)}$$

funksiya D sohada meromorf bo‘lsin.

3-ta‘rif: f akslantirishga nisbatan $H(Z)$ meromorf funksiyaning *izi* deb ushbu

$$[TrH](W) = \sum H\big(Z(W)\big), \qquad W \in G \setminus f\big(\psi(Z) = 0\big)$$

funksiyaga aytiladi. Bu yerda yig‘indi

$$f(Z) = W$$

tenglamaning ildizlari (karralisining tartibi ham e‘tiborga olingan) bo‘yicha olinadi.

Ushbu ishda birinchi tur klassik sohalarning dekart ko‘paytmasi orqali hosil qilingan matritsaviy poliedrda Bishop integral formulasi olingan.

Teorema: Agar

$$h(Z) \in \mathcal{O}(\Pi_{f,r}) \cap C(\overline{\Pi}_{f,r})$$

bo‘lib, $Z \in \Pi_{f,r}$ nuqtalarda yakobian

$$J_f(Z) \neq 0$$

bo‘lsa, u holda h/J_f meromorf funksiya uchun ushbu

$$\frac{h(Z)}{J_f(Z)} = c_{m,n} \int_{\Gamma_{f,r}} \frac{h(X)\Psi(Z,X)\,dX}{\Psi(Z,Z)\prod\limits_{k=1}^{n} \det\big(f(X) - f(Z)\big)}$$

integral formula o‘rinli bo‘ladi. Bu yerda

$$\Psi(Z, X) \neq 0$$

va

$$\overline{\Pi}_{f,r} \times \overline{\Pi}_{f,r}$$

to‘plamda golomorf bo‘lgan vazn funksiya.

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METRIC CLASSIFICATION AND GAUSSIAN CURVATURE OF QUADRATIC MONGE SURFACES IN THREE-DIMENSIONAL MINKOWSKI SPACE

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Let ${}^1R_3 = (R_3, \langle\cdot,\cdot\rangle_1)$ be the three-dimensional Minkowski space of index one, endowed with the scalar product

$$\langle X, Y\rangle_1 = x_1y_1 + x_2y_2 - x_3y_3, \qquad X = (x_1, x_2, x_3), \quad Y = (y_1, y_2, y_3).$$

For a nonzero vector X, its causal character is determined by the sign of $\langle X, X\rangle_1$: the vector is spacelike, timelike, or lightlike according as $\langle X, X\rangle_1 > 0$, $\langle X, X\rangle_1 < 0$, or $\langle X, X\rangle_1 = 0$, respectively [1,2].

Consider the quadratic Monge surface

$$\mathbf{r}(x, y) = (x, y, f(x, y)), \qquad f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T A\mathbf{x} + \ell^T\mathbf{x} + c, \qquad \mathbf{x} = (x, y)^T,$$

where $A \in R_{2\times 2}$ is a constant real symmetric matrix, $\ell \in R_2$ is a constant vector, and $c \in \mathbb{R}$. Then

$$\nabla f(\mathbf{x}) = A\mathbf{x} + \ell, \qquad \operatorname{Hess} f = A.$$

The matrix of the first fundamental form is

$$g = I_2 - \nabla f\,(\nabla f)^T, \qquad \Delta(\mathbf{x}) = \det g = 1 - \|A\mathbf{x} + \ell\|^2,$$

where $\|\cdot\|$ denotes the Euclidean norm in R_2. Hence a point of the surface is spacelike, timelike, or lightlike according as $\Delta > 0$, $\Delta < 0$, or $\Delta = 0$, respectively.

Theorem 1. Let

$$\Sigma_0 = \{\mathbf{x} \in \mathbb{R}_2 : \Delta(\mathbf{x}) = 0\}$$

be the lightlike locus of the quadratic Monge surface. Its form is determined by the rank of A as follows.

(a) If $\operatorname{rank} A = 2$ (equivalently, $\det A \neq 0$), then Σ_0 is an ellipse centered at

$$\mathbf{x}_0 = -A^{-1}\ell.$$

If λ_1 and λ_2 are the eigenvalues of A, its semiaxes are $1/|\lambda_1|$ and $1/|\lambda_2|$. This conclusion is independent of whether A is positive definite, negative definite, or indefinite.

(b) If $\operatorname{rank} A = 1$, let e_0 be a unit vector spanning $\ker A$ and set $\beta = \ell^T e_0$. Then Σ_0 consists of two parallel straight lines when $|\beta| < 1$, one straight line when $|\beta| = 1$, and the empty set when $|\beta| > 1$.

(c) If $\operatorname{rank} A = 0$, then $A = 0$ and

$$\Delta = 1 - \|\ell\|^2$$

is constant. Thus the entire surface is spacelike if $\|\ell\| < 1$, timelike if $\|\ell\| > 1$, and lightlike if $\|\ell\| = 1$.

At every non-lightlike point, a unit normal vector may be chosen as

$$N = \frac{(f_x, f_y, 1)}{\sqrt{|\Delta|}}, \qquad \langle N, N\rangle_1 = -\operatorname{sgn}\Delta.$$

The matrix of the second fundamental form is

$$h = -\frac{A}{\sqrt{|\Delta|}}.$$

Theorem 2. At every non-lightlike point ($\Delta \neq 0$), the Gaussian curvature of the quadratic Monge surface is

$$K(\mathbf{x}) = -\frac{\det A}{\Delta(\mathbf{x})^2}.$$

Consequently, $K < 0$ if $\det A > 0$, $K > 0$ if $\det A < 0$, and $K \equiv 0$ on the non-lightlike part of the surface if $\det A = 0$. Moreover, if $\det A \neq 0$, then

$$|K(\mathbf{x})| \longrightarrow \infty \qquad \text{as} \qquad \Delta(\mathbf{x}) \longrightarrow 0.$$

Conclusion. The proof of Theorem 1 is based on the orthogonal diagonalization of the symmetric matrix A and the reduction of

$$\Delta(\mathbf{x}) = 1 - \|A\mathbf{x} + \ell\|^2 = 0$$

to canonical form. If $\operatorname{rank} A = 2$, translating the origin to $\mathbf{x}_0 = -A^{-1}\ell$ yields

$$\lambda_1^2 u^2 + \lambda_2^2 v^2 = 1,$$

which is an ellipse. If $\operatorname{rank} A = 1$, decomposing ℓ into components parallel and orthogonal to $\ker A$ reduces the equation to

$$\lambda^2 u^2 + \beta^2 = 1,$$

which gives two parallel lines, one line, or the empty set according as $|\beta| < 1$, $|\beta| = 1$, or $|\beta| > 1$. The case $A = 0$ follows immediately from the constancy of Δ.

Theorem 2 follows from $\det g = \Delta$, the identity $h = -A/\sqrt{|\Delta|}$, and the Gauss equation

$$K = \langle N, N\rangle_1 \frac{\det h}{\det g}.$$

Thus the metric classification is governed by the rank and spectral properties of A, together with the component of ℓ along $\ker A$, whereas the sign of the Gaussian curvature is determined by $\det A$.

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SUPPORT-BASED FUZZY DENSITY AND MAPPING PROPERTIES IN LOWEN FUZZY TOPOLOGICAL SPACES

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Abstract. We study a support-based notion of fuzzy density and fuzzy weight in Lowen fuzzy topological spaces. Two equivalent descriptions of fuzzy density are considered: one based on supports of fuzzy sets meeting every nonzero fuzzy open set, and the other on ordinary subsets meeting every nonzero fuzzy open set in a positive degree. We characterize this invariant by the ordinary density of the associated support topology. We prove monotonicity for subspaces open in the support topology, and hence for open fuzzy subspaces, as well as under surjective fuzzy continuous mappings. The subspace inequality may fail without the support-openness assumption. For surjective mappings that are both fuzzy continuous and fuzzy open, we obtain a reverse estimate involving the cardinalities of the fibers. Applications to finite products, group quotients with controlled orbit cardinalities, and fuzzy symmetric powers are derived. We also compare fuzzy density with fuzzy weight, compute these invariants for the Lowen fuzzy topology of lower semicontinuous fuzzy sets, and study the behavior of fuzzy weight under surjective fuzzy continuous and fuzzy open mappings.

Keywords: Lowen fuzzy topology; support-based fuzzy density; support topology; fuzzy weight; fuzzy open mapping; group quotient; fuzzy symmetric power.

Let $I = [0, 1]$. A family $\mathcal{S} \subseteq I^X$ is a *Lowen fuzzy topology* if it contains every constant fuzzy set and is closed under arbitrary pointwise suprema and finite pointwise infima [1]. For $\mu \in I^X$, put

$$\operatorname{supp}(\mu) = \{x \in X : \mu(x) > 0\}.$$

A fuzzy set $\nu \in I^X$ is called *fuzzy dense* if $\mu \wedge \nu \neq 0$ for every nonzero $\mu \in \mathcal{S}$. Its associated cardinal invariant is

$$d_f(X, \mathcal{S}) = \omega + \min\left\{|\operatorname{supp}(\nu)| : \nu \in I^X \text{ is fuzzy dense}\right\}. \tag{1}$$

The supports of fuzzy open sets determine the ordinary topology

$$\tau_{\mathrm{supp}}(\mathcal{S}) = \{\mathrm{supp}(\mu) : \mu \in \mathcal{S}\}, \tag{2}$$

called the *support topology*. The following result gives the fundamental bridge between fuzzy density and ordinary density.

Theorem 1. *Let $(X, \mathcal{S})$ be a Lowen fuzzy topological space. Then*

$$d_f(X, \mathcal{S}) = \omega + d\big(X, \tau_{\mathrm{supp}}(\mathcal{S})\big), \tag{3}$$

where $d(X, \tau)$ is the ordinary density of (X, τ).

Thus, fuzzy separability is equivalent to separability of the associated support topology. Moreover, fuzzy continuous maps induce continuous maps between support topologies. To formulate a sharper mapping result, for $f : X \to Y$ define

$$f_*(\mu)(y) = \bigvee\{\mu(x) : x \in f^{-1}(y)\}. \tag{4}$$

A mapping is *fuzzy open* if $f_*(\mu) \in \mathcal{S}_Y$ for every $\mu \in \mathcal{S}_X$.

Theorem 2. *Let $f : (X, \mathcal{S}_X) \to (Y, \mathcal{S}_Y)$ be a surjective fuzzy continuous and fuzzy open mapping between nonempty Lowen fuzzy topological spaces. Put*

$$\kappa(f) = \sup\{|f^{-1}(y)| : y \in Y\}. \tag{5}$$

Then

$$d_f(Y, \mathcal{S}_Y) \le d_f(X, \mathcal{S}_X) \le \max\{d_f(Y, \mathcal{S}_Y), \kappa(f)\}. \tag{6}$$

In particular, if all fibers are at most countable, then

$$d_f(X, \mathcal{S}_X) = d_f(Y, \mathcal{S}_Y). \tag{7}$$

The support-topology construction is also compatible with finite fuzzy products. This leads to an exact product formula.

Theorem 3. *Let $n \ge 1$ and let $(X_1, \mathcal{S}_1), \dots, (X_n, \mathcal{S}_n)$ be nonempty Lowen fuzzy topological spaces. Then*

$$d_f\left(\prod_{i=1}^{n}(X_i, \mathcal{S}_i)\right) = \max_{1 \le i \le n} d_f(X_i, \mathcal{S}_i). \tag{8}$$

Consequently, a finite fuzzy product is fuzzy separable if and only if each factor is fuzzy separable. These results show that support topology transfers classical density arguments effectively to the Lowen fuzzy setting and gives precise preservation laws for natural mappings and product constructions.

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PRESERVATION OF SOFT INJECTIVE AND SOFT SURJECTIVE MAPPINGS UNDER FINITE SOFT SYMMETRIC POWERS

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Soft set theory provides a parameter-dependent extension of classical set and topological structures. It was introduced by Molodtsov and subsequently adapted to topological spaces and mappings between soft classes [1–4]. In the present note we consider the finite symmetric-power construction for soft topological spaces and focus on one of its basic preservation properties: the preservation of soft injective, soft surjective, and hence soft bijective mappings. The covariant-functor formulation of the construction has been considered separately; here the emphasis is placed only on the mapping properties stated below.

Let X be a set and let $E \neq \varnothing$ be a parameter set. A *soft set* over X with respect to E is a mapping

$$F : E \longrightarrow \mathcal{P}(X),$$

and is denoted by F_E. The family of all soft sets over X with parameter set E is denoted by $\mathrm{SSoft}(X, E)$. A family $\tau \subseteq \mathrm{SSoft}(X, E)$ is a *soft topology* if it contains the null and absolute soft sets and is closed under arbitrary soft unions and finite soft intersections. The triple (X, τ, E) is then called a soft topological space [2,3].

A soft mapping

$$f_\psi : (X, E) \longrightarrow (Y, K)$$

is determined by two mappings

$$f : X \longrightarrow Y, \qquad \psi : E \longrightarrow K.$$

For a soft set G_K over Y, its inverse image is defined by

$$\big(f_\psi^{-1}(G_K)\big)(e) = f^{-1}\big(G(\psi(e))\big), \qquad e \in E.$$

If (X, τ, E) and (Y, υ, K) are soft topological spaces, then f_ψ is called *soft continuous* if

$$G_K \in \upsilon \quad \Longrightarrow \quad f_\psi^{-1}(G_K) \in \tau.$$

Definition 1. A soft mapping f_ψ is called *soft injective* if both $f : X \to Y$ and $\psi : E \to K$ are injective. It is called *soft surjective* if both f and ψ are surjective. If both components are bijective, then f_ψ is called *soft bijective* [4,5].

Let $n \geq 1$. The symmetric group S_n acts on X^n and E^n by permuting coordinates. The corresponding orbit sets are

$$\mathrm{SP}^n X = X^n / S_n, \qquad \mathrm{SP}^n E = E^n / S_n.$$

Their elements are written as

$$[x_1, \ldots, x_n] \in \mathrm{SP}^n X, \qquad [e_1, \ldots, e_n] \in \mathrm{SP}^n E.$$

If $\pi_X : X^n \to \mathrm{SP}^n X$ and $\rho_E : E^n \to \mathrm{SP}^n E$ denote the orbit mappings, the quotient soft topology $\mathrm{SP}^n \tau$ is defined by

$$H_{\mathrm{SP}^n E} \in \mathrm{SP}^n \tau \quad \Longleftrightarrow \quad (\pi_X)^{-1}_{\rho_E}(H_{\mathrm{SP}^n E}) \in \tau^n.$$

Thus

$$(\mathrm{SP}^n X, \mathrm{SP}^n \tau, \mathrm{SP}^n E)$$

is the n-th soft symmetric power of (X, τ, E). For a soft mapping f_ψ, define

$$\mathrm{SP}^n f([x_1, \ldots, x_n]) = [f(x_1), \ldots, f(x_n)]$$

and

$$\mathrm{SP}^n \psi([e_1, \ldots, e_n]) = [\psi(e_1), \ldots, \psi(e_n)].$$

These two mappings determine the induced soft mapping

$$(\mathrm{SP}^n f)_{\mathrm{SP}^n \psi} : (\mathrm{SP}^n X, \mathrm{SP}^n \tau, \mathrm{SP}^n E) \longrightarrow (\mathrm{SP}^n Y, \mathrm{SP}^n \upsilon, \mathrm{SP}^n K).$$

Whenever f_ψ is soft continuous, the induced mapping is also soft continuous [5].

Proposition. Let $n \geq 1$, and let

$$f_\psi : (X, \tau, E) \longrightarrow (Y, \upsilon, K)$$

be a soft continuous mapping.

1. If f_ψ is soft injective, then
$$(\mathrm{SP}^n f)_{\mathrm{SP}^n \psi}$$
is soft injective.

2. If f_ψ is soft surjective, then
$$(\mathrm{SP}^n f)_{\mathrm{SP}^n \psi}$$
is soft surjective.

Consequently, the finite soft symmetric-power construction preserves soft bijective mappings.

The proposition records a preservation property of the induced mapping; no proof is included in this conference note.

Example. Equip all soft classes below with discrete soft topologies.

Injective case. Let

$$X = \{a, b\}, \qquad Y = \{0, 1, 2\}, \qquad E = \{e_1, e_2\}, \qquad K = \{k_1, k_2, k_3\},$$

and define

$$f(a) = 0, \qquad f(b) = 1,$$
$$\psi(e_1) = k_1, \qquad \psi(e_2) = k_2.$$

Then f_ψ is soft injective. For $n = 2$,

$$[a, a] \mapsto [0, 0], \qquad [a, b] \mapsto [0, 1], \qquad [b, b] \mapsto [1, 1],$$

and

$$[e_1, e_1] \mapsto [k_1, k_1], \qquad [e_1, e_2] \mapsto [k_1, k_2], \qquad [e_2, e_2] \mapsto [k_2, k_2].$$

Hence $(\mathrm{SP}^2 f)_{\mathrm{SP}^2\psi}$ is soft injective, although it is not soft surjective because the orbits containing 2 or k_3 are not attained.

Surjective case. Let

$$X' = \{a, b, c\}, \qquad Y' = \{0, 1\}, \qquad E' = \{e_1, e_2, e_3\}, \qquad K' = \{k_1, k_2\},$$

and define

$$g(a) = g(b) = 0, \qquad g(c) = 1,$$
$$\eta(e_1) = \eta(e_2) = k_1, \qquad \eta(e_3) = k_2.$$

Then g_η is soft surjective. Every orbit in $\mathrm{SP}^2 Y'$ has a preimage; for instance,

$$[0,0] = (\mathrm{SP}^2 g)([a,b]), \qquad [0,1] = (\mathrm{SP}^2 g)([a,c]), \qquad [1,1] = (\mathrm{SP}^2 g)([c,c]),$$

and the same coordinatewise choice proves surjectivity of $\mathrm{SP}^2\eta$. Thus $(\mathrm{SP}^2 g)_{\mathrm{SP}^2\eta}$ is soft surjective, although it is not soft injective.

This example separates the two assertions of the main result. In particular, preservation of a soft bijection follows by applying both parts, but a soft continuous bijection is not automatically a soft homeomorphism; soft continuity of its inverse is a separate condition.

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Geometry orbits of vector field in the pursuit problem

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The concepts and methods of differential geometry are increasingly being utilized in the formulation and solution of control theory problems. In recent years, several significant, original, and comprehensive works have emerged, providing an assessment of the key directions in the development of control theory for nonlinear continuous systems based on the application of differential-geometric and group-theoretic methods.

A characteristic feature of works in this area is the systematic application of differential-geometric and grouptheoretical methods. The possibility of applying these methods arises because only smooth dynamical systems are considered, and their qualitative properties are studied, namely, the properties of the corresponding reachable sets-in other words, issues more related to the controllability of systems than to obtaining optimality conditions. A differentiable manifold in the neighborhood of each of its points is structured as R^n, i.e., there is a diffeomorphic mapping of the neighborhood of each point in R^n, where the diffeomorphisms of the neighborhoods of close points are coordinated in such a way that by means of smooth changes of variables it is possible to pass from one neighborhood to another.

In this paper, the control system is a finite set of smooth (class C^∞) vector fields defined on an infinitely differentiable manifold M dimensions n. Control system S is called symmetric if the vector field $X \in S$, it follows that and $-X \in S$ [1].

A multitude S is Li algebra over the field of real numbers in which the product of vector fields $X, Y \in S$ serves as their Li bracket $[X, Y]$. If vector fields X and Y considered as an operation of differentiation of smooth functions, then the Li bracket $[X, Y]$ is defined as a mapping $[X, Y] : F(M) \to F(M)$, $[X, Y](F) = Y(X(F)) - X(Y(F))$, where $F(M)-$ set of smooth functions. It is known [2] that if vector fields X and Y everywhere concerns the submanifold $N \subset M$, then vector field $[X, Y]$ also applies everywhere to the submanifold N.

Definition 1. Management system S is called completely controllable if for any two points $x, y \in M$ there is such a trajectory for it $\alpha : [0, T] \to M$, $0 < T < \infty$, where $\alpha(0) = x$, $\alpha(T) = y$.

Let us recall that the trajectory of the control system S is called a continuous mapping $\alpha : [0, T] \to M$, for which there is a partition of the segment $[0, T] : 0 = t_0 < t_1 < t_2 < ... < t_k = T$, what is the display limitation α for the interval $[t_i, t_{i+1}]$, $i = 0, 1, ..., k - 1$, is the trajectory of one of the vector fields included in S.

Consider the following pursuit problem. Let $M-$ smooth manifold of dimension n, $Q_x, Q_y \subset T_xM-$ compacts where T_xM- tangent space to M at the point x, $S-$ a family of smooth vector fields defined on a set $M \times Q_x \times P_x$. Family S may contain a finite and infinite number of smooth vector fields. For a point $x \in M$ and $v_x \in Q_x$, $u_x \in P_x$, through $t \to X^t(x, v_x, u_x)$ let us denote the trajectory (integral curve) of the vector field, X passing through a point x at $t = 0$. $v_x, u_x -$ control parameters, respectively, of the fleeing (pursued) player and the pursuing player, which the players choose at the point x at the moment t. Display $t \to X^t(x, v_x, u_x)$ defined in some domain, $I(x)$ which in general depends not only on the field, X but also from the starting point x and control parameters v_x, u_x. In what follows, everywhere in formulas of the form $X^t(x, v_x, u_x)$ let's assume that $t \in I(x)$.

Orbit $L(x)$ family S vector fields passing through a point $x \in M$, is defined as a set of such points $y \in M$, for which real numbers exist $0 = t_0 < t_1 < t_2 < ... < t_k = T$ and vector fields $X_{i_1}, X_{i_2}, ..., X_{i_k}$ from S (where k arbitrary natural number) such that [3]

$$y = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_1}^{t_1}(x, v_x, u_x))...)), v_x \in Q_x, u_x \in P_x \tag{1}$$

In addition, the terminal set is distinguished $N \subset M$. The pursuit task starts from the initial position x_0 and is considered complete if for some k switching on is in progress $y \in N$. The pursuer tries to bring the phase point to the set N, and the object being pursued, generally speaking, has the opposite goal.

Next, through $X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1})$ let's designate such points $x_1 \in M$ that, with any choice of the fleeing player $v_{x_1} \in Q_{x_1}$, the pursuing player can pick up such $u_{x_1} \in P_{x_1}$, what is the trajectory $X^{t_1}(x_1, v_{x_1}, u_{x_1})$ vector field, X_{i_1} passing through a point x_1 at $t = t_1$ hits some point of the terminal set $N : X_{i_1}^{t_1}(x_1, v_{x_1}, u_{x_1}) \in N$ in time t_1. Through $X_{i_2}^{t_2}(X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1}))$ let's designate such points $x_2 \in M$ that, with any choice of the fleeing player $v_{x_2} \in Q_{x_2}$. The pursuing player can

pick up such $u_{x_2} \in P_{x_2}$, what is the trajectory $X^{t_2}(x_2, v_{x_2}, u_{x_2})$ vector field, X_{i_2} passing through a point x_2 at $t = t_2$ hits some point in the set $X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1}) : X_{i_2}^{t_2}(x_2, v_{x_2}, u_{x_2}) \in X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1})$ in time $t_2 - t_1$ and so on. Through $X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...X_{i_2}^{t_2}(X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1}))...))$ let's designate such points $x_k \in M$ that, with any choice of the fleeing player $v_{x_k} \in Q_{x_k}$, the pursuing player can pick up such $u_{x_k} \in P_{x_k}$, what is the trajectory $X^{t_k}(x_k, v_{x_k}, u_{x_k})$ vector field, X_{i_k} passing through a point x_k at $t = t_k = T$ hits some point in the set

$$X_{i_{k-1}}^{t_{k-1}}(...(X_{i_2}^{t_2}(X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1})))...) : X_{i_k}^{t_k}(x_k, v_{x_k}, u_{x_k})) \in$$

$$X_{i_{k-1}}^{t_{k-1}}(...(X_{i_2}^{t_2}(X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1})))...)$$

in time $t_k - t_{k-1} = T - t_{k-1}$. We will denote the last set by

$$\Omega = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_2}^{t_2}(X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1})))...)). \tag{2}$$

Definition 2. We will say that in the game (1), (2) from the initial position $y \in M \setminus N$ it is possible to complete the pursuit if there are numbers $T = T(y)$ and piecewise constant functions $u_x : u_{x_i}(v_{x_i}, x_i, t_i) \in P_{x_i}$, $v_{x_i} \in Q_{x_i}$, $x_i \in M$, $t = t_1, t_2, ..., t_k = T$, such that for an arbitrary piecewise constant function $v_x : v_{x_i} \in Q_{x_i}$, $x_i \in M$, $t = t_1, t_2, ..., t_k = T$, trajectory of vector fields $X_{i_1}, X_{i_2}, ..., X_{i_k}$, where $u_{x_i} = u_{x_i}(v_{x_i}, x_i, t_i)$, $v_{x_i} = v_{x_i}(x_i, t_i)$ the condition is met $y = x_k = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_1}^{t_1}(x_1, v_{x_1}, u_{x_1}))...)) \in N$, at $t = t_k = T$.

Theorem. Let for some natural number k oswitching on is in progress

$$x_k \in \Omega = X_{i_k}^{t_k}(X_{i_{k-1}}^{t_{k-1}}(...(X_{i_2}^{t_2}(X_{i_1}^{t_1}(N, v_{x_1}, u_{x_1})))...)). \tag{3}$$

Then in the game (1) from the initial position $y \in M \setminus N$ it is possible to complete the pursuit within the time T.

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Threshold Analysis of the Schrodinger Operator for a Two-Fermion System on a Two-Dimensional Triangular Lattice

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We consider the operator $H_\lambda(k), k \in \mathbb{T}^2$, acting in the Hilbert space $L_2^o(\mathbb{T}^2) = \{f \in L_2(\mathbb{T}^2) : f(-q) = -f(q)\}$ by the formula

$$H_\lambda(k) = H_0(k) - \lambda V,$$

where

$$(H_0(k)f)(p) = E_k(p)f(p), \quad (Vf)(p) = (2\pi)^{-2}\int_{\mathbb{T}^2} V(p-s)f(s)ds, \quad f \in L_2^o(\mathbb{T}^2),$$

$$E_k(p) = \varepsilon(\frac{1}{2}k + p) + \varepsilon(\frac{1}{2}k - p)$$

Since

$$V(p-s) = \sum_{i=1}^{2} \cos p_i \cos s_i + \cos(p_1 - p_2)\cos(s_1 - s_2) + \sum_{i=1}^{2} \sin p_i \sin s_i + \sin(p_1 - p_2)\sin(s_1 - s_2),$$

the interaction operator (the perturbation operator) V acts in the Hilbert space $L_2^o(\mathbb{T}^2)$, by the formula

$$(Vf)(p) = \frac{1}{(2\pi)^2}[\sum_{i=1}^{2} \sin p_i \int_{\mathbb{T}^2} \sin s_i f(s)ds +$$

$$+ \sin(p_1 - p_2)\int_{\mathbb{T}^2} \sin(s_1 - s_2)f(s)ds], f \in L_2^o(\mathbb{T}^2)$$

The Main Results. According to the Weyl theorem [3] on the essential spectrum, the essential spectrum $\sigma_{ess}(H_\lambda(k))$ of $H_\lambda(k), k \in \mathbb{T}^2$, coincides with the spectrum $\sigma(H_0(k))$ of $H_0(k)$. i.e., $\sigma_{\text{ess}}(H_\lambda(k)) = \sigma(H_0(k)) = \big[m_k, M_k\big]$ with

$$m_k = \min_{p\in\mathbb{T}^2} E_k(p), \quad M_k = \max_{p\in\mathbb{T}^2} E_k(p).$$

We present $L_2^o(\mathbb{T}^2)$ into the direct sums of subspaces $L_2^o(\mathbb{T}^3) = L_1 \oplus L_2$
where

$$L_1 := \{f \in L_2^o(\mathbb{T}^3) : f(x_1, x_2) = f(x_2, x_1)\},$$

$$L_2 := \{f \in L_2^o(\mathbb{T}^3) : f(x_1, x_2) = -f(x_2, x_1)\}.$$

We introduce the integral operators $V_i : L_i \to L_i, \quad i \in \{1, 2\}$,

$$(V_1 f)(p) = \frac{1}{4\pi}(\sin p_1 + \sin p_2)\int_{\mathbb{T}^2} (\sin t_1 + \sin t_2)f(s)ds,$$

$$(V_2 f)(p) = \frac{\sin p_1 - \sin p_2}{4\pi}\int_{\mathbb{T}^2} (\sin t_1 - \sin t_2)f(s)ds + \frac{\sin(p_1 - p_2)}{2\pi}\int_{\mathbb{T}^2} \sin(s_1 - s_2)f(s)ds.$$

The identity

$$2(\sin p_1 \sin s_1 + \sin p_2 \sin s_2) = (\sin p_1 + \sin p_2)(\sin s_1 + \sin s_2) + (\sin p_1 - \sin p_2)(\sin s_1 - \sin s_2)$$

allows to represent the operator V as $V = V_1 + V_2$. Therefore, we obtain the following lemma.

Lemma. For $k = \mathbf{0}$ the operator $H_\lambda(\mathbf{0})$, is invariant with respect to L_i, $i \in \{1, 2\}$ and the operator $H_\lambda^i(\mathbf{0}) := H_\lambda(\mathbf{0})$ acts in L_i in the form $H_\lambda^i(\mathbf{0}) = \mathbf{H_0}(\mathbf{0}) - \lambda\mathbf{V_i}$,] [2].

Note that the point $p = \mathbf{0}$, $\mathbf{0} := (0,0) \in \mathbb{T}^2$ is the unique non-degenerate minimum of the function $E_{\mathbf{0}}(p) = 2[3 - (\sum_{i=1}^{2} \cos(p_i) + \cos(p_1 - p_2))]$. Therefore, the following integrals exist and are finite

$$a = \frac{1}{4\pi} \int_{\mathbb{T}^2} \frac{(\sin p_1 - \sin p_2)^2}{E_0(p)} dp, b = \frac{1}{4\pi} \int_{\mathbb{T}^2} \frac{(\sin p_1 - \sin p_2)\sin(p_1 - p_2)}{E_0(p)} dp, \tag{1}$$

$$c = \frac{1}{4\pi} \int_{\mathbb{T}^3} \frac{\sin^2(p_1 - p_2)}{E_0(p)} dp, d = \frac{1}{4\pi} \int_{\mathbb{T}^2} \frac{(\sin p_1 + \sin p_2)^2}{E_0(p)} dp. \tag{2}$$

We set

$$\lambda_1 = \frac{1}{d}, \quad \lambda_2 = \frac{a + b - \sqrt{(a-c)^2 + 4b^2}}{2(ac - b^2)}, \quad \lambda_3 = \frac{a + b + \sqrt{(a-c)^2 + 4b^2}}{2(ac - b^2)}, \tag{3}$$

where according to the Cauchy inequality $ac - b^2 > 0$.

The main result is the following theorem.

Theorem 1. For $\lambda > 0$ the following statements are true:

1. If $\lambda \in [0, \lambda_1]$, then the operator $H_\lambda^1(0)$ has no eigenvalue lying to the left of the essential spectrum.

2. If $\lambda \in (\lambda_1, \infty)$, then operator $H_\lambda^1(0)$ has an unique eigenvalue lying to the left of the essential spectrum.

Theorem 2. For $\lambda > 0$ the following statements are true:

1. If $\lambda \in [0, \lambda_2]$, then the operator $H_\lambda^2(0)$ has no eigenvalue lying to the left of the essential spectrum.

2. If $\lambda \in (\lambda_2, \lambda_3]$, then operator $H_\lambda^2(0)$ has an unique eigenvalue lying to the left of the essential spectrum.

3. If $\lambda \in (\lambda_3, \infty)$, then the operator $H_\lambda^2(0)$ has two eigenvalues lying to the left of the essential spectrum.

Numerical calculations show that the order of $\lambda_i, \quad i \in \{1,2,3\}$, is $\lambda_1 < \lambda_2 < \lambda_3$. Therefore, to describe the main results, we introduce the following set. Let

$$I_0 := (0, \lambda_1], \quad I_i := (\lambda_i, \lambda_{i+1}], \, i = 1,2, \quad I_3 := (\lambda_3, \infty).$$

The following statement derives from Theorems Theorem 1 and Theorem 2.

Theorem 3. For any $\lambda \in I_n$, $n \in \{0,1,2,3\}$ the operator $H_\lambda(0)$ has n eigenvalues taking into account the multiplicity, lying to the left of the essential spectrum.

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BINAR MUNOSABAT MATRITSASI YORDAMIDA BINAR MUNOSABATLARNING XOSSALARINI TEKSHIRISH

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Binar munosabatlar diskret matematika va algebra fanlarining muhim tushunchalaridan biri bo'lib, ularni matritsalar yordamida ifodalash ko'plab masalalarni qulay va algoritmik tarzda yechish imkonini beradi. Ushbu ishda chekli to'plamda berilgan binar munosabatning asosiy xossalarini uning matritsasi yordamida tekshirish masalasi qaraladi.

$A = \{a_1, a_2, \ldots, a_n\}$ chekli to'plam va unda aniqlangan R binar munosabat berilgan bo'lsin. Ushbu munosabatga mos matritsa $M_R = (m_{ij})$ quyidagicha aniqlanadi:

$$m_{ij} = \begin{cases} 1, & \text{agar } a_i R a_j, \\ 0, & \text{agar } a_i \not R a_j. \end{cases}$$

Binar munosabatning refleksivlik, simmetriklik, antisimmetriklik va tranzitivlik xossalarini matritsaning elementlari orqali bevosita aniqlash mumkin.

Agar R refleksiv munosabat bo'lsa, har bir $a_i \in A$ uchun $a_i R a_i$ munosabat o'rinli bo'ladi. Shuning uchun refleksivlik sharti matritsaning bosh diagonalidagi barcha elementlarning birga tengligi bilan ifodalanadi:

$$R \text{ — refleksiv} \quad \Longleftrightarrow \quad m_{11} = m_{22} = \cdots = m_{nn} = 1.$$

Munosabat simmetrik bo'lishi uchun har qanday $a_i, a_j \in A$ uchun $a_i R a_j$ dan $a_j R a_i$ kelib chiqishi kerak. Matritsaviy ko'rinishda bu shart

$$M_R = M_R^T$$

tenglik bilan ifodalanadi. Demak, munosabat matritsasi bosh diagonalga nisbatan simmetrik bo'lsa, munosabat simmetrik bo'ladi.

Antisimmetriklikni tekshirishda $i \neq j$ bo'lgan indekslar uchun m_{ij} va m_{ji} elementlarning bir vaqtda 1 ga teng bo'lmasligi talab qilinadi:

$$i \neq j \quad \Longrightarrow \quad (m_{ij}, m_{ji}) \neq (1, 1).$$

Binar munosabatning tranzitivlik xossasini tekshirishda Bul matritsalar ko'paytmasidan foydalanish samarali hisoblanadi. Agar M_R^2 matritsa Bul algebra qoidalari asosida hisoblangan bo'lsa, tranzitivlik sharti

$$M_R^2 \leq M_R$$

ko'rinishida ifodalanadi. Bunda tengsizlik matritsalarning elementlari bo'yicha tushuniladi.

Masalan,

$$M_R = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

matritsa berilgan bo'lsin. Uning Bul ko'paytmasi

$$M_R^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

bo'ladi. Bu yerda

$$(M_R^2)_{13} = 1, \qquad (M_R)_{13} = 0.$$

Shuning uchun $M_R^2 \leq M_R$ sharti bajarilmaydi va berilgan binar munosabat tranzitiv emas. Shu bilan birga, matritsaning bosh diagonalidagi barcha elementlar 1 ga teng bo'lgani sababli munosabat refleksivdir. Matritsa simmetrik emas, chunki $m_{12} = 1$, ammo $m_{21} = 0$.

Shunday qilib, binar munosabat matritsasining tuzilishini tahlil qilish orqali munosabatning asosiy xossalarini tez va aniq aniqlash mumkin. Ayniqsa, tranzitivlikni tekshirishda Bul matritsalar ko'paytmasidan foydalanish hisoblash jarayonini sezilarli darajada soddalashtiradi. Mazkur yondashuv diskret matematika, algebra va matematik mantiq masalalarini algoritmlashtirishda muhim ahamiyatga ega.

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ON THE GEOMETRY OF COMPLETELY INTEGRABLE HAMILTONIAN SYSTEMS

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Hamilton (1834) and Jacobi (1837) developed a general method for integrating dynamical equations based on the introduction of special canonical coordinates. This concept was further advanced by Bour and Liouville in 1855 [1,2,3]. Utilizing the Hamilton-Jacobi method, it was proven that Hamiltonian equations with n degrees of freedom can be integrated if n independent integrals in involution are known (i.e., their mutual Poisson brackets vanish identically). Within this framework, Jacobi, Liouville, Kovalevskaya, Clebsch, and other authors resolved a number of non-trivial dynamical problems, notably the rotation of a rigid body around a fixed point.

Systems possessing a complete set of integrals in involution are currently termed "completely integrable." In such problems, the Euler–Poisson system, which describes the motion of a heavy rigid body around a fixed point, plays a fundamental role. S.V. Kovalevskaya devoted her 1888 research to the integrability of the Euler–Poisson equations. Her groundbreaking results were highly esteemed by the Paris Academy of Sciences, which awarded her the Bordin Prize in 1888. Subsequently, A.T. Fomenko, who developed an analog of Morse theory for functions with degenerate singularities, played a significant role in studying the geometry of Liouville foliations

generated by completely integrable Hamiltonian systems. The continuation of this topological approach is Bott's theory, to which the present work is dedicated.

Definition 1 A skew-symmetric differential 2-form ω is called a *symplectic structure* on a smooth manifold M if it satisfies two conditions:

1) ω is closed, meaning $d\omega = 0$;

2) ω is non-degenerate at every point of the manifold, meaning that in local coordinates $\det \Omega(x) \neq 0$, where $\Omega(x) = (\omega_{ij}(x))$ is the matrix of the form.

A manifold equipped with a symplectic structure is called a *symplectic manifold*. All symplectic manifolds are of even dimension and are inherently orientable.

Definition 2 Let M be a symplectic manifold and $H : M \to \mathbb{R}$ be a smooth function. The *Hamiltonian vector field* associated with H is the unique smooth vector field $\operatorname{sgrad} H$ on M that satisfies

$$\omega(\xi, \operatorname{sgrad} H) = \xi(H),$$

where ξ is an arbitrary tangent vector, and $\xi(H)$ is the directional derivative of the function H along ξ.

The equations governing the flow of $\operatorname{sgrad} H$ are known as the *Hamiltonian system* for the *Hamiltonian* H. The Hamiltonian vector field corresponding to $H(p, q) = H(x_1, \ldots, x_n, x_{n+1}, \ldots, x_{2n})$ is expressed as

$$\operatorname{sgrad} H = \sum_{i,j=1}^{2n} \omega^{ij} \frac{\partial H}{\partial x_j} \frac{\partial}{\partial x_i},$$

where ω^{ij} are the components of the inverse matrix to Ω.

Hamiltonian vector fields generate Hamiltonian systems. Questions about the geometry of vector fields and the structure of orbits of vector fields have been considered in numerous studies.

Definition 3. The *Poisson bracket* is a bilinear operation on the space of all smooth functions on a symplectic manifold M, assigning a smooth real-valued function $\{F, H\}$ to each pair $F, H \in C^\infty(M)$ according to the rule:

$$\{F, H\} = \omega(\operatorname{sgrad} F, \operatorname{sgrad} H) = \operatorname{sgrad} F(H).$$

Darboux's Theorem For any point of a symplectic manifold M^{2n}, there exists an open neighborhood with local regular coordinates $p_1, \ldots, p_n, q_1, \ldots, q_n$ in which the symplectic structure ω assumes the canonical form $\omega = \sum dp_i \wedge dq_i$.

In these canonical coordinates $\{p, q\}$, the Hamiltonian system is represented by the following system of ordinary differential equations:

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}, \quad \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \quad i = 1, \ldots, n.$$

Remark Let M be a symplectic manifold and $F, H : M \to \mathbb{R}$ be smooth functions with corresponding Hamiltonian vector fields $\operatorname{sgrad} F$, $\operatorname{sgrad} H$. The Hamiltonian vector field associated with the Poisson bracket of F and H corresponds, up to a sign, to the Lie bracket of the two vector fields:

$$\operatorname{sgrad}\{F, H\} = [\operatorname{sgrad} F, \operatorname{sgrad} H].$$

Definition 4. Let M^{2n} be a symplectic manifold and $\operatorname{sgrad} H$ a Hamiltonian vector field with a smooth Hamiltonian H. The system is called *completely integrable in the Liouville sense* if there exists a set of smooth functions $F_1, \ldots, F_n$ such that:

1) $F_1, \ldots, F_n$ are first integrals of the vector field $\operatorname{sgrad} H$;

2) They are functionally independent on M (i.e., almost everywhere on M, their gradients are linearly independent);

3) $\{F_i, F_j\} = 0$ for all i and j;

4) The vector fields $\operatorname{sgrad} F_i$ are complete, meaning their integral trajectories are defined over the entire real line.

Definition 5. The decomposition of the manifold M^{2n} into connected components of the common level surfaces of the integrals $F_1, \ldots, F_n$ is called the *Liouville foliation* corresponding to the integrable system $v = \operatorname{sgrad} H$.

Since $F_1, \ldots, F_n$ are preserved by the flow of v, every leaf of the Liouville foliation is an invariant surface. This foliation consists of regular leaves (which densely fill M almost everywhere) and singular leaves (which occupy a set of measure zero).

0.1 Geometry of the Regular Leaf

In the Goryachev-Chaplygin case of rigid body dynamics, the center of mass is constrained to lie within the equatorial plane of the ellipsoid of inertia. Moreover, the initial conditions are selected such that the area integral constant equals zero. This permits the introduction of an additional partial integral, reducing the equations of motion to quadratures [**?**]. In terms of special canonical variables L, G, l, g, the Hamiltonian takes the form:

$$H = \frac{G^2 + 3L^2}{8C} + \mu\left(\frac{L}{G}\cos l \sin g + \sin l \cos g\right), \quad \mu = \epsilon r, \quad C = \text{const}.$$

Applying the coordinate transformations

$$L = -p_1 - p_2, \quad G = p_2 - p_1, \quad q_1 = -l - g, \quad q_2 = g - l,$$

yields the Hamiltonian:

$$H(p_1, p_2, q_1, q_2) = \frac{p_1^3 - p_2^3}{2C(p_1 - p_2)} - \mu\left(\frac{p_1 \sin q_1}{p_1 - p_2} + \frac{p_2 \sin q_2}{p_1 - p_2}\right). \tag{1}$$

The corresponding Hamiltonian system $v = \operatorname{sgrad} H$ is:

$$\begin{aligned} p_1' &= \frac{\mu p_1 \cos q_1}{p_1 - p_2}, \quad q_1' = \frac{2p_1 + p_2}{2C} + \mu\left(\frac{p_1 \sin q_1}{(p_1 - p_2)^2} + \frac{p_2 \sin q_2}{(p_1 - p_2)^2}\right), \\ p_2' &= \frac{\mu p_2 \cos q_2}{p_1 - p_2}, \quad q_2' = \frac{p_1 + 2p_2}{2C} - \mu\left(\frac{p_1 \sin q_1}{(p_1 - p_2)^2} + \frac{p_2 \sin q_2}{(p_1 - p_2)^2}\right). \end{aligned} \tag{2}$$

Theorem. Every regular leaf of the Liouville foliation generated by the Hamiltonian system (2) (corresponding to the energy function of a rigid body rotating around a fixed point in the Goryachev-Chaplygin case), subject to the conditions

$$\begin{aligned} &p_1 \neq 0, \quad q_1 \neq \frac{\pi}{2} + \pi k, \\ &p_2 \neq 0, \quad q_2 \neq \frac{\pi}{2} + \pi k, \quad k \in \mathbb{N}, \end{aligned}$$

is a two-dimensional torus with non-zero Gaussian torsion.

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ON THE BOTT PROPERTY OF THE SECOND INTEGRAL IN THE GENERALIZED HÉNON–HEILES SYSTEM

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The global topology of the Liouville foliation of an integrable Hamiltonian system with two degrees of freedom is determined by the structure of the critical set of the momentum map. A key assumption of the Fomenko–Bolsinov theory is that the additional first integral F, restricted to a regular isoenergy surface $Q^3 = \{H = h\}$, is a *Bott function*, i.e. its critical set is a disjoint union of nondegenerate critical submanifolds [1]. In this work we verify the Bott property for the Sawada–Kotera case of the generalized Hénon–Heiles system [2].

Let $M^4 = \mathbb{R}^4$ be equipped with the canonical symplectic form $\omega = dp_1 \wedge dq_1 + dp_2 \wedge dq_2$ and consider the Hamiltonian

$$H(p,q) = \frac{1}{2}\big(p_1^2 + p_2^2\big) + \lambda\big(q_1^2 + q_2^2\big) + \mu\Big(q_1^2 q_2 + \frac{1}{3}q_2^3\Big), \qquad \lambda, \mu \in \mathbb{R}, \quad \lambda\mu \neq 0. \tag{1}$$

In the Sawada–Kotera case the system $\operatorname{sgrad} H$ (Hamiltonian vector field) possesses the second functionally independent first integral

$$F(p,q) = p_1 p_2 + 2\lambda q_1 q_2 + \mu q_1 q_2^2 + \frac{\mu}{3}q_1^3, \qquad \{F, H\} \equiv 0. \tag{2}$$

Hence the momentum map $\mathcal{F} = (H, F)\colon M^4 \to \mathbb{R}^2$ defines a Liouville foliation on M^4. Denote by K_1 the set of points at which $\operatorname{rank} d\mathcal{F} = 1$. At $x \in K_1$ there exist $\alpha, \beta \in \mathbb{R}$, unique up to proportionality, with $\alpha\, dF(x) + \beta\, dH(x) = 0$.

Let L be one dimensional subspace of tangent space at the point x of M^4 generated by vectors $sgradH$ and $sgradF$, L' be symplectic ortogonal space to L. By the criterion of Bolsinov and Fomenko [1], the point x is nondegenerate if and only if the rank of the symmetric 2-form $\alpha\, d^2F(x) + \beta\, d^2H(x)$, restricted to the symplectic transversal L' to the orbit through x, equals 2.

Writing the derivatives in the order (p_1, p_2, q_1, q_2) and putting $H_3 = 2q_1(\lambda + \mu q_2)$, $H_4 = 2\lambda q_2 + \mu(q_1^2 + q_2^2)$, we obtain $dH = (p_1, p_2, H_3, H_4)$ and $dF = (p_2, p_1, H_4, H_3)$, which gives the following structural relation.

Lemma 1. *In the Sawada–Kotera case the differential of F is obtained from the differential of H by the transpositions $(1 \leftrightarrow 2)$ and $(3 \leftrightarrow 4)$ of its components.*

Substituting Lemma 1 into $\alpha\,dF + \beta\,dH = 0$ yields $(\alpha^2 - \beta^2)p_i = 0$, so that $\alpha = \pm\beta$. For $\alpha = \beta$ the system reduces to $p_1 + p_2 = 0$, $H_3 + H_4 = (q_1 + q_2)[2\lambda + \mu(q_1 + q_2)] = 0$, and for $\alpha = -\beta$ it reduces to $p_1 = p_2$, $H_3 - H_4 = (q_1 - q_2)[2\lambda - \mu(q_1 - q_2)] = 0$. Consequently the critical set of $\mathcal{F}$ is the union of four two-dimensional planes

$$\begin{aligned} &N_1 = \{p_1 + p_2 = 0,\ q_1 + q_2 = 0\}, \quad N_2 = \{p_1 + p_2 = 0,\ q_1 + q_2 = -2\lambda/\mu\}, \\ &N_3 = \{p_1 = p_2,\ q_1 = q_2\}, \qquad\qquad N_4 = \{p_1 = p_2,\ q_1 - q_2 = 2\lambda/\mu\}. \end{aligned} \tag{3}$$

Their projections onto the configuration plane (q_1, q_2) are four straight lines intersecting at the points $O_1 = (0, 0)$, $O_2 = (\lambda/\mu, -\lambda/\mu)$, $O_3 = (-\lambda/\mu, -\lambda/\mu)$, $O_4 = (0, -2\lambda/\mu)$, which are exactly the projections of the rank-zero stratum K_0 and coincide with the minimum and the three saddle points of the potential of (1).

The Hessians of H and F in the coordinates (p_1, p_2, q_1, q_2) are

$$d^2H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2(\lambda + \mu q_2) & 2\mu q_1 \\ 0 & 0 & 2\mu q_1 & 2(\lambda + \mu q_2) \end{pmatrix}, \quad d^2F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2\mu q_1 & 2(\lambda + \mu q_2) \\ 0 & 0 & 2(\lambda + \mu q_2) & 2\mu q_1 \end{pmatrix}. \tag{4}$$

Theorem 1. *Let $Q^3_h = \{H = h\} \subset M^4$ be a regular compact isoenergy surface which meets the smooth arcs of the bifurcation diagram $\Sigma \subset \mathbb{R}^2$ transversally and avoids the singular points of Σ. Then the integral F restricted to Q^3_h is a Bott function whose connected critical submanifolds are circles.*

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Evklid va Pogorelov aksiomatik tizimlarining o'ziga xos jihatlari, boshlang'ich tushunchalari hamda ular orasidagi munosabatlar ilmiy asosda tahlil qilinadi. Har bir aksiomatik sistema matematikani aksiomatik qurishda turli yondashuvlarni taklif etadi va ularning mantiqiy qat'iyligi hamda qo'llanish sohalari bir-biridan farq qiladi. Tadqiqot davomida geometrik tushunchalar - nuqta, to'g'ri chiziq, tekislik, uzunlik va burchak kabi asosiy obyektlarning qaysi aksiomatik tizimga mansubligi aniqlanadi. Shuningdek, uchburchak, to'rtburchak,

kvadrat, aylana kabi tushunchalari va Fales, Pifogar kabi teoremalari kabilar boshlang'ich tushunchalar, munosabatlar va ta'riflar yordamida aksiomatik yondashuv asosida ko'rib chiqiladi. Aksiomalar sistemalarining o'zaro taqqoslanishi orqali ularning afzalliklari va kamchiliklari yoritilgan. Xususan, Evklid aksiomatikasi intuitivlikka asoslangan. Olingan natijalar matematik tushunchalarni chuqurroq anglash va ularni ta'lim jarayonida samarali qo'llash imkonini beradi. Matematikaning nazariy asoslarini mustahkamlashda aksiomatik yondashuv alohida ahamiyat kasb etadi. Aksiomatik metod yordamida matematik tushunchalar minimal boshlang'ich tushunchalar va isbot talab qilinmaydigan aksiomalar asosida qat'iy-mantiqiy tizim sifatida quriladi. Bu esa matematik nazariyalarning izchilligi, ziddiyatsizligi va umumlashganligini ta'minlaydi. Geometriya va ehtimollar nazariyasining rivojlanish jarayonida turli olimlar tomonidan turlicha aksiomatik tizimlar ishlab chiqilgan bo'lib, ular orasida Evklid va Pogorelov aksiomatikalari muhim o'rin egallaydi. Ushbu tizimlar nafaqat boshlang'ich tushunchalar to'plami bilan, balki ular orasidagi munosabatlarni ifodalovchi aksiomalar majmuasi bilan ham bir-biridan farq qiladi. Natijada, har bir aksiomatik sistema matematik obyektlarni turlicha talqin qilish imkonini beradi.

№	Evklid aksiomatikasi	Pogorelov aksiomatikasi
Boshlang'ich tushunchalar	1) nuqta, 2) chiziq, 3) tekislik	1) Nuqta, 2) to'g'ri chiziq, 3) tekislik, 4) tegishli, 5) orasida yotadi, 6) uzunlik, 7) burchakning gradus o'lchovi
Boshlang'ich tushunchalar orasidagi munosabatlar	Bog'liqlik, Tartib, Kongruentlik, Uzluksizlik, Paralellik.	Tegishlilik, Tartib, Kesma va burchaklarning o'lchovi, Berilgan burchakka teng uchburchakning mavjudligi, Uzunligi berilgan kesmaning mavjudligi, Paralellik
Aksiomalar sistemasini taqqoslash	Intuitiv va geometrik tasavvurlarga asoslangan, qat'iylik pastroq	Maktab geometriyasiga mos, sodda va amaliy
Boshlang'ich tushunchalar ular orasidagi munosabatlarning va aksiomalar sistemasi yordamida geometrik tushunchalarning tariflanishi	Uchburchak, To'rtburchak	To'rtburchak, Kvadrat, Vektor, Aylana
Yuqoridagi hammasini isbot talab qiladigan jumlalarning isbotlanish holatini taqqoslash	Ba'zi isbotlar to'liq qat'iy emas	Maktab darajasida yetarli, lekin to'liq formal emas
	Yutug'i: bog'liqlik, tartib, tenglik, paralellik (tasavvurga asoslangan). **Kamchiligi:** uzluksizlik (yetarlicha aniq emas), masofa tushunchasi yo'q, burchaklar tengligi yo'q	**Yutug'i:** bog'liqlik, tartib, tenglik, masofa tushunchasi, burchaklar tengligi. **Kamchiligi:** uzluksizlik (yetarlicha aniq emas)

Evklid aksiomasi 1.1) Har qanday A va B nuqtalar orqali bitta va faqat bitta to'g'ri chiziq o'tadi. Pogorelov aksiomasi 1.1) To'g'ri chiziq qanday bo'lmasin, to'g'ri chiziqqa tegishli va tegishli bo'lmagan nuqtalar mavjud. Evklid aksiomasi: Kongruentlik aksiomalari
III1 aksioma. l1 to'g'ri chiziqda A va B nuqtalar shu to'g'ri chiziqda yoki boshqa l2 to'g'ri

chiziqda yotuvchi A' nuqta berilgan bo'lsin, u holda berilgan A' nuqtaga ko'ra l2 to'g'ri chiziqda yo'nalishida va A'B' kesma AB kesmaga kongreng bo'ladigan har doim bitta va faqat bitta B' nuqta toppish mumkin. Kesmalarning kongruenglik munosabati AB= A'B' kabi belgilandi.
III2 aksioma. Bir kesmaga kongrueng bo'lgan kesmalar o'zaro kongrueng.
Natija . Har bir kesma o'ziga kongrueng. III1 va III2 aksiomalardan kongrueng munosabatining simmetrikligi va tranzitivligi kelib chiqadi.
III3 aksioma. To'g'ri chiziqda umumiy ichki nuqtalarga ega bo'lmagan AB va BC kesmalar berilgan bo'lsin. A'B' va B'C' kesmalar shu to'g'ri chiziqda yoki boshqa to'g'ri chiziqda yotuvchi va umumiy ichki nuqtalarga ega bo'lmagan kesmalar bo'lsin. Agar bunda AB=A'B' va BC=B'C' munosabatlar o'rinli bo'lsa, u holda AC=A'C' munosabat o'rinli. Xulosa qilib aytganda, turli aksiomatik tizimlar matematikaning turli yo'nalishlarini shakllantirish va rivojlantirishda muhim nazariy asos vazifasini bajaradi. Evklid aksiomatikasi o'zining soddaligi va intuitivligi bilan ajralib tursa-da, uning mantiqiy qat'iyligi zamonaviy talablar nuqtai nazaridan to'liq mukammal emas. Pogorelov aksiomatikasi esa maktab geometriyasiga mosligi va amaliy jihatdan qulayligi bilan ahamiyatlidir.

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FINDING THE MINKOWSKI DIFFERENCE OF CUBES GIVEN IN THREE-DIMENSIONAL EUCLIDEAN SPACE

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In convex geometry, the Minkowski operations on sets, which have very important applications, are geometric operations in the sense that they are independent of the nature of the elements that make up the sets. These operations are used to solve problems in fields such as robotics, computer-aided design, image processing, collision detection, geometric computation, and optimal control.

Definition 1 [1]. Let non-empty sets $X, Y \subset \mathbb{R}^n$ be given in the space $\mathbb{R}^n$. The geometric (or Minkowski) sum and difference of the sets X and Y are respectively defined as the following sets:

$$X + Y = \{x + y : x \in X,\ y \in Y\}, \quad X \dot{-} Y = \{z \in \mathbb{R}^n : z + Y \subset X\}.$$

The computation of the Minkowski difference of certain sets and the properties of Minkowski operations are discussed in [2, 3, 4]. Below, we develop an algorithm for finding the Minkowski difference of cubes given in the Euclidean space $\mathbb{R}^3$.

To uniquely define a three-dimensional cube in the Euclidean space $\mathbb{R}^3$, it is sufficient to specify four vertices that are not coplanar and lie on the edges emanating from a single vertex. Suppose the cubes C^A and C^B are given by the vertices A, A_1, A_2, A_3 and B, B_1, B_2, B_3, respectively (Fig. 1). To find the geometric dimensions and exact coordinates of the shape resulting from the difference $C^A \dot{-} C^B$, we perform the following sequence of steps. 1) first, from

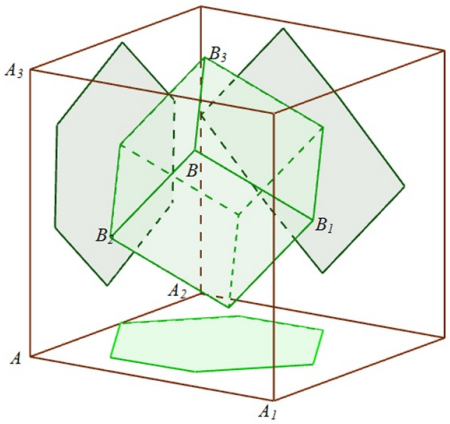


Fig. 1. Minkowski difference of cubes

the given vertices of the cubes C^A and C^B, the coordinates of three vectors corresponding to their edges are determined. That is, if $A = \{\alpha^1, \alpha^2, \alpha^3\}$, $A_1 = \{\alpha_1^1, \alpha_1^2, \alpha_1^3\}$, $A_2 = \{\alpha_2^1, \alpha_2^2, \alpha_2^3\}$, $A_3 = \{\alpha_3^1, \alpha_3^2, \alpha_3^3\}$ are the vertices of the cube C^A, and $B = \{\beta^1, \beta^2, \beta^3\}$, $B_1 = \{\beta_1^1, \beta_1^2, \beta_1^3\}$, $B_2 = \{\beta_2^1, \beta_2^2, \beta_2^3\}$, $B_3 = \{\beta_3^1, \beta_3^2, \beta_3^3\}$ are the vertices of the cube C^B, then we find the vectors:

$$\vec{a}_1 = \overrightarrow{AA_1} = \{\alpha_1^1 - \alpha^1, \alpha_1^2 - \alpha^2, \alpha_1^3 - \alpha^3\},$$

$$\vec{a}_2 = \overrightarrow{AA_2} = \{\alpha_2^1 - \alpha^1, \alpha_2^2 - \alpha^2, \alpha_2^3 - \alpha^3\},$$

$$\vec{a}_3 = \overrightarrow{AA_3} = \{\alpha_3^1 - \alpha^1, \alpha_3^2 - \alpha^2, \alpha_3^3 - \alpha^3\},$$

and

$$\vec{b}_1 = \overrightarrow{BB_1} = \{\beta_1^1 - \beta^1, \beta_1^2 - \beta^2, \beta_1^3 - \beta^3\},$$

$$\vec{b}_2 = \overrightarrow{BB_2} = \{\beta_2^1 - \beta^1, \beta_2^2 - \beta^2, \beta_2^3 - \beta^3\},$$

$$\vec{b}_3 = \overrightarrow{BB_3} = \{\beta_3^1 - \beta^1, \beta_3^2 - \beta^2, \beta_3^3 - \beta^3\};$$

2) Using the vectors $\vec{b}_1, \vec{b}_2, \vec{b}_3$, we find the vectors corresponding to all diagonals of the cube C^B as follows:

$$\vec{d}_1 = \vec{b}_1 + \vec{b}_2 + \vec{b}_3,$$

$$\vec{d}_2 = -\vec{b}_1 - \vec{b}_2 + \vec{b}_3,$$

$$\vec{d}_3 = -\vec{b}_1 + \vec{b}_2 + \vec{b}_3,$$

$$\vec{d}_4 = \vec{b}_1 - \vec{b}_2 + \vec{b}_3.$$

3) We compute the projections of these vectors onto the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$. The projection is calculated using the following formula:

$$\left|\mathrm{proj}_{\vec{a}_j} \vec{d}_i\right| = \frac{\left|\langle \vec{a}_j, \vec{d}_i \rangle\right|}{|\vec{a}_j|}, \quad i = \overline{1,4}, \quad j = \overline{1,3}.$$

4) At this step, a verification is performed: the calculated lengths of the projections are compared with the length of the vector $\vec{a}_1$. If any of the projections is greater than the length of the vector $\vec{a}_1$, then the Minkowski difference is empty set, and the process is terminated.

If the length of any projection is equal to the vector $\vec{a}_1$, then the Minkowski difference consists of a single point, whose coordinates are:

$$D_0 = \left\{ \frac{\sum(\alpha_i^1 - \beta_i^1) - (\alpha^1 - \beta^1)}{2}, \frac{\sum(\alpha_i^2 - \beta_i^2) - (\alpha^2 - \beta^2)}{2}, \frac{\sum(\alpha_i^3 - \beta_i^3) - (\alpha^3 - \beta^3)}{2} \right\},$$

where $i = 1, 2, 3$.

If all projections are smaller than the length of the vector $\vec{a}_1$, the process of finding the difference continues with the next step.

5) If all projections are smaller than the length of the vector $\vec{a}_1$, then the resulting vectors $\vec{c}_1, \vec{c}_2, \vec{c}_3$ corresponding to the sides of the difference $C^A \dot{-} C^B$ are directed the same as the vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ of the cube C^A, and their lengths are found using the following formulas:

$$|\vec{c}_1| = |\vec{a}_1| - \max_{i=\overline{1,4}} \left| \text{proj}_{\vec{a}_1} \vec{d}_i \right|,$$

$$|\vec{c}_2| = |\vec{a}_2| - \max_{i=\overline{1,4}} \left| \text{proj}_{\vec{a}_2} \vec{d}_i \right|,$$

$$|\vec{c}_3| = |\vec{a}_3| - \max_{i=\overline{1,4}} \left| \text{proj}_{\vec{a}_3} \vec{d}_i \right|.$$

6) To place the constructed vectors, it is necessary to find the vertex D of the difference $C^A \dot{-} C^B$ corresponding to the vertex A. For this purpose, the distances from all 8 vertices of the cube C^B (the coordinates of the remaining vertices are found from the points B, B_1, B_2, B_3) to the planes AA_1A_3, AA_1A_2, and AA_2A_3 are computed, and the points at the shortest distance from these planes are determined.

7) Through the point closest to the plane AA_1A_3, we construct the plane σ_1 parallel to AA_1A_3. Similarly, we construct the planes σ_2 and σ_3 corresponding to the planes AA_1A_2 and AA_2A_3, respectively.

8) The common intersection point of the planes $\sigma_1, \sigma_2, \sigma_3$ is found and denoted by $\tilde{B}$.

9) To find the coordinates of the vertex D of the difference $C^A \dot{-} C^B$, the coordinates of the point $\tilde{B}$ are subtracted from the coordinates of the point A.

$$D = A - \tilde{B}$$

The remaining vertices of the difference are found by placing the vectors obtained in step 5) at the point D.

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Geometry of Foliations Generated by Orbits of Hamiltonian Vector Fields

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Let $V(M)$ is the set of all smooth (class C^∞) vector fields defined on M. The set $V(M)$ is a linear space over the field of real numbers and Lie algebra in which a binary operation is Lie bracket $[X, Y]$ of vector fields $X, Y \in V(M)$.

Let us consider a set $D \subset V(M)$ and denote the smallest Lie subalgebra containing D by $A(D)$. The family D may contain finitely or infinitely many smooth vector fields.

For a point $x \in M$, by $t \to X^t(x)$ we denote integral curve of the vector field X passing through the point x for $t = 0$. The map $t \to X^t(x)$ is defined in some region $I(x)$, which in the general case depends not only from the field X, but also from the starting point x.

Definition 1. The orbit $L(x)$ of a system D of vector fields through a point x is the set of points y in M such that there exist $t_1, t_2, ..., t_k \, \epsilon \, R$ and vector fields $X_1, X_2, ..., X_k \, \epsilon \, D$ such that

$$y = X_k^{t_k}(X_{k-1}^{t_k-1}(...(X_1^{t_1}(x)))),$$

where k is an arbitrary positive integer.

Definition 2. A *Poisson bracket* on a smooth manifold M is an operation that assigns a smooth real-valued function $\{F, H\}$ on M to each pair F, H of smooth, real-valued functions, with the basic properties.

(a) Bilinearity

$$\{\lambda F + \mu P, H\} = \lambda\{F, H\} + \mu\{P, H\},$$

$$\{F, \lambda H + \mu P\} = \lambda\{F, H\} + \mu\{F, P\}, \ \lambda, \mu \in \mathbb{R};$$

(b) Skew-Symmetry

$$\{F, H\} = -\{H, F\};$$

(c) Jacobi Identity

$$\{\{F, H\}, P\} + \{\{P, F\}, H\} + \{\{H, P\}, F\} = 0;$$

(d) Leibniz' Rule

$$\{F, H \cdot P\} = \{F, H\} \cdot P + H \cdot \{F, P\}.$$

(Here $[\cdot]$ denotes the ordinary multiplication of real-valued functions.)

Definition 3. Let M be a Poisson manifold and $H : M \to R$ a smooth function. The *Hamiltonian vector field* associated with H is the unique smooth vector field $sgradH$ on M satisfying

$$sgradH(F) = \{F, H\} = -\{H, F\}$$

for every smooth function $F : M \to R$.

The equations governing the flow of $sgradH$ are referred to as *Hamilton's equations* for the *Hamiltonian function* H.

In the case of the canonical Poisson bracket on R^m $(m = 2n+l)$, the Hamiltonian vector field to any $H(p, q, z)$, as clearly, corresponds

$$sgradH = \sum_{i=1}^{n} \left(\frac{\partial H}{\partial p^i} \frac{\partial}{\partial q^i} - \frac{\partial H}{\partial q^i} \frac{\partial}{\partial p^i} \right).$$

Let us consider Hamiltonian $H : \mathbb{R}^4 \to \mathbb{R}$ on the Euclidean four dimensional space $\mathbb{R}^4$ with cartezian coordinates p_1, p_2, q_1, q_2 which are given by the formulas

$$H(p_1, p_2, q_1, q_2) = \frac{p_1^2 + p_2^2}{2} + \frac{\omega_1^2 q_1^2 + \omega_2^2 q_2^2}{2} + \frac{k_1}{q_1^2} + \frac{k_2}{q_2^2} \tag{1}$$

Here, $\omega_1, \omega_2 > 0$ denote the frequencies of the harmonic oscillators, while $k_1, k_2 > 0$ represent the coupling constants of the centrifugal potential barriers. The presence of these centrifugal terms prevents the particle from reaching the origin $q_i = 0$, $(i = 1, 2)$ thereby restricting the physical motion to the non-singular domains.

Lemma 1. Let $M_C = \{(p_1, p_2, q_1, q_2) \in \mathbb{R}^2 \times (\mathbb{R} \setminus \{0\})^2 \mid H(p, q) = C\}$ be the energy level hypersurface (an isoenergy surface)defined by the Hamiltonian:

where $C > 0$, $\omega_i > 0$, and $k_i > 0$ $(i = 1, 2)$. Then, the level set M_C is a compact subset of $\mathbb{R}^4$.

Let us consider the Hamiltonian

$$H_1(p_1, p_2, q_1, q_2) = \frac{p_1^2}{2} + \frac{\omega_1^2 q_1^2}{2} + \frac{k_1}{q_1^2},$$

and Hamiltonian vector fields corresponding to H and H_1:

$$X_H = \left(-\omega_1^2 q_1 + \frac{2k_1}{q_1^3},\ -\omega_2^2 q_2 + \frac{2k_2}{q_2^3},\ p_1,\ p_2 \right),\ X_{H_1} = \left(-\omega_1^2 q_1 + \frac{2k_1}{q_1^3},\ 0,\ p_1,\ 0 \right). \tag{2}$$

Theorem 1. The orbits of the Hamiltonian vector fields given in (2) generate a singular foliation, the regular leaves of which are two-dimensional surfaces with zero Gauss curvature and zero normal torsion.

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KELI DARAXTIDAGI p-ADIK SPIN FAZOSIGA EGA POTTS MODELIDA OPERATORIY TENGLAMALAR BIFURKATSIYASI VA ASOSIY HOLATLAR TASNIFI

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1. KIRISH VA MATEMATIK ASOSLAR

Keli daraxtidagi panjara modellari statistik fizika va operatorlar nazariyasida muhim o'rin tutadi. Klassik modellardan farqli ravishda, spinlar fazosi p-adik sonlar maydonida ($\mathbb{Q}_p$) yoki uning butun sonlar halqasida ($\mathbb{Z}_p$) qaralganda, tizim ultrametrik va Arximedcha bo'lmagan (non-Archimedean) geometriya xossalariga ega bo'ladi. Bu holat modelning energetik strukturasi va asosiy holatlarining (ground states) tabiatini keskin o'zgartiradi.

Keli daraxtining $\Gamma^k = (V, E)$ iyerarxik tuzilishi p-adik sonlar maydonining topologik strukturasi bilan tabiiy ravishda uyg'undir. Har bir $x \in V$ uchga mos keluvchi spin qiymatlari $\sigma(x) \in \Phi$ topologik kompakt fazodan olinadi, bu yerda $\Phi \subset \mathbb{Q}_p$ (masalan, $\Phi = \mathbb{Z}_p$ yoki $\Phi = \{1, 2, \dots, p\}$).

2. GAMILTONIAN VA ULTRAMETRIK O'ZARO TA'SIR

Chekli $\Lambda \subset V$ sohadagi p-adik spinli Potts modelining Gamiltoniani quyidagicha aniqlanadi:

$$H(\sigma) = -J \sum_{\langle x,y\rangle \subset V} \delta_p(\sigma(x), \sigma(y)) - h \sum_{x \in V} \sigma(x) \tag{1}$$

Bu yerda $J, h \in \mathbb{Q}_p$ — parametrlar bo'lib, $\delta_p(u, v)$ o'zaro ta'sir potensiali p-adik normaga bog'liq ravishda aniqlanadi:

$$\delta_p(u, v) = \begin{cases} 1, & \text{agar } |u - v|_p \le p^{-\gamma} \text{ bo'lsa,} \\ 0, & \text{aks holda.} \end{cases}$$

Ultrametrik kuchli uchburchak tengsizligi $|x + y|_p \le \max(|x|_p, |y|_p)$ sababli, spinlar orasidagi o'zaro ta'sir cheksiz o'lchamli potensiallar fazosida invariant o'lchovlarning bifurkatsiyasini keltirib chiqaradi.

3. REKURRENT OPERATORLAR VA QO'ZG'ALMAS NUQTALAR

Daraxtning tarmoqlanish xossasiga ko'ra, $h_x \in \mathbb{Q}_p^p$ chegara shartlari potensiali kiritiladi. Tizimning asosiy holatlarini topish $T \to 0$ (ya'ni p-adik harorat darajasi $\beta \to \infty$) limitda quyidagi rekurrent operatorning qo'zg'almas nuqtalarini (fixed points) aniqlashga keltiriladi:

$$h_x = F(h_{x_1}, h_{x_2}, \dots, h_{x_k}) \tag{2}$$

$$h_x(i) = \sum_{j=1}^{k} \ln_p \left(\frac{\sum_{l \in \Phi} \exp_p(J\delta_p(i, l) + h_{x_j}(l))}{\sum_{l \in \Phi} \exp_p(h_{x_j}(l))} \right) \tag{3}$$

4. ASOSIY NATIJALAR: BIFURKATSIYA VA TASNIF

4.1. Binary Tree ($k = 2$) Holatida Operator Tahlili

$k = 2$ bo'lganda F operatori kvadratik ko'rinishga keladi: $h(i) = 2 \cdot f(h(i))$.

1. **Bir jinsli (Translatsion-invariant) Holatlar:** $|J|_p > 0$ bo'lganda p ta bir jinsli konstant asosiy holatlar mavjud: $\sigma^{(i)}(x) = i, \forall x \in V$.

2. **Kuchsiz Davriy (Weakly Periodic) Holatlar:** Juft va toq qatlamli simmetriyada $\frac{p(p-1)}{2}$ ta 2-davriy antiferromagnit asosiy holatlar shakllanadi.

4.2. Ternary Tree ($k = 3$) Holatida Bifurkatsiya

$k = 3$ bo'lganda F operatori kubik ko'rinishga keladi: $h_x = f(h_{x_1}) + f(h_{x_2}) + f(h_{x_3})$.

$\theta = \exp_p(J) - 1$ parametrining p-adik normasiga ko'ra bifurkatsiya nuqtasi quyidagicha aniqlanadi:

- $|\theta|_p \le p^{-\frac{1}{p-1}}$ holatida: Tizimda faqat standart chekli davriy asosiy holatlar mavjud.
- $|\theta|_p > p^{-\frac{1}{p-1}}$ holatida: Kubik operatorda p-adik bifurkatsiya sodir bo'ladi. Natijada Keli daraxtining simmetriyalar guruhiga ($\mathrm{Aut}(\Gamma^3)$) nisbatan invariant bo'lmagan, kontinium quvvatga ega no-davriy (chaotic) asosiy holatlar to'plami hosil bo'ladi.

Teorema 1. *Γ^k Keli daraxtida p-adik spinlar fazosiga ega Potts modeli berilgan bo'lsin.*

1. *$k = 2$ bo'lganda har qanday $J \in \mathbb{Q}_p$ parametr uchun davriy va kuchsiz davriy asosiy holatlarning umumiy soni ko'pi bilan $p + \frac{p(p-1)}{2}$ tani tashkil qiladi.*
2. *$k = 3$ bo'lganda p-adik parametrning $|\theta|_p > p^{-\frac{1}{p-1}}$ shartida operatoriy tenglamada bifurkatsiya yuzaga keladi hamda $Aut(\Gamma^3)$ guruhiga nisbatan invariant bo'lmagan kontinium quvvatli no-davriy asosiy holatlar to'plami mavjud bo'ladi.*

Доказательство. Teoremaning isboti (2)-(3) rekurrent operatorlarning p-adik tahlili hamda Shauder va Banaxning qo'zg'almas nuqta haqidagi teoremalarini ultrametrik fazolarga tatbiq etish orqali amalga oshiriladi.

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Analysis of Free Energy for Constructive Gibbs Measures of the Ising Model on the Cayley Tree of Order three

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Let $\Gamma^k = (V, L), k \ge 1$ be the Cayley tree of order k, where V and L are the set of vertices and the set of all edges of the tree Γ^k, respectively. Γ^k can be represented as a group G_k, which is the free product of $k+1$ cyclic groups of the second order (see, e.g. [1], [2]).

Two vertices x and y are called nearest neighbors if there exists an edge $l \in L$ connecting them $l = \langle x, y\rangle$. A collection of nearest neighbor pairs $\langle x, x_1\rangle$, $\langle x_1, x_2\rangle, ..., \langle x_m, y\rangle$ is called path from x to y. By the path, one can define distance d on the tree. The distance between vertices x and y is the number of edges of the shortest path from x to y. Let us fix a vertex $x^0 \in V$ and call it as a root of the tree. Then for any natural number n, we introduce the following set:

$$W_n = \{x \in V \mid d(x, x^0) = n\}, \quad V_n = \bigcup_{m=0}^{n} W_m, \quad L_n = \{\langle x, y\rangle \in L : \ x, y \in V_n\},$$

The sets W_n and V_n are called a sphere and a ball with radius n, respectively.

For a given $x \in W_n$ we denote

$$S(x) = \{y \in W_{n+1} : d(x, y) = 1\}, \quad x \in W_n,$$

which is called a set of direct successors of x.

For $A \subseteq V$ a spin configuration on A is defined as a function

$$x \in A \to \sigma_A(x) \in \Phi = \{-1, 1\}.$$

The set of all configurations coincides with $\Omega_A = \Phi^A$. We denote $\Omega = \Omega_V$ and $\sigma = \sigma_V$.

We consider the Hamiltonian of the Ising model

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} \sigma(x)\sigma(y), \tag{1}$$

where $J \in R$, $\sigma(x) \in \Phi$ and $\langle x, y\rangle$ are nearest neighbors.

For every n, we then define a measure μ_n on Ω_{V_n} by

$$\mu_n(\sigma_n) = Z_n^{-1} \exp\{-\beta H(\sigma_n) + \sum_{x \in W_n} h_x \sigma(x)\}, \tag{2}$$

where $\beta = \frac{1}{T}$ (T is temperature, $T > 0$), $\sigma_n = \{\sigma(x), x \in V_n\} \in \Omega_{V_n}$, Z_n^{-1} is the normalizing factor, and

$$H(\sigma_n) = -J \sum_{\langle x,y\rangle \in L_n} \sigma(x)\sigma(y).$$

The compatibility condition for the measures $\mu_n, n > 1$ is

$$\sum_{\sigma^{(n)}} \mu_n(\sigma_{n-1}, \sigma^{(n)}) = \mu_{n-1}(\sigma_{n-1}), \tag{3}$$

where $\sigma^{(n)} = \{\sigma(x), x \in W_n\}$.

Let $\mu_n, n \geq 1$ be a sequence of measures on the sets Ω_{V_n} that satisfy compatibility condition (0.7). By the Kolmogorov theorem, we then have a unique limit measure μ on Ω such that

$$\mu(\{\sigma \mid_{V_n} = \sigma_n\}) = \mu_n(\sigma_n),$$

for every $n = 1, 2, ...$ and $\sigma_n \in \Phi^{V_n}$. Such a measure is called a splitting Gibbs measure corresponding to the Hamiltonian (0.7) and function $h_x, x \in V$.

Theorem 1 [1] The measures $\mu_n(\sigma_n), n = 1, 2, ...$, in (0.7) are compatible iff for any $x \in V$ the following equation holds:

$$h_x = \sum_{y \in S(x)} f(h_y, \theta), \tag{4}$$

where $f(x, \theta) = \arctan h(\theta \tanh x)$, $\theta = \tanh(J\beta)$.

Definition 1 Let K be a subgroup of G_k, $k \geq 1$. We say that a function $h = \{h_x \in R : x \in G_k\}$ is K- periodic if $h_{yx} = h_x$ for all $x \in G_k$ and $y \in K$. A G_k-periodic function h is called translation-invariant.

Definition 2 A Gibbs measure is called K-periodic if it corresponds to K- periodic function h.

Definition 3 A set of quantities $h = \{h_x, x \in G_k\}$ is called K- weakly periodic, if $h_x = h_{ij}$, for any $x \in H_i, x_\downarrow \in H_j$, where $x_\downarrow$-ancestor of x.

The theorem 1 establishes a one-to-one correspondence: a boundary condition solving the functional equation (4) uniquely determines a Gibbs measure and conversely. This means that finding all Gibbs measures is equivalent to finding all solutions to equation (4).

We aim to investigate the relationship between the boundary condition and the resulting free energy, specifically when the free energy exists:

$$F(\beta, h) = -\lim_{n\to\infty} \frac{1}{\beta \mid V_n \mid} \ln Z_n(\beta, h). \tag{5}$$

We are going to describe new Gibbs measures of the Ising model on the semi-infinite Cayley tree of order k = 3. We define the set of quantities $h = \{h_x, x \in V\}$ g as follows:

(A) If $h_x = h_2$, then we put h_1 on all direct successors of x.

If $h_x = h_1$, then we put h_2 and 0 on an arbitrary vertex and the other two vertices of direct successors of x, respectively.

If $h_x = 0$, then we put 0 on all direct successors of x.

The set of the boundary condition $\{0, h_1, h_2\}$ defined by the rule **(A)** must satisfy the boundary condition (4):

$$\begin{cases} h_1 = f(h_2, \theta), \\ h_2 = 3f(h_1, \theta). \end{cases} \tag{6}$$

It is clear that $h_1 = h_2 = 0$ is a solution of (6).

Using the formula $f(x, \theta) = \operatorname{arctan} h(\theta \tanh x) = \frac{1}{2} \ln \frac{(1+\theta)e^{2h}+(1-\theta)}{(1-\theta)e^{2h}+(1+\theta)}$, and introducing the notations $\alpha = \frac{1-\theta}{1+\theta}$, $z_i = e^{2h_i}$, $i = 1, 2$, we have $\alpha \in (0,\ 1) \cup (1,\ +\infty)$. By the last notations, (6) yields the following system of equations:

$$\begin{cases} z_1 = \frac{z_2+\alpha}{\alpha z_2+1}, \\ z_2 = \left(\frac{z_1+\alpha}{\alpha z_1+1}\right)^3. \end{cases} \tag{7}$$

By analyzing system of equation (7), we get the following:

Theorem 2 For the Gibbs measures for the Ising model corresponding to $h = \{h_x, x \in V\}$ which defined by rule **(A)** on the Cayley tree of order three the following statements hold:

1. if $\alpha \in (0; 2-\sqrt{3}) \cup (2+\sqrt{3}; \infty)$, then there are three Gibbs measures, moreover, two of them are non-translation-invariant Gibbs measures;
2. if $\alpha \in (2-\sqrt{3}; 1) \cup (1; 2+\sqrt{3})$, then there exists a unique translation-invariant Gibbs measure.

Remark 1 The translation-invariant measure corresponding to $(z_1, z_2) = (1, 1)$ found in Theorem 3.1 was studied [2], [3], [4].

We calculate the free energy corresponding to the obtained Gibbs measures

Theorem 3 The free energy of the Gibbs measures which are defined by rule **(A)** for the Ising model on the Cayley tree of order three is defined the following formula:

$$F_A(\beta, h_A) = -\frac{3}{4\beta} \ln(2\cosh(\beta J)), \tag{8}$$

where h_A are constructed according to the rule **(A)**.

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On Gibbs measures for the three-state Potts-SOS model with an external field

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We consider the following Potts-SOS model with an external field on a Cayley tree of order two [1]:

$$H(\sigma) = -J_P \sum_{\langle x,y\rangle\in L} \delta_{\sigma(x)\sigma(y)} - J_S \sum_{\langle x,y\rangle\in L} \mid \sigma(x) - \sigma(y) \mid + \alpha \sum_{x\in V} \sigma(x), \tag{1}$$

where $\sigma \in \{0,1,2\}^V$.

Remark 1. Note that model (1) coincides with the standard Potts-SOS model when $\alpha = 0$. Moreover, when $J_S = 0$, model (1) reduces to the Potts model with an external field, which was studied in [2]. On the other hand, when $J_P = 0$, model (1) coincides with the SOS model with an external field (see [3]).

Let

$$h : x \mapsto h_x = (h_{0,x}, h_{1,x}, h_{2,x}) \in \mathbb{R}^3, \qquad x \in V \setminus \{x^0\},$$

be a real-valued vector function. For each $n = 1, 2, \ldots$, consider the probability distributions $\mu^{(n)}$ on Φ^{V_n}, where $\Phi = \{0,1,2\}$, defined by

$$\mu^{(n)}(\sigma_n) = \frac{1}{Z_n} \exp\left\{-\beta H_n(\sigma_n) + \sum_{x\in W_n} h_{\sigma(x),x}\right\}, \qquad \sigma_n \in \Phi^{V_n}, \tag{2}$$

where $\beta = 1/T$ is the inverse temperature and Z_n is the partition function

$$Z_n = \sum_{\widetilde{\sigma}_n\in\Phi^{V_n}} \exp\left\{-\beta H_n(\widetilde{\sigma}_n) + \sum_{x\in W_n} h_{\widetilde{\sigma}(x),x}\right\}. \tag{3}$$

The family of probability distributions $\mu^{(n)}{}_{n\geq 1}$ is said to be compatible if, for every $n \geq 2$ and every $\sigma_{n-1} \in \Phi^{V_{n-1}}$,

$$\sum_{\omega_n\in\Phi^{W_n}} \mu^{(n)}\big(\sigma_{n-1} \vee \omega_n\big) = \mu^{(n-1)}(\sigma_{n-1}), \tag{4}$$

where $\sigma_{n-1} \vee \omega_n \in \Phi^{V_n}$ denotes the concatenation of the configurations σ_{n-1} and ω_n.

Definition 1. If the family of probability distributions $\mu^{(n)}{}_{n\geq 1}$ satisfies the compatibility condition (4), then there exists a unique probability measure μ on Φ^V such that, for every $n \geq 1$ and $\sigma_n \in \Phi^{V_n}$,

$$\mu\left(\{\sigma \in \Phi^V : \sigma \mid_{V_n} = \sigma_n\}\right) = \mu^{(n)}(\sigma_n).$$

The measure μ is called a splitting Gibbs measure (SGM) corresponding to the Hamiltonian H and the boundary function $h : x \mapsto h_x$.

The following theorem gives a necessary and sufficient condition on the function h for the family $\mu^{(n)}{}_{n\geq 1}$ to satisfy the compatibility condition.

Theorem 1. The probability distributions $\mu^{(n)}$, defined by (2), satisfy the compatibility condition (4) if and only if

$$h_x^* = \sum_{y \in S(x)} F\left(h_y^*, J_P, J_S, \alpha\right), \qquad x \in V \setminus \{x^0\}, \tag{5}$$

where $\beta = 1/T$ is the inverse temperature. Here,

$$h_x^* = \left(h_{0,x} - h_{2,x}, h_{1,x} - h_{2,x}\right)$$

is the normalized boundary-law vector. The vector-valued function

$$F(\cdot, J_P, J_S, \alpha) : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

is defined by

$$F(h, J_P, J_S, \alpha) = \big(F_0(h, J_P, J_S, \alpha), F_1(h, J_P, J_S, \alpha)\big),$$

where, for $h = (h_0, h_1) \in \mathbb{R}^2$,

$$F_i(h, J_P, J_S, \alpha) = \ln \frac{\sum\limits_{j \in \Phi} \exp\left(\beta J_P \delta_{ij} + \beta J_S \mid i - j \mid -\beta\alpha j + h_j\right)}{\sum\limits_{j \in \Phi} \exp\left(\beta J_P \delta_{2j} + \beta J_S \mid 2 - j \mid -\beta\alpha j + h_j\right)}, \qquad i = 0, 1. \tag{6}$$

Thus, for every collection of boundary functions satisfying the functional equation (5), there exists a unique splitting Gibbs measure. Conversely, every splitting Gibbs measure is associated with a solution of (5).

Theorem 2. For the three-state Potts-SOS model with an external field on a Cayley tree of order two, the number N_{SGM} of splitting Gibbs measures satisfies

$$N_{\mathrm{SGM}} \geq \begin{cases} 1, & \text{if } Q < 0, \\ 2, & \text{if } Q = 0, \\ 3, & \text{if } Q > 0, \end{cases}$$

where

$$Q := -\alpha_1\big(\alpha_1^2 + 1\big)\Big(4\alpha_1^4\theta^3 - \alpha_1^3\theta^4 - 18\alpha_1^3\theta^2 + 12\alpha_1^2\theta^3 - \alpha_1\theta^4 + 27\alpha_1^3 - 18\alpha_1\theta^2 + 4\theta^3 + 27\alpha_1\Big),$$

$$\theta := \exp(\beta J_S), \qquad J_P := 2J_S, \qquad \alpha_1 := \exp(-\beta\alpha).$$

Remark 2. Using the methods presented in [3] and the Gibbs measures obtained in Theorem 2, one can construct infinitely many distinct Gibbs measures for the model.

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Constructive Gibbs Measures for the Ising-Potts Model on the binary tree

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In this work, we use a constructive method for the Ising and Potts models as in paper [1] and [2], respectively. We find conditions for the existence of new non-translation-invariant Gibbs measures for the Ising-Potts model. Using known measures, we construct non-translation-invariant Gibbs measures by a constructive method.

Let the vectors $h_x = h_i = (x_i, x_i)$, $i = 1, 2, 3$, which correspond to the solutions found in the work [3]. This vectors correspond to translation-invariant Gibbs measures on the Cayley tree of order two.

Using the vectors h_i, we construct non-translation-invariant solutions of the equation

$$h_x^* = \sum_{y \in S(x)} F(h_y^*, \nu, \eta), \tag{1}$$

where $h_x^* = (h_{-1,x}^*, h_{+1,x}^*)$.

Consider the Cayley tree of order $k = 5$. Denote the set of vertices by V^5. For any $x \in V$, let $S_{k_0}(x)$ be an arbitrary subset of k_0 elements from $S(x)$.

First, using h_1 and h_3, we construct a collection of vectors h_x on V^5 satisfying equation (1) as follows:

(I) If at vertex x we have $h_x = h_1$, then on vertices $S_3(x)$ we assign $h_x = h_1$, and on the remaining vertices $S_2(x)$ we assign $h_x = h_3$. If at vertex x we have $h_x = h_3$, then on vertex $S_1(x)$ we assign $h_x = h_1$, and on the remaining vertices $S_4(x)$ we assign $h_x = h_3$.

From equation (1) we obtain the system:

$$\begin{cases} h_1 = 3F(h_1, \nu, \eta) + 2F(h_3, \nu, \eta), \\ h_3 = F(h_1, \nu, \eta) + 4F(h_3, \nu, \eta). \end{cases} \tag{2}$$

Since we consider the case $\eta = \frac{1}{2}$, we obtain

$$\frac{1}{2} \ln x_1 + \ln x_3 = 0. \tag{3}$$

Note that $x_i = x_i(\nu)$, $i = 1, 3$, hence the left-hand side depends only on ν. For values of ν satisfying (3), the constructed vectors h_x satisfy equation (1).

Now, using h_1, h_2, and h_3, define another collection:

(II) If at vertex x we have $h_x = h_2$, then on vertices $S_2(x)$ assign $h_x = h_2$, on $S_1(x)$ assign $h_x = h_1$, and on the remaining vertices assign $h_x = h_3$. If $h_x = h_3$, then assign h_1 on $S_1(x)$ and h_3 on $S_4(x)$. If $h_x = h_1$, assign h_1 on $S_3(x)$ and h_3 on $S_2(x)$.

Then we obtain:

$$\begin{cases} h_1 = 3F(h_1,\nu,\eta) + 2F(h_3,\nu,\eta), \\ h_2 = F(h_1,\nu,\eta) + 2F(h_2,\nu,\eta) + 2F(h_3,\nu,\eta), \\ h_3 = F(h_1,\nu,\eta) + 4F(h_3,\nu,\eta). \end{cases} \tag{4}$$

Analyzing systems (4) and (3), we obtain:

Theorem 1. For the Ising-Potts model at $\nu = \nu_c^* (\approx 11.44)$ on the Cayley tree of order five, there exist at least two constructive Gibbs measures obtained by schemes I and II.

Now let $k = p + q + 2$, $p, q \in \mathbb{N}$. Using $h_i = (x_i, x_i)(i = 1, 2, 3)$, construct new solutions:

(III) If $h_x = h_1$, assign h_1 on $S_{p+2}(x)$ and h_3 on $S_q(x)$. If $h_x = h_3$, assign h_1 on $S_p(x)$ and h_3 on $S_{q+2}(x)$.

Then:

$$\begin{cases} h_1 = (p+2)F(h_1,\nu,\eta) + qF(h_3,\nu,\eta), \\ h_3 = pF(h_1,\nu,\eta) + (q+2)F(h_3,\nu,\eta). \end{cases} \tag{5}$$

For $\eta = \frac{1}{2}$:

$$x_1^p \cdot x_3^q = 1. \tag{6}$$

Using the equality obtained from (III), we construct the fourth condition as follows:

(IV) If at vertex x we have $h_x = h_2$, then for every vertex in $S_2(x)$ we set the value $h_x = h_2$, for every vertex in $S_p(x)$ we set the value $h_x = h_1$, and for the remaining vertices in $S_q(x)$ we set the value $h_x = h_3$. If at vertex x we have $h_x = h_3$, then for every vertex in $S_p(x)$ we set the value $h_x = h_1$, and for the remaining vertices in $S_{q+2}(x)$ we set the value $h_x = h_3$. If at vertex x we have $h_x = h_1$, then for every vertex in $S_{p+2}(x)$ we set the value $h_x = h_1$, and for the remaining vertices in $S_q(x)$ we set the value $h_x = h_3$. The following equation will be added to the system:

$$h_2 = pF(h_1,\nu,\eta) + 2F(h_2,\nu,\eta) + qF(h_3,\nu,\eta).$$

Analysing (I)-(IV) we get the following theorem:

Theorem 2. For the Ising–Potts model with $a \in T(p,q)$ on the Cayley tree of order $k = p+q+2$, there exist at least two constructive Gibbs measures obtained by schemes III and IV, where

$$T(p,q) = \{a \in \mathbb{R} : \nu > \nu_c (\approx 11.44)\} \cap \{a \in \mathbb{R}_+ : (x_1(\nu))^p (x_3(\nu))^q = 1\}.$$

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GEOMETRY OF ORBITS OF ROTATIONAL VECTOR FIELDS

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Geometric methods for studying vector fields and the transformation groups generated by them play an essential role in modern differential geometry and the theory of nonlinear differential equations. One of the natural objects arising within this approach is the orbits of families of vector fields. According to Sussmann's theorem, the orbits of a family of smooth vector fields possess the structure of immersed submanifolds, which makes it possible to study them using the methods of Riemannian and differential geometry [1].

The relevance of studying the geometry of orbits is related, in particular, to the fact that symmetries of a differential equation make it possible to identify invariant variables, reduce the number of independent variables, and pass to simpler reduced equations. At the same time, a geometric description of the orbits themselves provides additional information that cannot be obtained solely from an algebraic analysis of symmetries: the metric properties of the orbits, their curvature, geodesics, and the features of their embedding in the ambient space characterize the geometry of the action of the corresponding transformation group [2,3].

Consider the family of vector fields $D = \{X, Y\}$ on $\mathbf{R^4}$, where

$$X = 3t\partial_t + x_1\partial_{x_1} + x_2\partial_{x_2} + u\partial_u, \qquad Y = -x_2\partial_{x_1} + x_1\partial_{x_2}. \tag{1}$$

For these fields, $[X, Y] = 0$, and the joint action of their flows has the form

$$(t, x, u) \mapsto (e^{3a}t, e^a R_b x, e^a u).$$

Definition. The orbit $L(x)$ of the family D of vector fields passing through a point x is defined as the set of all points $y \in \mathbf{R^4}$ for which there exist real numbers $t_1, t_2, \ldots, t_k$ and vector fields $X_1, X_2, \ldots, X_k$ from D (where k is an arbitrary positive integer) such that

$$y = X_k^{t_k}\Big(X_{k-1}^{t_{k-1}}\big(\cdots(X_1^{t_1}(x))\cdots\big)\Big).$$

We now proceed to study the geometry of the orbits of these vector fields.

Theorem. The orbits of the family D of vector fields generate a singular foliation whose regular leaves are surfaces with nonnegative Gaussian curvature.

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GAMILTON–KELI TEOREMASI VA UNING TATBIQLARI

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Zamonaviy matematikaning muhim sohalaridan biri bo‘lgan chiziqli algebra nazariyasi differensial tenglamalar sistemalarini integrallash, matematik fizika tenglamalarida operatorlar spektrini o‘rganish hamda hisoblash usullarini ixcham shaklda bayon etishda asosiy apparat hisoblanadi. Matritsalar nazariyasida xarakteristik ko‘phad va Gamilton–Keli teoremasi alohida o‘rin egallaydi. Mazkur teorema matritsaning algebraik xossalarini uning xos qiymatlari orqali ifodalash, matritsalarning yuqori darajalarini soddalashtirish, xosmas matritsalarning teskarisini aniqlash hamda matritsalar algebrasi elementlari ustida muhim ayniyatlarni qisqa usulda isbotlash imkonini beradi [1–5].

Faraz qilaylik, $A \in M_n(\mathbb{C})$ kompleks sonlar maydoni ustidagi n-tartibli kvadrat matritsa bo‘lsin. Uning xarakteristik ko‘phadi quyidagi ko‘rinishda aniqlanadi:

$$p_A(\lambda) = \det(\lambda I_n - A) = \lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0.$$

Bu yerda I_n birlik matritsa bo‘lib, ko‘phad koeffitsiyentlari matritsaning muhim spektral invariantlari bilan bevosita bog‘langan:

$$c_{n-1} = -\operatorname{tr}(A), \qquad c_0 = (-1)^n \det(A).$$

Xususan, 2×2 o‘lchamli matritsalar uchun xarakteristik ko‘phad

$$p_A(\lambda) = \lambda^2 - \operatorname{tr}(A)\lambda + \det(A)$$

ko‘rinishida ifodalanadi.

Teorema (Gamilton–Keli). Har qanday $A \in M_n(\mathbb{C})$ kvadrat matritsa o‘zining xarakteristik ko‘phadini qanoatlantiradi, ya’ni

$$p_A(A) = O_n.$$

Isbot. Schur uchburchaklashtirish teoremasiga binoan, ixtiyoriy $A \in M_n(\mathbb{C})$ matritsa uchun shunday unitar U va yuqori uchburchakli T matritsa mavjudki, $A = UTU^*$ munosabat bajariladi. Bunda T matritsaning bosh diagonalida A matritsaning barcha xos sonlari $\lambda_1, \lambda_2, \ldots, \lambda_n$ algebraik karraliklari bilan joylashadi. Shuning uchun xarakteristik ko‘phad quyidagicha ko‘paytuvchilarga ajraladi:

$$p_A(\lambda) = \prod_{j=1}^{n} (\lambda - \lambda_j).$$

Matritsali ko‘phadlar xossasiga asosan $p_A(A) = U p_A(T) U^*$ tenglik o‘rinlidir. Yuqori uchburchakli T matritsa uchun $p_A(T) = O_n$ ekanligini matritsa tartibi n bo‘yicha matematik induksiya usulida ko‘rsatamiz. Matritsani blokli ko‘rinishda ifodalaymiz:

$$T = \begin{pmatrix} T_{n-1} & v \\ 0 & \lambda_n \end{pmatrix},$$

bu yerda $T_{n-1} \in M_{n-1}(\mathbb{C})$, $v \in \mathbb{C}^{n-1}$. $q(\lambda) = \prod_{j=1}^{n-1}(\lambda - \lambda_j)$ deb belgilasak, induksiya faraziga binoan $q(T_{n-1}) = O_{n-1}$ bo‘ladi. Natijada

$$q(T) = \begin{pmatrix} q(T_{n-1}) & w \\ 0 & q(\lambda_n) \end{pmatrix} = \begin{pmatrix} O & w \\ 0 & q(\lambda_n) \end{pmatrix}.$$

Bundan foydalanib:

$$p_A(T) = q(T)(T - \lambda_n I_n) = \begin{pmatrix} O & w \\ 0 & q(\lambda_n) \end{pmatrix} \begin{pmatrix} T_{n-1} - \lambda_n I_{n-1} & v \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} O & O \\ 0 & 0 \end{pmatrix} = O_n.$$

Demak, $p_A(A) = U p_A(T) U^* = U O_n U^* = O_n$. Teorema to‘liq isbotlandi.

Gamilton–Keli teoremasining eng asosiy tatbiqlaridan biri xosmas ($c_0 \neq 0$) matritsalar uchun teskari matritsani osongina aniqlashdir. Haqiqatan ham, teorema tengligidan:

$$A^n + c_{n-1}A^{n-1} + \cdots + c_1 A + c_0 I_n = O_n \implies A\left(A^{n-1} + c_{n-1}A^{n-2} + \cdots + c_1 I_n\right) = -c_0 I_n.$$

Ushbu munosabatning har ikki tomonini $-\frac{1}{c_0}$ skalyarga ko‘paytirib, quyidagi formulani hosil qilamiz:

$$A^{-1} = -\frac{1}{c_0}\left(A^{n-1} + c_{n-1}A^{n-2} + \cdots + c_1 I_n\right). \tag{1}$$

1-masala. $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ matritsaning teskarisini Gamilton–Keli teoremasi orqali hisoblang.

Yechim. Berilgan matritsaning xarakteristik ko‘phadi $p_A(\lambda) = \lambda^2 - 4\lambda + 3$ ko‘rinishida bo‘lib, $c_1 = -4$ va $c_0 = 3$ ga teng. U holda $A^2 - 4A + 3I_2 = O_2$. (1) formulani qo‘llasak:

$$A^{-1} = \frac{1}{3}(4I_2 - A) = \frac{1}{3}\left(\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\right) = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$$

Bundan tashqari, matritsaning xarakteristik ko‘phadi uning izi va determinanti bilan uzviy bog‘liq bo‘lganligi bois, Gamilton–Keli teoremasi matritsalar ustidagi turli kommutatorli ayniyatlarni isbotlashda kuchli apparat vazifasini bajaradi.

2-masala. Agar $A, B \in M_2(\mathbb{C})$ va $\det A = \det B = 1$ bo‘lsa, quyidagi tenglikni isbotlang:

$$\operatorname{tr}(AB) - \operatorname{tr}(A)\operatorname{tr}(B) + \operatorname{tr}(AB^{-1}) = 0.$$

Yechim. B matritsaning xarakteristik ko‘phadi $p_B(\lambda) = \lambda^2 - \operatorname{tr}(B)\lambda + 1$ bo‘lgani uchun, teoremaga ko‘ra $B^2 - \operatorname{tr}(B)B + I_2 = O_2$. Ushbu tenglikni o‘ng tomondan B^{-1} ga ko‘paytirsak:

$$B - \operatorname{tr}(B)I_2 + B^{-1} = O_2.$$

Hosil bo‘lgan ifodani chap tomondan A matritsaga ko‘paytiramiz:

$$AB - \operatorname{tr}(B)A + AB^{-1} = O_2.$$

Iz operatorining chiziqlilik xossasidan foydalanib, har ikkala tomondan iz olamiz:

$$\operatorname{tr}(AB) - \operatorname{tr}(B)\operatorname{tr}(A) + \operatorname{tr}(AB^{-1}) = 0.$$

Tenglik isbotlandi.

3-masala. Agar $A, B \in M_3(\mathbb{C})$ bo‘lsa, u holda

$$\det(AB - BA) = \frac{1}{3}\operatorname{tr}\left((AB - BA)^3\right)$$

munosabat o‘rinli bo‘lishini isbotlang.

Yechim. $C = AB - BA$ kommutator matritsani qaraymiz. Izning siklik xossasiga asosan:

$$\operatorname{tr}(C) = \operatorname{tr}(AB) - \operatorname{tr}(BA) = 0.$$

3-tartibli C matritsaning xarakteristik ko‘phadini $p_C(\lambda) = \lambda^3 - c_1\lambda^2 + c_2\lambda - c_3$ deb yozsak, bu yerda $c_1 = \operatorname{tr}(C) = 0$ va $c_3 = \det(C)$. Gamilton–Keli teoremasiga ko‘ra $C^3 + c_2C - c_3I_3 = O_3$. Har ikki tomondan iz operatorini olamiz:

$$\operatorname{tr}(C^3) + c_2 \operatorname{tr}(C) - 3c_3 = 0.$$

$\operatorname{tr}(C) = 0$ ekanligini hisobga olsak, $\operatorname{tr}(C^3) - 3\det(C) = 0$ tenglik kelib chiqadi. Bundan esa:

$$\det(AB - BA) = \frac{1}{3}\operatorname{tr}\big((AB - BA)^3\big).$$

Xulosa qilib aytganda, Gamilton–Keli teoremasi chiziqli algebraning nazariy qonuniyatlari bilan amaliy hisoblashlari o‘rtasidagi eng muhim vositalardan biridir. U matritsali funksiyalarni hisoblash, operatorlar spektral nazariyasi va differensial tenglamalar sistemasini yechishda samarali qo‘llaniladi.

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UDK 511.5

EKSPONENSIAL DIOFANT TENGLAMALARI

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Eksponensial Diofant tenglamalari noma’lumlarning kamida bittasi daraja ko‘rsatkichida qatnashadigan va yechimlari butun yoki natural sonlarda izlanadigan tenglamalardir. Ularning muhim sinflari

$$a^x - b^y = c, \qquad x^m - y^n = k, \qquad x^2 + D = y^n$$

ko‘rinishida ifodalanadi. Bunday tenglamalar oddiy algebraik Diofant tenglamalaridan noma’lumlarning eksponensial o‘sishi bilan farq qiladi. Shu sababli yechimlarni aniqlashda bo‘linuvchanlik, qoldiq sinflari, tub ko‘paytuvchilarga ajratish, daraja ko‘rsatkichlarini kamaytirish hamda algebraik sonlar logarifmlarining chiziqli shakllariga oid baholardan foydalaniladi [1,2]. Eksponensial Diofant tenglamalarini o‘rganishning asosiy maqsadi nafaqat mavjud yechimlarni topish, balki boshqa yechimlar mavjud emasligini qat’iy isbotlashdan iborat.

Dastlab elementar arifmetik usullarning imkoniyatini ko‘rib chiqamiz. Quyidagi tenglama buning sodda, ammo mazmunli namunasidir:

$$2^x - 3^y = 1, \qquad x, y \in \mathbb{N}.$$

Theorem 1. $2^x - 3^y = 1$ tenglamaning musbat butun sonlardagi yagona yechimi

$$(x, y) = (2, 1)$$

dan iborat.

Proof. Tenglamani 3 modul bo‘yicha qarasak,

$$2^x \equiv 1 \pmod 3.$$

$2 \equiv -1 \pmod 3$ bo‘lganligi uchun x juft. Shuning uchun $x = 2k$ deb yozamiz. Natijada

$$3^y = 2^{2k} - 1 = (2^k - 1)(2^k + 1).$$

Bu ikki ko‘paytuvchi toq son bo‘lib,

$$\text{EKUB}(2^k - 1, 2^k + 1) = 1.$$

Haqiqatan, ularning umumiy bo‘luvchisi ayirmasi 2 ni ham bo‘lishi kerak, lekin ikkala son ham toq. Ko‘paytma 3 ning darajasiga teng bo‘lganligi va ko‘paytuvchilar o‘zaro tub bo‘lganligi sababli ulardan biri 1 ga teng:

$$2^k - 1 = 1.$$

Bundan $k = 1$, demak $x = 2$. Dastlabki tenglamadan $3^y = 3$ va $y = 1$ kelib chiqadi.

Mazkur misolda taqqoslama daraja ko‘rsatkichining juftligini, ko‘paytuvchilarga ajratish esa uning aniq qiymatini aniqladi. Biroq

$$x^m - y^n = 1, \qquad x, y, m, n > 1,$$

tenglama uchun bunday elementar usullar yetarli emas. Bu masala Catalan gipotezasi nomi bilan ma’lum bo‘lib, uzoq vaqt davomida sonlar nazariyasining muhim ochiq masalalaridan biri bo‘lgan. R. Tijdeman algebraik sonlar logarifmlarining chiziqli shakllariga oid Baker nazariyasidan foydalanib, mazkur tenglamadagi daraja ko‘rsatkichlarining chegaralanganligini isbotlagan [3]. P. Mihailescu esa keyinchalik masalani to‘liq hal qilgan [4].

Theorem 2 (Mihailescu). Agar $x, y, m, n > 1$ butun sonlar uchun

$$x^m - y^n = 1$$

tenglik bajarilsa, u holda

$$x = 3, \qquad m = 2, \qquad y = 2, \qquad n = 3.$$

Demak,

$$3^2 - 2^3 = 9 - 8 = 1$$

ushbu tenglamaning yagona yechimidir.

Bu teoremaning natijasiga ko‘ra, birdan katta darajali mukammal darajalar orasida ketma-ket joylashgan yagona ikkita son 8 va 9 dir. Natija eksponensial Diofant tenglamalarida yechimlar juda kam uchrashi mumkinligini ko‘rsatadi. Masalan,

$$5^2 - 2^4 = 9, \qquad 3^3 - 2^4 = 11,$$

bo‘lib, darajalar o‘sishi bilan ikki eksponensial ifodaning ayirmasi kichik doimiy songa teng bo‘lishi keskin cheklanadi.

Bunday cheklanishni logarifmik usul orqali tushuntirish mumkin. Faraz qilaylik,

$$a^x - b^y = c, \qquad a, b > 1, \quad c > 0.$$

Tenglamani

$$a^x = b^y \left(1 + \frac{c}{b^y}\right)$$

ko‘rinishda yozib, logarifm olamiz:

$$x \log a - y \log b = \log \left(1 + \frac{c}{b^y}\right).$$

Quyidagi chiziqli shaklni qaraymiz:

$$\Lambda = x \log a - y \log b.$$

$\log(1 + t) < t$ tengsizlikdan

$$0 < \Lambda < \frac{c}{b^y}$$

kelib chiqadi. Shunday qilib,

$$\log |\Lambda| < \log c - y \log b. \tag{1}$$

Boshqa tomondan, a va b multiplikativ jihatdan bog‘liq bo‘lmasa, Baker nazariyasi Λ uchun noldan farqli samarali quyi baho beradi. Umumiy ko‘rinishda u

$$\log |\Lambda| > -C(1 + \log B), \qquad B = \max\{x, y\}, \tag{2}$$

shaklida ifodalanadi. Bu yerda $C > 0$ o‘zgarmas son a va b ga bog‘liq [1,2]. (1) va (2) tengsizliklarni birlashtirish natijasida

$$y \log b < C(1 + \log B) + \log c$$

hosil bo‘ladi. Chap tomon y ga nisbatan tezroq, o‘ng tomon esa $\log B$ tartibida o‘sadi. Shu sababli x va y uchun yuqori chegara mavjud bo‘ladi. Mazkur tamoyil zamonaviy eksponensial Diofant tenglamalarini yechishning asosiy vositalaridan biridir.

Eksponensial Diofant tenglamalarining yana bir klassik namunasi Ramanujan–Nagell tenglamasidir:

$$x^2 + 7 = 2^n. \tag{3}$$

Ushbu tenglamaning barcha musbat butun yechimlari ma’lum [5].

Theorem 3 (Ramanujan–Nagell). (3) tenglamaning musbat butun sonlardagi yechimlari

$$(x, n) = (1, 3),\ (3, 4),\ (5, 5),\ (11, 7),\ (181, 15)$$

dan iborat.

Mazkur natijaning n juft bo‘lgan holatini elementar usul bilan aniqlash mumkin. $n = 2k$ bo‘lsin. U holda

$$x^2 + 7 = 2^{2k}$$

dan

$$(2^k - x)(2^k + x) = 7$$

kelib chiqadi. Har ikkala ko‘paytuvchi musbat toq son bo‘lganligi sababli

$$2^k - x = 1, \qquad 2^k + x = 7.$$

Tenglamalarni qo‘shsak,

$$2^{k+1} = 8,$$

bundan $k = 2$. Demak,

$$n = 4, \qquad x = 3.$$

Shunday qilib, n juft bo‘lgan holatda yagona yechim $(3, 4)$ dir. Toq daraja ko‘rsatkichlarini tekshirish esa ancha chuqur usullarni talab qiladi.

Ramanujan–Nagell tenglamasi

$$x^2 + D = y^n \tag{4}$$

ko‘rinishdagi umumiy eksponensial Diofant tenglamalarining xususiy holidir. (4) tenglamani o‘rganishda daraja ko‘rsatkichini kamaytirish muhim usullardan biri hisoblanadi. Agar n murakkab son va p uning toq tub bo‘luvchisi bo‘lsa, $n = pt$ deb yozib,

$$y^n = (y^t)^p$$

ko‘rinishga o‘tiladi. Shu sababli ko‘p hollarda n ni toq tub son deb qarash kifoya qiladi. Agar $4 \mid n$ bo‘lsa, alohida

$$x^2 + D = z^4$$

holati tekshiriladi. Bu kamaytirish noma’lum daraja ko‘rsatkichlarining cheksiz to‘plamini tub ko‘rsatkichli holatlarga ajratish imkonini beradi [6].

Ayrim $x^2+D = y^n$ tenglamalarini mos kvadratik sonlar maydonida ko‘paytuvchilarga ajratish natijasida

$$\alpha^n - \beta^n$$

ko‘rinishdagi ifodalar hosil bo‘ladi. Ular

$$U_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

Lucas turidagi rekurrent ketma-ketliklar bilan bog‘lanadi. Shu sababli eksponensial Diofant tenglamalari bilan rekurrent ketma-ketliklar nazariyasi o‘rtasida tabiiy bog‘lanish mavjud.

Bilu, Hanrot va Voutier tomonidan Lucas va Lehmer ketma-ketliklarining yangi tub bo‘luvchilari haqidagi muhim natija isbotlangan [7]. Tegishli shartlar bajarilganda $n > 30$ bo‘lgan had yangi tub bo‘luvchiga ega bo‘ladi. Bunday tub sonning mavjudligi $x^2 + D = y^n$ kabi tenglamalarda katta n qiymatlarini chiqarib tashlash uchun qo‘llanadi. Natijada avval cheksiz ko‘p bo‘lgan daraja ko‘rsatkichlari chekli to‘plamgacha qisqartiriladi.

Shunday qilib, eksponensial Diofant tenglamalarini o‘rganishda birinchi navbatda qoldiq sinflari va bo‘linuvchanlikdan foydalanib mahalliy cheklovlar olinadi. Keyin ko‘paytuvchilarga ajratish va daraja ko‘rsatkichlarini kamaytirish amalga oshiriladi. Katta noma’lumlarni chegaralashda esa logarifmlarning chiziqli shakllari, algebraik sonlar nazariyasi va rekurrent ketma-ketliklarning yangi tub bo‘luvchilari haqidagi natijalardan foydalaniladi. Mihailescu hamda Ramanujan–Nagell teoremalari ushbu usullarning eksponensial Diofant tenglamalaridagi samaradorligini ko‘rsatadigan klassik natijalardir.

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UDK 511.34

TUB MODUL BO‘YICHA KVADRATIK CHEGIRMA VA CHEGIRMAMASLARNING TAQSIMOTI HAQIDA

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Analitik sonlar nazariyasida tub modul bo‘yicha kvadratik chegirmalar va kvadratik chegirmamaslarning oraliqlarda taqsimlanishi, xususan, eng kichik musbat kvadratik chegirmamas $n_1(p)$ ning miqdoriy tartibini yuqoridan baholash fundamental masalalardan biri hisoblanadi. $p > 2$ toq tub son va $(a, p) = 1$ butun soni uchun $x^2 \equiv a \pmod p$ taqqoslama yechimga ega bo‘lsa, a soni p moduli bo‘yicha *kvadratik chegirma*, aks holda *kvadratik chegirmamas* deyiladi.

Ushbu tushunchalar Eyler mezoni hamda Lejandr simvoli $\left(\frac{a}{p}\right)$ orqali analitik ifodalanadi:

$$\left(\frac{a}{p}\right) \equiv a^{\frac{p-1}{2}} \pmod p, \quad \left(\frac{a}{p}\right) \in \{-1, 1\}.$$

Ma’lumki, $1, 2, \ldots, p-1$ qisqargan chegirmalar sistemasida kvadratik chegirmalar va kvadratik chegirmamaslar soni teng taqsimlangan bo‘lib, har birining soni aynan $\frac{p-1}{2}$ tadan iborat. Biroq ularning kichik oraliqlardagi taqsimoti nihoyatda murakkab bo‘lib, xarakterlar yig‘indisi usulini jalb etishni talab qiladi.

$1 \le x \le H$ oraliqda joylashgan kvadratik chegirmalar soni $R(H)$ va chegirmamaslar soni $N(H)$ quyidagi ortogonal munosabat orqali ifodalanadi:

$$R(H) - N(H) = \sum_{x=1}^{H} \left(\frac{x}{p}\right). \tag{1}$$

Klassik I. M. Vinogradov [1] va G. Polya tengsizligiga asosan, har qanday non-trivial Dirixle xarakteri, xususan Lejandr simvoli uchun to‘liq bo‘lmagan xarakterlar yig‘indisi quyidagicha baholanadi:

$$\left|\sum_{x=1}^{H} \left(\frac{x}{p}\right)\right| \le \sqrt{p} \ln p. \tag{2}$$

Agar (1) tenglikda $H > \sqrt{p}\ln p$ deb tanlansa, $R(H) - N(H) < H$ bo‘lishi kafolatlanadi, bu esa $1, 2, \ldots, H$ oraliqda kamida bitta kvadratik chegirmamas mavjudligini ko‘rsatadi. Natijada eng kichik musbat kvadratik chegirmamas $n_1(p)$ uchun I. M. Vinogradovning mashhur bahosi hosil bo‘ladi:

$$n_1(p) \le \sqrt{p}\ln p + 1. \tag{3}$$

Keyinchalik D. A. Burgess xarakterlar yig‘indisi uchun chuqur baho olib, ixtiyoriy $\varepsilon > 0$ uchun $n_1(p) \ll_\epsilon p^{\frac{1}{4\sqrt{e}}+\varepsilon}$ ekanligini isbotlagan bo‘lsa-da, mazkur bahoda noma’lum asimptotik o‘zgarmaslar qatnashadi.

Ushbu ishda qisqa oraliqlarda kvadratik chegirmalar va chegirmamaslarning ketma-ket uchrash intervallari tahlil qilinadi.

Teorema. *Faraz qilaylik, $p \ge 3$ tub son bo‘lsin. Har qanday $H \ge 2\sqrt{p}\ln p$ uzunlikdagi intervalda kvadratik chegirmalar soni $R(H)$ va kvadratik chegirmamaslar soni $N(H)$ uchun quyidagi qat’iy asimptotik taqsimot o‘rinli:*

$$\left|R(H) - \frac{H}{2}\right| \le \frac{1}{2}\sqrt{p}\ln p, \quad \left|N(H) - \frac{H}{2}\right| \le \frac{1}{2}\sqrt{p}\ln p. \tag{4}$$

Isbot sxemasi. Oraliqdagi sonlar uchun $R(H) + N(H) = H$ ayniyat o‘rinli. (1) formuladagi ayirmani ushbu yig‘indi bilan kombinatsiya qilsak:

$$R(H) = \frac{1}{2}\left(H + \sum_{x=1}^{H}\left(\frac{x}{p}\right)\right), \quad N(H) = \frac{1}{2}\left(H - \sum_{x=1}^{H}\left(\frac{x}{p}\right)\right).$$

(2) dagi Vinogradov–Polya tengsizligini qo‘llash orqali bevosita (4) dagi qat’iy chegaralarga ega bo‘lamiz. Bu esa $H > \sqrt{p}\ln p$ bo‘lganda oraliqda har ikki toifadagi chegirmalar mavjudligini hamda ularning zichligi $\frac{1}{2}$ ga yaqinlashishini to‘liq ko‘rsatadi.

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GOLDBAXNING BINAR MUAMMOSI BO‘YICHA SO‘NGGI NATIJALAR

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Tub sonlarning additiv xossalarini o‘rganish sonlar nazariyasining eng muhim yo‘nalishlaridan biridir. Ushbu yo‘nalishdagi eng mashhur masalalardan biri Goldbax muammosi bo‘lib, u XVIII asrdan buyon matematiklarning e’tiborini tortib kelmoqda. Goldbaxning binar gipotezasi quyidagicha ifodalanadi:

$$N > 2, N = 2k, N = p + q$$

bu yerda p va q — tub sonlar. Masalan, 4=2+2,6=3+3,10=3+7,4=2+2, va 100=3+97=11+89=17+83=29+71.

Ko‘plab hisoblash tajribalari gipotezaning to‘g‘riligini qo‘llab-quvvatlaydi. Biroq chekli sondagi hisoblash natijalari cheksiz ko‘p juft sonlar uchun matematik isbot vazifasini bajara olmaydi.[1]

Goldbax muammosining qiyinligi tub sonlarning additiv taqsimotini juda aniq baholash zarurati bilan bog‘liq. Tub sonlarning multiplikativ xususiyatlari yaxshi o‘rganilgan bo‘lsa-da, ularning yig‘indilar ko‘rinishidagi taqsimoti ancha murakkabdir.

Goldbax muammosini o‘rganishda asosiy arifmetik funksiya

$$r_2(N) = \{(p,q) : p+q = N,\ p,q \text{ - tub}\}$$

orqali aniqlanadi.

Binar Goldbax gipotezasining isboti uchun barcha juft $N > 2$ lar uchun

$$r_2(N) > 0$$

ekanini ko‘rsatish yetarli.

Hardy va Littlewood tomonidan ishlab chiqilgan aylana metodi ushbu masalani o‘rganishda fundamental vositalardan biridir. Mazkur yondashuv tub sonlarni eksponensial yig‘indilar yordamida ifodalash va integralni katta hamda kichik yoylarga ajratishga asoslanadi. Goldbax muammosini tadqiq qilishda g‘alvirlash nazariyasi alohida ahamiyatga ega. G‘alvirlash usullari tub sonlarni boshqa “deyarli tub“ sonlardan ajratishda kuchli natijalar beradi. Ushbu yo‘nalishdagi eng mashhur natijalardan biri Chen Jingrun tomonidan olingan teorema hisoblanadi. Chen teoremasiga ko‘ra, yetarlicha katta har qanday juft son tub son va ko‘pi bilan ikkita tub ko‘paytuvchiga ega bo‘lgan son yig‘indisi sifatida ifodalanadi.[2] Sxematik ravishda:

$$N = p + P_2,$$

bu yerda $p-$ tub son, P_2 esa ko‘pi bilan ikkita tub ko‘paytuvchidan tashkil topgan son.

Mazkur natija klassik Goldbax gipotezasiga juda yaqin bo‘lsa-da, P_2 har doim tub son bo‘lishi shart emas. Shu sababli Chen teoremasi Goldbax gipotezasining to‘liq isboti hisoblanmaydi. Shunga qaramay, u binar Goldbax muammosi uchun eng muhim qisman natijalardan biri bo‘lib qolmoqda[3].

So‘nggi tadqiqotlarda Goldbax muammosining zaiflashtirilgan shakllarini tobora kuchaytirish tendensiyasi kuzatilmoqda.

J. Li va J. Liu tomonidan 2026-yilda e’lon qilingan natijada yetarlicha katta juft sonlar uchun

$$N = p + rq$$

ko‘rinishidagi tasvir o‘rganilgan. Bu yerda p va q — tub sonlar, r esa tub son bo‘lib, ma’lum yuqori chegara bilan chegaralangan.

Ularning natijasida r uchun

$$r \le q^{0.9}$$

chegaraga ega bo‘lgan tasvirning mavjudligi ko‘rsatiladi.

Mazkur natijaning konseptual ahamiyatini quyidagi zanjir orqali ko‘rsatish mumkin: $N = p+q \quad \leftarrow N = p+rq \leftarrow \quad N = p+P_2$. Klassik Goldbax gipotezasida r=1 bo‘lishi kerak. Yangi natijada esa r ning kengroq qiymatlariga yo‘l qo‘yiladi[4]. Shuning uchun ushbu teorema gipotezaning o‘zini isbotlamaydi, ammo unga olib boruvchi oraliq bosqich sifatida muhimdir.

Bundan tashqari, ushbu yo‘nalishdagi natijalar ma’lum qo‘shimcha analitik gipotezalar, jumladan Elliott–Halberstam tipidagi farazlar bilan bog‘liq holda yanada kuchaytirilishi mumkin. Goldbax muammosining yana bir muhim yo‘nalishi – maxsus to‘plamni o‘rganishdir. Agar

$$E(x) = \{N \leq x : N\ juft\ va\ Goldbax\ tasviriga\ ega\ emas\}$$

deb olinsa, asosiy maqsad $E(x)$ ning imkon qadar kichik ekanini ko‘rsatishdan iborat. ”Deyarli barcha” juft sonlar uchun Goldbax tasvirining mavjudligini ko‘rsatish barcha juft sonlar uchun isbot olishdan ancha oson. Biroq maxsus to‘plamning nolga tengligini isbotlash aynan muammoning eng qiyin qismidir.

2026-yildagi yangi tadqiqotlarda Hardy–Littlewood aylana metodining katta yoylari, eksponensial yig‘indilar va maxsus to‘plamlar bo‘yicha oldingi natijalar qayta ko‘rib chiqilib, aniqroq baholar olishga harakat qilinmoqda[5].

Bu natijalar Goldbax muammosining ”deyarli barcha” holatdan ”barcha“ holatga o‘tishidagi asosiy analitik to‘siqlarni yaxshiroq tushunish imkonini beradi. Goldbax gipotezasini tekshirishda yuqori unumdor hisoblash texnologiyalaridan foydalanish ham muhim yo‘nalish hisoblanadi.

2013-yilda T. Oliveira e Silva, S. Herzog va S. Pardi tomonidan amalga oshirilgan katta hajmdagi hisoblashlar Goldbax gipotezasini $4 \times 10184 \cdot 10^{18}$ gacha tekshirish imkonini bergan[6].

Keyingi yillarda taqsimlangan hisoblash loyihalari yordamida ushbu chegarani yanada kengaytirish bo‘yicha ishlar olib borilmoqda.

Kompyuter tekshiruvlarining asosiy afzalligi shundaki, juda katta sonlar diapazonida qarshi misol mavjud emasligini amaliy ravishda tekshirish mumkin. Biroq bu usulning fundamental cheklovi ham mavjud: har qanday chekli hisoblash cheksiz ko‘p sonlar uchun nazariy isbotning o‘rnini bosa olmaydi.

Shuning uchun zamonaviy tadqiqotlarda hisoblash usullari analitik usullar bilan birgalikda qo‘llanadi. Kompyuterlar kichik va o‘rta qiymatli qoldiq holatlarni tekshirish, nazariy baholarning konstantalarini aniqlash va katta hajmdagi ma’lumotlarni tahlil qilishda samarali vosita hisoblanadi.

2026-yil holatiga ko‘ra binar Goldbax gipotezasi to‘liq isbotlangan emas. Shunga qaramay, uning atrofida quyidagi muhim natijalar mavjud [7]:

- juda katta sonlar diapazonida gipotezaning hisoblash yo‘li bilan tasdiqlanishi;
- Chen teoremasi orqali ”tub son + deyarli tub son” tasvirining mavjudligi;
- “deyarli barcha” juft sonlar uchun Goldbax tasvirlari bo‘yicha kuchli natijalar;
- aylana metodi orqali Goldbax tasvirlari sonining kutilayotgan asimptotik baholari;
- 2026-yilda $N = p + rq$ ko‘rinishidagi kuchaytirilgan oraliq natijalarning olinishi;
- maxsus to‘plamni yanada kichraytirishga qaratilgan yangi tadqiqotlar.

Kelgusida muammoni hal qilish uchun bir nechta yo‘nalish ayniqsa istiqbolli hisoblanadi. Birinchidan, tub sonlarning qisqa intervallardagi taqsimoti haqidagi natijalarni kuchaytirish zarur. Ikkinchidan, kichik yoylarda paydo bo‘ladigan eksponensial yig‘indilarni yanada aniq baholash kerak. Uchinchidan, galvirlash usullarining ”deyarli tub” sonlardan haqiqiy tub sonlarga o‘tishdagi cheklovlarini bartaraf etish muhim.

Goldbaxning binar muammosi ikki yuz yildan ortiq vaqt davomida sonlar nazariyasining markaziy ochiq masalalaridan biri bo‘lib kelmoqda. Uning formulirovkasi juda sodda bo‘lsa-da, tub sonlarning additiv taqsimotini boshqarish zarurati masalani nihoyatda murakkablashtiradi.

So‘nggi natijalar Goldbax gipotezasiga turli tomondan yaqinlashayotganini ko‘rsatmoqda. Aylana metodi, g‘alvirlash nazariyasi, maxsus to‘plamlar bo‘yicha baholar va yuqori unumdor hisoblash usullari bir-birini to‘ldiruvchi asosiy vositalarga aylangan.

Ayniqsa, 2026-yildagi $N = p + rq$ ko‘rinishidagi natijalar klassik Goldbax gipotezasiga olib boruvchi oraliq bosqichlarni yanada aniqlashtirdi. Biroq $r = 1$ holatini barcha juft sonlar uchun isbotlash masalasi ochiq qolmoqda.

Shunday qilib, hozirgi bosqichda Goldbax muammosining asosiy vazifasi mavjud ”deyarli barcha” va zaiflashtirilgan natijalarni kuchaytirib, barcha juft sonlar uchun

$$N = p + q$$

tasvirining mavjudligini qat’iy isbotlashdan iborat. Ushbu maqsadga erishish analitik sonlar nazariyasi rivojida fundamental ahamiyatga ega bo‘lib, tub sonlarning additiv xususiyatlari haqidagi bilimlarimizni sezilarli darajada kengaytiradi.

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Van der Corput Lemmas for Mittag-Leffler Functions With L^p Amplitude Functions

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The classical van der Corput lemma, established by the Dutch mathematician J. G. van der Corput [1], provides sharp decay estimates for one-dimensional oscillatory integrals. In its standard formulation, if φ is a real-valued smooth function on $[a, b]$ with $|\varphi^{(k)}(x)| \geq 1$ for all $x \in (a, b)$, then for ψ smooth:

$$\left| \int_a^b e^{i\lambda\phi(x)}\, \psi(x)\, dx \right| \leq C_k\, \lambda^{-\frac{1}{k}}, \qquad \lambda \to \infty, \tag{1}$$

for $k = 1$ with ϕ' monotone, or $k \geq 2$. This estimate and its many generalizations are fundamental in harmonic analysis, dispersive PDE, and analytic number theory.

Recent years have seen growing interest in oscillatory integrals where the classical exponential e^{iz} is replaced by the two-parameter Mittag-Leffler function.

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \qquad \alpha > 0,\ \beta \in \mathbb{R}, \tag{2}$$

which satisfies $E_{1,1}(z) = e^z$ and arises naturally as the fundamental solution of time-fractional differential equations. The systematic study of van der Corput lemmas for such integrals was initiated in [2], where integrals of the form

$$\tilde{I}_{\alpha,\beta}(\lambda) = \int_{\mathbb{R}} E_{\alpha,\beta}\big(i^{\alpha}\lambda\,\phi(x)\big)\,\psi(x)\,dx, \qquad 0 < \alpha \leq 1, \tag{3}$$

were treated, and continued in [2] for the α -direction integrals

$$I_{\alpha,\beta}(\lambda) = \int_{\mathbb{R}} E_{\alpha,\beta}\big(i^{\alpha}\lambda\,\phi(x)\big)\,\psi(x)\,dx, \qquad 0 < \alpha \leq 2, \tag{4}$$

with amplitude $\psi \in L^1(\mathbb{R})$. The integrals (4) appear in the study of decay estimates for solutions of time-fractional Schrödinger and wave equations.

We establish van der Corput-type decay estimates for oscillatory integrals of the form

$$I_{\alpha,\beta}(\lambda) = \int_{\mathbb{R}} E_{\alpha,\beta}\big(i^{\alpha}\lambda\,\phi(x)\big)\,\psi(x)\,dx$$

where the amplitude function ψ belongs to $L^p(\mathbb{R})$ for $1 < p < \infty$, extending the classical L^1 framework developed by Ruzhansky and Torebek. The key tool is a combination of the known asymptotic estimates for Mittag–Leffler functions with Hölder's inequality and a refined splitting argument. For the range $0 < \alpha < 2$ with $\beta > 1 + \frac{\alpha}{q}$ (where $q = \frac{p}{p-1}$ is the conjugate exponent), we obtain the bound

$$|I_{\alpha,\beta}(\lambda)| \leq M(1 + \lambda m)^{-\frac{\beta-1}{\alpha}+\frac{1}{q}} \|\psi\|_{L^p(\mathbb{R})}, \quad \lambda \geq 1$$

where $m = \operatorname{ess\,inf}_{x\in\mathbb{R}} |\phi(x)| > 0$ and M is independent of ϕ, ψ, and λ.Analogous results are derived for $\alpha = 2$. As decay rates decrease smoothly from the L^1 case ($p \to 1$) to the L^∞ limit ($p \to \infty$), these estimates reflect a natural interpolation structure. Applications to dispersive estimates for time-fractional Klein–Gordon and Schrödinger equations, as well as a generalized Riemann–Lebesgue lemma, are presented.

We consider the integral $I_{\alpha,\beta}(\lambda) = \int_{\mathbb{R}} E_{\alpha,\beta}\big(i^{\alpha}\lambda\,\phi(x)\big)\,\psi(x)\,dx, 0 < \alpha \leq 2$ with $\lambda \geq 1$ (for small λ the integral is trivially bounded by $\|\psi\|_{L^p}$ times a constant).

Theorem 1(Case $\beta > 1$, general phase). Let $\phi : \mathbb{R} \to \mathbb{R}$ be measurable with $m = \operatorname{ess\,inf}_{x\in\mathbb{R}} |\phi(x)| > 0$, and let for some $1 < p < \infty$. Set $q = \frac{p}{p-1}$. Suppose $0 < \alpha \leq 2$ and $\beta > 1$.Then:

(i) If $0 < \alpha < 2$ and $\beta \geq \alpha + 1$, then

$$|I_{\alpha,\beta}(\lambda)| \leq M_1\,(1 + \lambda m)^{-\frac{\beta-1}{\alpha}+\frac{1}{q}}\,\|\psi\|_{L^p(\mathbb{R})}, \qquad \lambda \geq 1, \tag{5}$$

where $M_1 = M_1(\alpha, \beta, p)$ is independent of ϕ, ψ, and λ.

(ii) If $0 < \alpha < 2$ and $\beta > 1 + \frac{\alpha}{q}$, then

$$|I_{\alpha,\beta}(\lambda)| \le M_2 \left(1 + \lambda m\right)^{-\frac{\beta-1}{\alpha}+\frac{1}{q}} \|\psi\|_{L^p(\mathbb{R})}, \qquad \lambda \ge 1, \tag{6}$$

where $M_2 = M_2(\alpha, \beta, p)$ is independent of ϕ, ψ, and λ.

(iii) If $\alpha = 2$ and $\beta > 1 + \frac{2}{q}$, then

$$|I_{2,\beta}(\lambda)| \le M_3 \left(1 + \lambda m\right)^{-\frac{\beta-1}{2}+\frac{1}{q}} \|\psi\|_{L^p(\mathbb{R})}, \qquad \lambda \ge 1, \tag{7}$$

where $M_3 = M_3(\beta, p)$ is independent of ϕ, ψ, and λ.

Remark. a) When $p \to 1$ (so $1/q \to 0$) the bounds in Theorem 1 recover exactly the L^1 estimates of [2]. When $p \to \infty$ (so $1/q \to 1$) one obtains the weakest possible decay, consistent with the fact that L^∞ functions need not be integrable against any fixed kernel.

b)The constraint $\beta > 1 + \frac{\alpha}{q}$ is necessary to ensure that the Hölder dual function $x \mapsto (1 + \lambda|\phi(x)|)^{-\frac{\beta-1}{\alpha}}$ lies in L^q_{loc} that the Hölder pairing is finite. When $\beta \le 1 + \frac{\alpha}{q}$ additional integrability on ϕ is required.

Theorem 2(Case $\beta = 1$: invertible phase). Let $\phi : \mathbb{R} \to \mathbb{R}$ be invertible and differentiable with $m = \inf_{x\in\mathbb{R}} |\phi'(x)| > 0$, and let $\psi \in L^p(\mathbb{R})$ for some $1 < p < \infty$. Suppose $0 < \alpha \le 2$ and $\beta = 1$. Then

$$|I_{\alpha,1}(\lambda)| \le M \left(\lambda m\right)^{-1+\frac{1}{q}} \|\psi\|_{L^p(\mathbb{R})}, \qquad \lambda \ge 1, \tag{8}$$

where $M = M(\alpha, p)$ is independent of ϕ, ψ, and λ.

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Lobachevskiy geometriyasining ${}^1R_3$-Minkovskiy fazosida sfera ustidagi talqini

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N. I. Lobachevskiy “Tasavvurdagi geometriya” nomli asarida birinchi marta 1826-yilda noevklid geometriya haqida ma’ruza qilgan. Shundan so’ng, Lobachevskiy geometriyasining yaratilishi haqidagi ma’ruzalar bir qator manbalarda bayon qilindi. Lobachevskiy geometriyasi o’z o’rniga va metodlariga ega bo’lishi uchun, uning bir nechta talqini paydo bo’ldi. Ayniqsa, Keli-Kleyn va Puankare nazariyalari alohida ahamiyatga ega.

Ma’lumki, Lobachevskiy geometriyasining asosini uning parallellik aksiomasi orqali ko’rish mumkin. Bunda Yevklid geometriyasidagi parallellik aksiomasidan farqli bo’lgan aksiomalari Lobachevskiy geometriyasi uchun ham o’rinlidir.

Lobachevskiy aksiomasini tasavvur orqali anglash va tushunish talabalar uchun shu vaqtgacha egallagan bilimlari yetarli bo’lmaydi. Ya’ni biz bilgan tekislik orqali bu aksiomani tasavvur qilish mumkin emas.

Lobachevskiy tekisligining talqinini Minkovskiy fazodagi mavhum radiusli sfera, aniqrog'i ikki pallali giperboloidning bir pallasi va unga mos izotrop konus (nol radiusli sfera) dan foydalanamiz. Bu paragraf davomida konus deb Minkovskiy fazodagi izotrop konusni, giperboloid deb fazodagi mavhum radiusli sferani tushunamiz.

Koordinatalar sistemasidan konus uchini koordinatalar boshiga keltiramiz. Oz o'qi esa, konusning simmetriya markazida bo?lsin

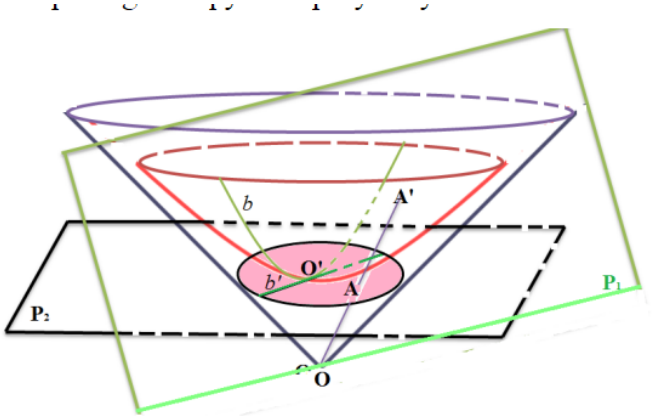


Рис. 3: Lobachevskiy tekisligi

Ma'lumki, Yevklid geometriyasida ikki tekislik biror to'g'ri chiziq orqali kesishadi. Asimptotik konus uchidan o'tuvchi ikki tekislik albatta biror u to'g'ri chiziq (Yevklid ma'nosida) bo'ylab kesishadi. Bunda to'g'ri chiziq konus tashqarisida konus ichida, konus ustida joylashgan bo'lishi mumkin.

Lobachevskiy tekisligining Keli-Kleyn modeli

Ikki pallali giperboloidning (bir pallasi) ustida bajarilgan Lobachevskiy geometriyasidagi aksiomalarni Keli -Kleyn modeli bilan bog'laymiz. Rasmda konusni kesib o'tgan P_2 tekislik giperboloidning O' nuqtadagi urinma tekisligi bo'lib, konus bilan chegaralangan qismi L doirani Lobachevskiy tekisligi deb ataymiz.

Giperboloidning A' nuqtasi L doira ichidagi A nuqtaga, giperboloid va P_1 tekislikning kesishishidan hosil bo'lgan b chiziq esa, shu doira ichidagi b' vatarga mos keladi. Natijada, Minkovskiy fazosidagi sfera ustidagi geometriya, ya'ni Lobachevskiy tekisligi $z = 1$ tekislikning izotrop konus bilan kesishishidan hosil bo'lgan doira ichki nuqtalariga bir qiymatli proyeksiyalanadi.

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Some topological and geometric properties of the class of $(A)\,m$-convex functions

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The theory of convex functions plays an essential role in mathematical analysis, convex geometry, differential equations, and potential theory. The classical concept of convexity is typically characterized by the positive definiteness of the Hessian matrix of a function. However, contemporary research in the theory of functions of several variables has brought about the necessity of studying various generalizations of convexity, in particular, the classes of m-convex and $(A)m$-convex functions.

The theory of m-convex functions constitutes one of the important directions in real analysis and convex geometry, dealing with the study of function classes defined via Hessian operators and their eigenvalues. This class represents an intermediate state between classical convex functions and subharmonic functions. In particular, for $m = 1$, the class of m-convex functions coincides with the class of classical convex functions in $\mathbb{R}^n$, whereas for $m = n$, it coincides with the class of subharmonic functions.

The theory of convex functions has been extensively studied in the works of A.D. Aleksandrov [1], [2], I.Ya. Bakelman [3], [4], A.V. Pogorelov [8], and other scholars. For cases where $m > 1$, the theory of m-convex functions and related Hessian-type operators has been developed in the research of N.M. Ivochkina, N. Trudinger, X. Wang, R. Sharipov, M. Ismailov, and other authors (cf. [10], [11], [12]).

It is known that the class of m-convex functions possesses a number of natural algebraic and geometric properties. However, for the class of $(A)m$-convex functions, such properties do not always hold true. For instance, the sum of two $(A)m$-convex functions is not necessarily $(A)m$-convex in the general case. Likewise, the inclusion relation $(A)m - cv \subset (A)(m+1) - cv$ may not hold in general. This indicates that the class of $(A)m$-convex functions has a more complex structure compared to the class of m-convex functions. The main objective of this research work is to study certain algebraic, topological, and geometric properties of the class of $(A)m$-convex functions defined in the space $\mathbb{R}^n$.

Let $D \subset \mathbb{R}^n$ be a domain and $u \in C^2(D)$. We denote the eigenvalue vector of the Hessian matrix $G_u(x) = \left(\frac{\partial^2 u(x)}{\partial x_j \partial x_k}\right)_{j,k=1}^n$ by $\lambda(G_u(x)) = (\lambda_1(x), \lambda_2(x), \ldots, \lambda_n(x))$.

Defenition 1. A function u is said to be m-convex in the domain D if, for every $x \in D$, $H_k(\lambda(G_u(x))) \geq 0, \quad k = 1, 2, \ldots, n - m + 1$. Here,

$$H_k(\lambda) = \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n} \lambda_{i_1} \lambda_{i_2} \ldots \lambda_{i_k}$$

is the k-th elementary symmetric function of the vector $\lambda = (\lambda_1, \ldots, \lambda_n)$.

This definition can also be expressed as $\lambda(G_u(x)) \in \overline{\Gamma}_{n-m+1}$, where

$$\overline{\Gamma}_m = \{\lambda \in \mathbb{R}^n : H_k(\lambda) \geq 0,\ k = 1, 2, \ldots, m\}$$

denotes the closure of the Garding cone.

Defeniton 2. Let $D \subset \mathbb{R}^n$ be a domain and $u \in C^2(D)$. If, at every point $x \in D$, the conditions

$$\lambda_{j_1}(x) + \cdots + \lambda_{j_m}(x) \geq 0$$

for all indices $1 \le j_1 < \cdots < j_m \le n$, and

$$H_{n-m+1}(\lambda(x)) \ge 0$$

are satisfied, then the function u is called $(A)m$-convex in the domain D.

Now, we present certain algebraic, topological, and geometric properties of the class of $(A)m$-convex functions. First of all, unlike the class of classical m-convex functions, we show that the class of $(A)m$-convex functions is not closed under addition in the general case.

It is known that the class of m-convex functions is closed under positive linear combinations (cf. [11]). However, for the class of $(A)m$-convex functions, such a property does not hold in general.

From this, the following conclusion follows: the class of $(A)m$-convex functions is not closed under addition in general. In particular, this class does not form a linear space. Therefore, determining under what additional conditions the class of $(A)m$-convex functions remains closed under addition is a natural problem. Even if one of the summands is a convex function in the usual sense, the sum may still fail to belong to the class of $(A)m$-convex functions.

Now, we modify the condition imposed on the second function to an affinity condition.

Theorem 1. Let $D \subset \mathbb{R}^n$ be a domain and $u \in (A)m - cv(D)$. If

$$\ell(x) = a_1x_1 + \cdots + a_nx_n + b$$

is an affine funkction, then $u + \ell \in (A)m - cv(D)$.

Theorem 2. If $u \in (A)m - cv(D)$ and $c \ge 0$, then $cu \in (A)m - cv(D)$.

Definition 3. Let $D \subset \mathbb{R}^n$ be a domain and $u \in C^2(D)$. If, for every compact set $K \Subset D$, there exists a number $\delta_K > 0$ such that for all $x \in K$ and all indices $1 \le j_1 < \cdots < j_m \le n$, the conditions $\lambda_{j_1}(x) + \cdots + \lambda_{j_m}(x) \ge \delta_K$ and $H_{n-m+1}(\lambda(x)) \ge \delta_K$ are satisfied, then the function u is called strictly $(A)m$-convex in the domain D.

Theorem 3. If the function u is strictly $(A)\,m$-convex in the domain D, then for every compact set $K \Subset D$ there exists an $\varepsilon > 0$ such that for any $\varphi \in C^2(D)$ satisfying the condition

$$\sup_{x\in K} \|G_\varphi(x)\| < \varepsilon$$

we have $u + \varphi \in (A)m - cv(K)$.

In addition to the algebraic properties of the class of $(A)m$-convex functions, it is also important to study its topological properties. In particular, the question naturally arises as to whether the limit function remains $(A)m$-convex when a sequence of functions converges in a certain sense. Below, we show that the class of $(A)m$-convex functions is closed in the C^2_{loc}-topology.

Defination 4. Let $D \subset \mathbb{R}^n$ be a domain. If, for a sequence of functions $\{u_j\} \subset C^2(D)$ and a function $u \in C^2(D)$, the condition

$$\|u_j - u\|_{C^2(K)} \to 0, \quad j \to \infty$$

holds for every compact set $K \Subset D$, then the sequence $u_j(x)$ is said to converge to $u(x)$ in the $C^2_{\text{loc}}(D)$ sense, where $\|u_j - u\|_{C^2(K)} = \max_{|\alpha|\le 2} \sup\limits_{x\in K} |D^\alpha u_j(x) - D^\alpha u(x)|$.

Theorem 4. Let $D \subset \mathbb{R}^n$ be a domain. Suppose that $u_j \in (A)m - cv(D)$, $j = 1, 2, \ldots$, and $u_j \to u$ in the sense of $C^2_{\text{loc}}(D)$. Then $u \in (A)m - cv(D)$.

Defination 5. (Similarity transformation). A transformation of the form$\Phi(x) = cOx + b$ in the space $\mathbb{R}^n$ is called a similarity transformation, where $O \in O(n)$ is an orthogonal matrix, $c > 0$ is a scale factor, and $b \in \mathbb{R}^n$ is a translation vector.

Theorem 5. (Invariance under similarity transformations). Let $D \subset \mathbb{R}^n$ be a domain and $u \in (A)m - cv(D)$. Then for any similarity transformation $\Phi(x) = cOx + b$, the function $v(x) = u(\Phi(x))$is $(A)m$-convex in the domain$\Phi^{-1}(D)$.

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Uch diagonalli blok operatorli matritsalarga mos Shur to‘ldiruvchilarining spektral xossalari

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Ma'lumki, uchta Hilbert fazosining to‘g‘ri yig‘indisida ta'sir qiluvchi ixtiyoriy chiziqli chegaralangan operator hamisha uchinchi tartibli operatorli matritsa ko‘rinishida ifodalanadi [1]. Bunday matritsalarning spektral xossalarini o‘rganishda diagonal bloklar bilan bir qatorda ular orasidagi bog‘lanishni ifodalovchi operatorlarni ham hisobga olish zarur. [2] ishda uch diagonalli 3×3 o‘z-o‘ziga qo‘shma operatorli matritsalar uchun kubik sonli soha yordamida yangi spektral baholar olingan. Xos qiymatlarning mavjudlik shartlarini aniqlashda Shur to‘ldiruvchisi usuli qo‘llaniladi. Bu usulda spektral tenglamalar sistemasidagi noma'lumlardan biri tegishli diagonal blokning rezolventasi yordamida ifodalanib, qolgan tenglamalarga qo‘yiladi. Natijada dastlabki spektral masalaga teng kuchli bo‘lgan operator tenglamasi hosil bo‘ladi. Mazkur maqolada bir o‘lchamli kompleks fazo hamda bir va ikki o‘lchamli torlarda aniqlangan kvadrati

bilan integrallanuvchi funksiyalar fazolarining to‘g‘ri yig‘indisida ta’sir qiluvchi uch diagonalli operatorli matritsa qaraladi. Dastlabki ikki fazoni birlashtirish orqali ushbu operator ikkinchi tartibli blok operatorli matritsa ko‘rinishida yoziladi. So‘ngra $z \in \rho(A_{22})$ shartida unga mos birinchi Shur to‘ldiruvchisi $S_1(z)$ aniqlanadi. Ushbu shartni qanoatlantiruvchi z soni dastlabki operatorning xos qiymati bo‘lishi uchun 0 soni $S_1(z)$ operatorning xos qiymati bo‘lishi zarur va yetarli ekanligi ko‘rsatiladi. Shuningdek, yordamchi blok operator va umumlashgan Fridrixs modellariga mos Fredgolm determinantlari kiritilib, ularning spektral masalalarni o‘rganishdagi o‘rni bayon etiladi. Mazkur fazolar hamda ularga mos operatorli matritsaning aniq ta’riflari quyida keltiriladi.

$\mathbb{T}^d$ orqali d o‘lchamli torni, $\mathcal{H}_0 := \mathbb{C}$ orqali bir o‘lchamli kompleks fazoni, shuningdek, $\mathcal{H}_1 := L_2(\mathbb{T})$ va $\mathcal{H}_2 := L_2(\mathbb{T}^2)$ orqali mos ravishda $\mathbb{T}$ hamda $\mathbb{T}^2$ torlarda aniqlangan kvadrati bilan integrallanuvchi funksiyalarning Gilbert fazolarini belgilaymiz. So‘ngra bu fazolarning to‘g‘ri yig‘indisini quyidagicha aniqlaymiz:

$$\mathcal{H}^{(0,2)} := \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2.$$

$\mathcal{H}^{(0,2)}$ fazoning ixtiyoriy elementi $f = (f_0, f_1, f_2)$ vektordan iborat, bunda $f_i \in \mathcal{H}_i$, $i = 0, 1, 2$.

Endi $\mathcal{H}^{(0,2)}$ Gilbert fazosida ta’sir qiluvchi quyidagi operatorli matritsani qaraymiz:

$$\mathcal{A} := \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{10} & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}.$$

$\mathcal{A}$ operatorli matritsaning elementlari quyidagicha aniqlanadi:

$$A_{00}f_0 = u_0 f_0, \qquad A_{01}f_1 = \int_{\mathbb{T}} v_0(t) f_1(t)\, dt,$$

$$(A_{11}f_1)(x) = u_1(x) f_1(x), \qquad (A_{12}f_2)(x) = \int_{\mathbb{T}} v_1(t) f_2(x,t)\, dt,$$

$$(A_{22}f_2)(x,y) = u_2(x,y) f_2(x,y),$$

bu yerda $u_0 \in \mathbb{R}$, $u_1(\cdot)$ va $u_2(\cdot,\cdot)$ haqiqiy qiymatli uzluksiz funksiyalar bo‘lib, ularning aniqlanish sohalari mos ravishda $\mathbb{T}$ hamda $\mathbb{T}^2$ hisoblanadi. $v_0(\cdot)$ va $v_1(\cdot)$ funksiyalar esa $\mathbb{T}$ hamda $\mathbb{T}^2$ fazolarda aniqlangan haqiqiy qiymatli uzluksiz funksiyalardir.

Funksional analizda asosiy tushunchalardan biri bo‘lgan qo‘shma operator ta’rifidan foydalanib

$$(A_{10}f_0)(x) = v_0(x) f_0, \qquad (A_{21}f_1)(x,y) = v_1(y) f_1(x)$$

tengliklarni olamiz.

Mazkur ishda keyingi natijalarni bayon qilish uchun qaralayotgan Gilbert fazosini quyidagi ko’rinishdagi ikkita Hilbert fazolarining to‘g‘ri yig‘indisi ko’rinishida ifodalaymiz:

$$\mathcal{H}^{(0,2)} = \mathcal{H}^{(0,1)} \oplus \mathcal{H}_2, \qquad \mathcal{H}^{(0,1)} = \mathcal{H}_0 \oplus \mathcal{H}_1.$$

Bu ajratishga nisbatan yuqoridagi $\mathcal{A}$ operatorli matritsani unga ekvivalent bo‘lgan

$$\mathcal{A} = \begin{pmatrix} \widehat{A}_{11} & \widehat{A}_{12} \\ \widehat{A}_{21} & \widehat{A}_{22} \end{pmatrix}$$

ko‘rinishdagi 2×2 blok operatorli matritsa ko‘rinishida ifodalash mumkin. Bu yerda

$$\widehat{A}_{11} := \begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{pmatrix}, \qquad \widehat{A}_{12} := \begin{pmatrix} 0 \\ A_{12} \end{pmatrix},$$

$$\widehat{A}_{21} := \begin{pmatrix} 0 & A_{21} \end{pmatrix}, \qquad \widehat{A}_{22} := A_{22}.$$

Avval mos Hilbert fazosida ta'sir etuvchi birlik operatorni I bilan belgilaymiz va quyidagi belgilashlarni kiritamiz:

$$m_1 := \min_{x\in\mathbb{T}} u_1(x), \qquad M_1 := \max_{x\in\mathbb{T}} u_1(x).$$

$$m_2(x) := \min_{y\in\mathbb{T}} u_2(x,y), \qquad M_2(x) := \max_{y\in\mathbb{T}} u_2(x,y).$$

$u_1(\cdot)$ va $u_2(x,\cdot)$ funksiyalar har bir $x \in \mathbb{T}$ uchun $\mathbb{T}$ torda uzluksiz bo'lganligidan, ularning qiymatlar sohalarini

$$\mathrm{Ran}u_1 = [m_1, M_1], \quad \mathrm{Ran}u_2(x,\cdot) = [m_2(x), M_2(x)]$$

ko'rinishda ifodalash mumkin. Endi $z \in \mathbb{C} \setminus [m_1, M_1]$ sohada analitik bo'lgan

$$\Delta_0(z) := u_0 - z - \int_{\mathbb{T}} \frac{v_0^2(t)}{u_1(t) - z}\, dt$$

funksiyani va har bir tayinlangan $x \in \mathbb{T}$ hamda $z \in \mathbb{C} \setminus [m_2(x), M_2(x)]$ uchun esa

$$\Delta_1(x,z) := u_1(x) - z - \int_{\mathbb{T}} \frac{v_1^2(t)}{u_2(x,t) - z}\, dt$$

funksiyani qaraymiz.

$\Delta_0(z)$ funksiya $\widehat{A}_{11}$ operatorli matritsaga mos Fredgolm determinanti deb ataladi.

Har bir tayinlangan x uchun $\Delta_1(x,\cdot)$ funksiyani umumlashgan Fridrixs modeliga mos Fredgolm determinanti sifatida qarash mumkin.

Shunday qilib, $\Delta_0(\cdot)$ va $\Delta_1(\cdot,\cdot)$ funksiyalar qaralayotgan operatorlar uchun samarali spektral parametrlashni ta'minlaydi. Ularning analitik xossalari nollarining joylashuvi va karraliligi bilan birgalikda diskret spektrni hamda uning x parametriga bog'liqligini tadqiq etish imkonini beradi. Bundan tashqari, ushbu Fredgolm determinantlarining nollari bilan mos operatorlarning xos qiymatlari orasidagi bog'lanishni Shur to'ldiruvchisi va Fredgolm alternativasi yordamida o'rnatish mumkin. Quyida bu yondashuv dastlabki operatorli matritsaning spektral tuzilishini tavsiflash uchun qo'llaniladi.

Faraz qilaylik, $z \in \rho(\widehat{A}_{22})$, bunda $\rho(\widehat{A}_{22})$ bilan $\widehat{A}_{22}$ operatorning rezolventa to'plami belgilanadi. U holda $\mathcal{A}$ operatorli matritsaga mos birinchi Shur to'ldiruvchisi quyidagicha aniqlanadi:

$$S_1(z) : \mathcal{H}^{(0,1)} \to \mathcal{H}^{(0,1)},$$

$$S_1(z) := \widehat{A}_{11} - zI - \widehat{A}_{12}(\widehat{A}_{22} - zI)^{-1}\widehat{A}_{21}.$$

Sodda algebraik almashtirishlar yordamida $\mathcal{A}$ operatorli matritsaga mos birinchi Shur to'ldiruvchisini quyidagi ko'rinishda ifodalash mumkin:

$$S_1(z) = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix},$$

bu yerda $z \in \rho(A_{22})$ sonlari uchun $S_1(z)$ operatorli matritsaning elementlari quyidagicha aniqlanadi:

$$S_{00}f_0 = (u_0 - z)f_0, \qquad S_{01}f_1 = \int_{\mathbb{T}} v_0(t) f_1(t)\, dt,$$

$$(S_{10}f_0)(x) = v_0(x)f_0, \qquad (S_{11}f_1)(x) = \Delta(x,z)f_1(x), \qquad (f_0,f_1) \in \mathcal{H}^{(0,1)}.$$

Shunday qilib, har bir tayinlangan $z \in \rho(A_{22})$ uchun birinchi Shur to‘ldiruvchisi $S_1(z)$ $\mathcal{A}$ operatorli matritsaga mos umumlashgan Fridrixs modelining operator ko‘rinishidagi tasviri sifatida talqin qilinishi mumkin.

Quyidagi natija $\mathcal{A}$ operatorli matritsaning xos qiymatlari bilan uning birinchi Shur to‘ldiruvchisining nol xos qiymati orasidagi bog‘lanishni o‘rnatadi.

1-teorema. $z \in \rho(A_{22})$ bo‘lsin. U holda z soni $\mathcal{A}$ operatorli matritsaning xos qiymati bo‘lishi uchun 0 soni $S_1(z)$ operatorning xos qiymati bo‘lishi zarur va yetarlidir.

Ushbu teorema operatorli matritsaning xos qiymatlarini aniqlash masalasini uning Shur to‘ldiruvchisi $S_1(z)$ operatorining spektral xossalarini o‘rganish orqali soddalashtirish imkonini beradi. Mazkur natija murakkab operatorli sistemalarning spektrini tadqiq qilishda, xususan, matematik fizika va kvant mexanikasi masalalarida muhim nazariy va amaliy ahamiyatga ega.

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Pro-nilpotency and explicit enumeration of a two-variable generalization of the positive Witt algebra

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The classical Witt algebra is one of the fundamental examples of an infinite-dimensional Lie algebra. Its positive part and various generalizations occur naturally in the theory of graded Lie algebras, polynomial vector fields and representation theory. Generalized Witt algebras were investigated, in particular, by Ree and Kawamoto; see [1,2]. In the present note we consider a positive two-variable analogue and describe its pro-nilpotent structure together with a concrete enumeration of its basis.

Let W_2 be the Lie algebra with basis elements

$$w((a,b),i) = t_1^a t_2^b \frac{\partial}{\partial t_i}, \qquad a,b \in \mathbb{Z}_{\geq 0}, \quad a+b \geq 1, \quad i \in \{1,2\}.$$

The multiplication is given by

$$[w((a,b),i), w((p,q),j)] \\ = (\delta_{1,j}a + \delta_{2,j}b)w((a+p,b+q),i) - (\delta_{1,i}p + \delta_{2,i}q)w((a+p,b+q),j),$$

where $\delta_{r,s}$ is the Kronecker delta. When the construction is restricted to one variable, one obtains the positive part of the classical Witt algebra. Thus W_2 may be regarded as its natural two-variable generalization.

For a Lie algebra L, denote its lower central series by

$$L^1 = L, \qquad L^{k+1} = [L^k, L], \quad k \geq 1.$$

The multiplication in W_2 shows that the total degree $a+b$ increases under brackets. In particular, the terms of the lower central series form a strictly descending sequence

$$W_2 \supsetneq W_2^2 \supsetneq W_2^3 \supsetneq \cdots,$$

and no nonzero basis vector belongs to all of them. Moreover, for every $k \geq 2$ the quotient W_2/W_2^k is finite-dimensional; more precisely,

$$\dim(W_2/W_2^k) = k(k+1).$$

Consequently we obtain the first main result.

Theorem 1. *The Lie algebra W_2 is pro-nilpotent.*

Proof. Since the total degree of every nonzero element occurring in W_2^k tends to infinity with k, we have

$$\bigcap_{k=1}^{\infty} W_2^k = \{0\}.$$

Hence W_2 is residually nilpotent. On the other hand,

$$\dim(W_2/W_2^k) = k(k+1) < \infty$$

for every $k \geq 2$. Therefore W_2 is pro-nilpotent. □

The natural basis of W_2 is described by three parameters (a, b, c). For computational purposes it is useful to replace this notation by a single natural index. Put

$$e_i = w((a_i, b_i), c_i),$$

and define

$$i = (a_i + b_i)(a_i + b_i + 1) + 2a_i + c_i - 2. \tag{1}$$

This formula orders the basis first by the total degree $a_i + b_i$, and inside each degree by a_i and by the value of c_i.

Lemma 1. *Formula (1) gives a one-to-one enumeration of the natural basis of W_2; hence $\{e_1, e_2, \ldots\}$ is a basis of W_2.*

Now let show how can we find out the appearance of the basis elements by using its index:

Theorem 2. *Let $e_i = w((a, b), c) \in W_2$ be a basis element indexed as in (1). Then the parameters a, b, c are determined by i according to the parity of i as follows.*

Case 1: *$i = 2k$ is even $k \in \mathbb{N}$. Set $p = [\sqrt{i}]$ and define $t = i - p^2$, $0 \leq t \leq 2p$. Then:*

- *If $t = 0$:*

$$a = \frac{p}{2}, \quad b = \frac{p}{2} - 1, \quad c = 2.$$

- *If $1 \leq t \leq p - 2$:*

$$a = \frac{1}{2}(i - p^2 + p), \quad b = \frac{1}{2}(p^2 + p - i - 2), \quad c = 2.$$

- *If* $t = p - 1$:
  No basis element exists for this t.

- *If* $p \le t \le 2p$*:*
$$a = \frac{1}{2}(i - p^2 - p), \quad b = \frac{1}{2}(p^2 + 3p - i), \quad c = 2.$$

Case 2: $i = 2k - 1$ *is odd* $k \in \mathbb{N}$. *Set* $q = [\sqrt{i+1}]$ *and define* $t = i + 1 - q^2, \quad 0 \le t \le 2q$. *Then:*

- *If* $t = 0$*:*
$$a = \frac{q}{2} \quad b = \frac{q}{2} - 1, \quad c = 1.$$

- *If* $1 \le t \le q - 2$*:*
$$a = \frac{1}{2}(i + 1 - q^2 + q), \quad b = \frac{1}{2}(q^2 + q - i - 3), \quad c = 1.$$

- *If* $t = q - 1$:
  No basis element exists for this t.

- *If* $q \le t \le 2q$*:*
$$a = \frac{1}{2}(i + 1 - q^2 - q), \quad b = \frac{1}{2}(q^2 + 3q - i - 1), \ c = 1.$$

For example, the first basis elements are

$$e_1 = w((0,1),1), \quad e_2 = w((0,1),2),$$
$$e_3 = w((1,0),1), \quad e_4 = w((1,0),2),$$
$$e_5 = w((0,2),1), \quad e_6 = w((0,2),2).$$

Thus W_2 admits two complementary descriptions: the realization by polynomial vector fields is well suited to its algebraic construction, while the sequence $\{e_i\}_{i\ge1}$ is convenient for explicit computations. Theorem 1 places W_2 in the class of pro-nilpotent Lie algebras, while Lemma 1 and Theorem 2 provide a direct correspondence between the natural multi-index basis and a single-index basis. This representation may be useful in subsequent studies of derivations, cohomology and pro-solvable extensions of W_2.

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Pro-solvable extensions of a generalized positive Witt algebra.

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The Witt algebra is one of the basic examples of an infinite-dimensional Lie algebra and appears naturally in geometry, representation theory and mathematical physics. Its positive part

$$\mathcal{W}_+ = \operatorname{span}\{e_1, e_2, \ldots\}, \qquad [e_i, e_j] = (j-i)e_{i+j}, \quad i, j \geq 1,$$

is pro-nilpotent. Motivated by the construction of solvable Lie algebras from a prescribed nilradical and by its infinite-dimensional analogues, we consider, for a fixed integer $s \geq 0$, the Lie algebra

$$\mathcal{W}(s) = \operatorname{span}\{e_1, e_2, \ldots\}, \qquad [e_i, e_j] = (j-i)e_{i+j+s}, \quad i, j \geq 1. \tag{1}$$

Clearly, $\mathcal{W}(0) = \mathcal{W}_+$. The parameter s therefore measures the shift from the ordinary positive Witt algebra.

A first structural relation between $\mathcal{W}(s)$ and $\mathcal{W}_+$ is obtained by means of δ-derivations. Recall that a linear map f is a δ-derivation if

$$f([x, y]) = \delta\big([f(x), y] + [x, f(y)]\big).$$

Define $f_s : \mathcal{W}_+ \to \mathcal{W}_+$ by $f_s(e_i) = e_{i+s}$. Then

$$2f_s([e_i, e_j]) = [f_s(e_i), e_j] + [e_i, f_s(e_j)],$$

so f_s is a $\frac{1}{2}$-derivation of $\mathcal{W}_+$. Moreover,

$$f_s([e_i, e_j]_{\mathcal{W}_+}) = (j-i)e_{i+j+s} = [e_i, e_j]_{\mathcal{W}(s)}.$$

Hence $\mathcal{W}(s)$ is a mutation of the positive Witt algebra realized by the $\frac{1}{2}$-derivation f_s. Thus the whole family $\{\mathcal{W}(s)\}_{s\geq 0}$ is directly linked to the classical positive Witt algebra by a simple shift operator.

The next step is to determine the derivations of $\mathcal{W}(s)$.

Proposition 1. A basis of $\operatorname{Der}(\mathcal{W}(s))$ consists of derivations $\{d_m\}_{m\in M}$ acting on basis elements e_j $(j \geq 1)$ as follows:

$$d_m(e_j) = \begin{cases} (s+j)e_j, & m = 0, \\ (j-1)e_{j+s+1}, & m = s+1, \\ \dfrac{s-m+j}{s-m+1}\, e_{j+m}, & 1 \leq m \leq s, \\ \dfrac{s-m+j}{s-m+1}\, e_{j+m}, & s+2 \leq m \leq t_0, \end{cases}$$

where $t_0 < \infty$ is a natural number, $M = \{0, 1, 2, \ldots t_0\} \subset \mathbb{N} \cup \{0\}$.

In particular, d_0 is diagonal in the natural basis, whereas every d_m with $m \geq 1$ strictly increases the index of a basis vector. Consider the descending chain

$$V_k = \operatorname{span}\{e_k, e_{k+1}, \ldots\}, \qquad k \geq 1.$$

Then $d_m(V_k) \subseteq V_{k+1}$ for every $m \geq 1$, while d_0 is not strictly triangularizable because all its diagonal coefficients $s + j$ are nonzero. Consequently, the maximal number of strictly triangularizable independent derivations of $\mathcal{W}(s)$ is equal to one.

This observation determines the possible size of a pro-solvable extension. Let

$$\mathcal{R}(\mathcal{W}(s)) = \mathcal{W}(s) \rtimes \mathcal{Q}$$

be a non-pro-nilpotent pro-solvable Lie algebra whose pro-nilpotent radical is $\mathcal{W}(s)$. The general extension method for pro-nilpotent radicals implies that $\dim \mathcal{Q}$ is bounded by the maximal number of strictly triangularizable independent derivations. Therefore

$$\dim \mathcal{Q} = 1.$$

Choosing $\mathcal{Q} = \langle x \rangle$ and normalizing the non-strictly triangularizable component of ad_x, one obtains the following classification.

Theorem 1. Every non-pro-nilpotent pro-solvable Lie algebra whose pro-nilpotent radical is $\mathcal{W}(s)$ admits a basis $\{x, e_1, e_2, \ldots\}$ in which

$$\begin{aligned} [e_i, e_j] &= (j - i)e_{i+j+s}, \qquad i, j \geq 1, \\ [e_j, x] &= \frac{s+j}{s+1} e_j + \sum_{i=1}^{s} \frac{s+j-i}{s+1-i}\, \alpha_{i+1} e_{j+i}, \qquad j \geq 1, \end{aligned} \tag{2}$$

where $\alpha_2, \ldots, \alpha_{s+1} \in \mathbb{C}$.

Formula (2) shows that, although $\mathcal{W}(s)$ is infinite-dimensional, its maximal pro-solvable extensions are controlled by only finitely many parameters.

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GALILEY FAZOSIDAGI HARAKATLAR OSTIDA TRAYEKTORIYA SIRTINING HAJMI

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Ushbu ishda uch o'lchovli Galiley fazo harakatlari natijasida qo'zg'almas nuqtalar hosil qiladigan trayektoriya sirtlarining hajmi o'rganiladi. Asosiy maqsad trayektoriya sirtining hajmini aniqlash va klassik Holditch teoremasini Galiley fazosida hajmlar uchun umumlashtirishdan iborat.

Galiley fazosida $a = (x, y, z)$ va $b = (x_1, y_1, z_1)$ vektorlarning skalyar ko'paytmasi

$$\langle a, b \rangle_G = xx_1$$

ko'rinishda aniqlanadi.

Agar $x \neq 0$ bo'lsa, vektor *noizotrop*, agar $a = (0, y, z)$ bo'lsa, vektor *izotrop* deyiladi. Izotrop vektorlar uchun maxsus skalyar ko'paytma

$$\langle p, q\rangle_I = yy_1 + zz_1$$

formula bilan aniqlanadi.

1-ta'rif. g_1 noizotrop, g_2 va g_3 izotrop vektorlar bo'lib,

$$\langle g_1, g_1\rangle_G = \langle g_2, g_2\rangle_I = \langle g_3, g_3\rangle_I = 1,$$

$$\langle g_1, g_2\rangle_G = \langle g_1, g_3\rangle_G = \langle g_2, g_3\rangle_I = 0$$

shartlarni qanoatlantirsa, $\{g_1, g_2, g_3\}$ Galiley fazosining *ortonormal reperi* deyiladi.

Uch o'lchovli Galiley harakati deb translatsiya vektori va reper vektorlarining t_1, t_2, t_3 parametrlariga bog'liq bo'lgan harakatiga aytiladi va B_3 bilan belgilanadi.

Galiley fazosidagi harakat uchun

$$dg_1 = \omega_3 g_2 + \omega_2 g_3, \qquad dg_2 = \omega_1 g_3, \qquad dg_3 = \omega_1 g_2.$$

Translatsiya vektori

$$du = \sigma_1 g_1 + \sigma_2 g_2 + \sigma_3 g_3$$

ko'rinishda beriladi.

Qo'zg'almas $X = (x_1, x_2, x_3)$ nuqtaning trayektoriya sirtining hajm elementi

$$dJ_X = \tau_1 \wedge \tau_2 \wedge \tau_3$$

va uning hajmi

$$J_X = \int_G dJ_X$$

formula bilan aniqlanadi.

1-teorema. B_3 harakat ostida $X = (x_1, x_2, x_3)$ qo'zg'almas nuqtaning trayektoriya sirtining hajmi koordinatalarga nisbatan kvadratik funksiya bo'ladi:

$$\boxed{J_X = J_O + ax_1^2 + b_2x_1x_2 + b_3x_1x_3 + c_1x_1 + c_2x_2 + c_3x_3}$$

Shuning uchun bir xil hajmli trayektoriya sirtlarini hosil qiluvchi barcha qo'zg'almas nuqtalar bitta kvadrik sirtga tegishli:

$$\boxed{ax_1^2 + b_2x_1x_2 + b_3x_1x_3 + c_1x_1 + c_2x_2 + c_3x_3 + J_O - J_0 = 0.}$$

2-teorema. X va Y nuqtalarni tutashtiruvchi kesmada Z nuqta

$$z_i = \lambda x_i + \mu y_i, \qquad \lambda + \mu = 1$$

shartni qanoatlantirsa, uning trayektoriya sirtining hajmi

$$\boxed{J_Z = \lambda^2 J_X + 2\lambda\mu J_{XY} + \mu^2 J_Y}$$

formula orqali aniqlanadi.

3-teorema. Galiley harakatiga bog'liq maxsus masofa

$$\boxed{D^2(X,Y) = \varepsilon\left[a(x_1-y_1)^2 + b_2(x_1-y_1)(x_2-y_2) + b_3(x_1-y_1)(x_3-y_3)\right],}$$

bu yerda $\varepsilon = \pm 1$.
U holda

$$\boxed{J_Z = \lambda J_X + \mu J_Y - \varepsilon\lambda\mu D^2(X,Y).}$$

Agar

$$J_X = J_Y,$$

bo'lsa,

$$\boxed{J_X - J_Z = \varepsilon D(Z,Y)D(X,Z).}$$

Bu natija klassik Holditch teoremasining Galiley fazosidagi hajm umumlashmasini beradi.

4-teorema. Agar Galiley fazosidagi uchburchakning uchlari bir xil hajmli trayektoriya sirtlarini hosil qilsa, uchburchak tekisligidagi boshqa nuqta hosil qiladigan trayektoriya sirtining hajmi uchlar va ushbu nuqta orasidagi Galiley harakatiga bog'liq masofalar orqali aniqlanadi.

Uch o'lchovli Galiley harakati ostida trayektoriya sirtining hajmi nuqta koordinatalarining kvadratik funksiyasi ekanligi aniqlandi. Teng hajmli trayektoriya sirtlarini hosil qiluvchi nuqtalar kvadrik sirtni tashkil etadi. Maxsus masofa yordamida Holditch teoremasining Galiley fazosidagi hajmiy umumlashmasi olindi.

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On main properties of max-plus gauges

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Note that a function k on $\mathbb{R}^n$ is called [1] a gauge or a Minkowski functional of a set C if k is a non-negative positively homogeneous convex function such that $k(0) = 0$, i. e. if epi k is a

convex cone in $\mathbb{R}^{n+1}$ containing the origin but not containing any vectors $(x,\, t)$ such that $t < 0$. Gauges are thus the functions k such that

$$k(x) = \gamma(x|C) = \inf\{t \geq 0 :\ x \in tC\} \tag{1}$$

for some non-empty convex set C.

In functional analysis and convex analysis, the Minkowski functional (or gauge functional) of a convex set is well known for its numerous remarkable theoretical and practical applications. These include:

1. When C is a convex set, the functional in (1) is sublinear, and for $\lambda \geq 0$ the equality $\gamma_C(\lambda x) = \lambda\gamma(x)$ holds. Consequently, this functional is regarded as one of the fundamental tools in functional analysis and convex analysis;

2. In the construction of seminorms and norms (which leads to the construction of locally convex topologies);

3. In convex optimization;

4. In proximal optimization and regularization problems [4], including signal denoising, sparse modeling, structured estimation, and atomic norm/gauge regularization;

5. If C is taken as the set of admissible controls, then $\gamma_C(x)$ can be interpreted as the minimum time required to reach the state x. Thus, the gauge functional can also be applied to minimum-time and reachability problems [5].

In [3], it is shown that the Minkowski functional induces a metric on the space of probability measures. We define [2] a max-plus-gauge ${}^{\oplus}k$ of a max-plus-convex subset F of $\mathbb{R}^n_{\max}$ by equality

$${}^{\oplus}k(x) =^{\oplus} \gamma\,(x|F) = \inf\{t \geq -\infty :\ x \in t \odot F\}, \qquad x \in \mathbb{R}^n_{\max}.$$

The main result of the present note establishes important properties of max-plus-gauges, and it can be formulated as the following statement.

Theorem 1. A max-plus-gauge ${}^{\oplus}\gamma$ of a max-plus-convex subset F of $\mathbb{R}^n_{\max}$ has the following properties:

1st **fundamental property:** max-plus positive homogeneity. For any $\lambda \in \mathbb{R}_{\max}$ and $x \in \mathbb{R}^n_{\max}$ equality

$${}^{\oplus}\gamma_F(\lambda \odot x) = \lambda \odot^{\oplus} \gamma_F(x)$$

is true.

2nd **fundamental property:** max-plus subadditivity. For each pair $x,\, y \in \mathbb{R}^n_{\max}$ we have

$${}^{\oplus}\gamma_F(x \oplus y) \leq^{\oplus} \gamma_F(x) \oplus^{\oplus} \gamma_F(y).$$

3rd **property:** max-plus sublinearity. For any $\lambda,\, \mu \in \mathbb{R}_{\max}$ and any $x,\, y \in \mathbb{R}^n_{\max}$ one has

$${}^{\oplus}\gamma_F(\lambda \odot x \oplus \mu \odot y) \leq \lambda \odot^{\oplus} \gamma_F(x) \oplus \mu \odot^{\oplus} \gamma_F(y).$$

4th **property:** monotonicity according to F. If $F_1 \subset F_2$ then

$${}^{\oplus}\gamma(x \mid F_2) \leq^{\oplus} \gamma(x \mid F_1).$$

5th **property:** values of the gauge at points in F. If $x \in F$ then ${}^{\oplus}\gamma_F(x) \leq 0$.

Corollary 1. A max-plus-gauge ${}^{\oplus}\gamma$ of a max-plus-convex subset F of $\mathbb{R}^n_{\max}$ is max-plus convex, i. e. for every couple $\alpha,\ \beta \in \mathbb{R}_{\max}$ with $\alpha \oplus \beta = 1$ we have

$$ {}^{\oplus}\gamma_F(\alpha \odot x \oplus \beta \odot y) \leq \alpha \odot^{\oplus} \gamma_F(x) \oplus \beta \odot^{\oplus} \gamma_F(y). $$

Corollary 1 yields an important conclusion: max-plus convexity of F implies max-plus convexity of its gauge ${}^{\oplus}\gamma_{_F}$. That is why gauges are naturally raised objects.

Remark 1. However, in the general case, it cannot be asserted that

$$ x \in F \quad \Rightarrow \quad {}^{\oplus}\gamma_F(x) = 0. $$

Actually, consider a set

$$ F = \left\{(x_1,\, x_2) \in \mathbb{R}^2_{\max}\colon\ -3 \leq x_1,\, x_2 \leq 3\right\}. $$

For $x = (1,\, 1) \in F$, $y = (3,\, 3) \in F$ we have ${}^{\oplus}\gamma_{_F}(x) = -2$, ${}^{\oplus}\gamma_{_F}(y) = 0$.

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EIGHT-ELECTRON SECOND SINGLET STATE IN THE HUBBARD MODEL

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We consider the energy operator of eight-electron systems in the Hubbard model and describe the structure of the essential spectra and discrete spectrum of the system for second singlet states in the $\nu-$ dimensional lattice. The Hamiltonian of the model has the form [1]

$$ H = A\sum_{m,\gamma} a^+_{m,\gamma} a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m,\gamma} a_{m+\tau,\gamma} + U\sum_{m} a^+_{m,\uparrow} a_{m,\uparrow} a^+_{m,\downarrow} a_{m,\downarrow}. \qquad (1) $$

Here A is the electron energy at a lattice site and B is the transfer integral between electrons at neighboring sites; we assume for convenience that $B > 0$. Further, $\tau = \pm e_j$, $j = 1, 2, ..., \nu$, where e_j are pairwise orthogonal unit vectors, the summation is taken over the nearest neighbors, U is the parameter of the on-site Coulomb interaction of two electrons in the lattice sites, γ is the spin index, $\gamma =\uparrow$ or $\gamma =\downarrow$, where $\uparrow$ and $\downarrow$ denote the spin values $\frac{1}{2}$ and $-\frac{1}{2}$, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $z \in Z^\nu$.

Hamiltonian (1) commutes with all components of the total spin operator $S = (S^+, S^-, S^z)$, and the structure of eigenfunctions and eigenvalues of the system therefore depends on S. The Hamiltonian H acts in the antisymmetric Fo'ck space $\mathcal{H}_{as}$.

Let φ_0 be the vacuum vector in the space $\mathcal{H}_{as}$. The second singlet state corresponds to the free motion of eight electrons and their interactions over the lattice with the basic functions ${}^2s^0_{m,n,r,k,p,q,t,l\in Z^\nu} = a^+_{m,\uparrow}a^+_{n,\downarrow}a^+_{r,\uparrow}a^+_{k,\downarrow}a^+_{p,\uparrow}a^+_{q,\downarrow}a^+_{t,\uparrow}a^+_{l,\downarrow}\varphi_0$. The subspace ${}^2\mathcal{H}^0_s$ corresponding to the second singlet state is the set of all vectors of the form

$$ {}^2\psi^0_s = \sum_{m,n,r,k,p,q,t,l\in Z^\nu} f(m,n,r,k,p,q,t,l)\,{}^2s^0_{m,n,r,k,p,q,t,l\in Z^\nu}, f \in l^{as}_2, $$

where l^{as}_2 is the subspace of antisymmetric functions in the space $l_2((Z^\nu)^8)$. We let ${}^2H^0_s$ denote the restriction of operator H to the subspace ${}^2\mathcal{H}^0_s$.

Theorem 1. (coordinate representation of the action of operator ${}^2H^0_s$). The subspace ${}^2\mathcal{H}^0_s$ is invariant under the operator H, and the restriction ${}^2H^0_s$ of operator H to the subspace ${}^2\mathcal{H}^0_s$ is a bounded self-adjoint operator. It generates a bounded self-adjoint operator ${}^2\overline{H}^0_s$ acting in the space l^{as}_2 as

$$ {}^2\overline{H}^0_s{}^2\psi^0_s = 8Af(m,n,r,k,p,q,t,l) + B\sum_\tau [f(m+\tau,n,r,k,p,q,t,l) + f(m,n+\tau,r,k,p,q,t,l) + $$

$$ +f(m,n,r+\tau,k,p,q,t,l)+f(m,n,r,k+\tau,p,q,t,l)+f(m,n,r,k,p+\tau,q,t,l)+f(m,n,r,k,p,q+\tau,t,l)+ $$

$$ +f(m,n,r,k,p,q,t+\tau,l)+f(m,n,r,k,p,q,t,l+\tau)]+U[\delta_{m,n}+\delta_{m,k}+\delta_{m,q}+\delta_{m,l}+\delta_{n,r}+\delta_{n,p}+\delta_{n,t}+\delta_{r,k}+ $$

$$ +\delta_{r,q}+\delta_{r,l}+\delta_{k,p}+\delta_{k,t}+\delta_{p,q}+\delta_{p,l}+\delta_{q,t}+\delta_{t,l}]f(m,n,r,k,p,q,t,l). \qquad (2) $$

The operator ${}^2H^0_s$ acts on a vector ${}^2\psi^0_s \in {}^2\mathcal{H}^0_s$ as

$$ {}^2H^0_s\,{}^2\psi^0_s = \sum_{m,n,r,k,p,q,t,l\in Z^\nu} ({}^2\overline{H}^0_s f)(m,n,r,k,p,q,t,l)\,{}^2s^0_{m,n,r,k,p,q,t,l\in Z^\nu}. \qquad (3) $$

Lemma 1. The spectra of the operators ${}^2H^d_{1/2}$ and ${}^1\overline{H}^d_{1/2}$ coincide.

Let $\mathcal{F} : l_2((Z^\nu)^8) \to L_2((T^\nu)^8) \equiv {}^2\widetilde{\mathcal{H}}^0_s$ be the Fourier transformation, where T^ν is the $\nu-$dimensional torus endowed with the normalized Lebesgue measure $d\lambda$, i.e., $\lambda(T^\nu) = 1$. We set ${}^2\widetilde{H}^0_s = \mathcal{F}{}^2\overline{H}^0_s\mathcal{F}^{-1}$.

Theorem 2. (quasimomentum representation of the action of operator ${}^2H^0_s$). The Fourier transform of operator ${}^2\overline{H}^0_s$ is an operator ${}^2\widetilde{H}^0_s$ acting in the space $L^{as}_2((T^\nu)^8)$ be the formula

$$ {}^2\widetilde{H}^0_s\,{}^2\psi^0_s = h(\lambda,\mu,\gamma,\theta,\eta,\xi,\alpha,\beta)f(\lambda,\mu,\gamma,\theta,\eta,\xi,\alpha,\beta) + U\int_{T^\nu}[f(s,\lambda+\mu-s,\gamma,\theta,\eta,\xi,\alpha,\beta)+ $$

$$ +f(s,\mu,\gamma,\lambda+\theta-s,\eta,\xi,\alpha,\beta)+f(s,\mu,\gamma,\theta,\eta,\lambda+\xi-s,\alpha,\beta)+f(s,\mu,\gamma,\theta,\eta,\xi,\alpha,\lambda+\beta-s)+ $$

$$ +f(\lambda,s,\mu+\gamma-s,\theta,\eta,\xi,\alpha,\beta)+f(\lambda,s,\gamma,\theta,\mu+\eta-s,\xi,\alpha,\beta)+f(\lambda,s,\gamma,\theta,\eta,\xi,\mu+\alpha-s,\beta)+ $$

$$+f(\lambda,\mu,s,\gamma+\theta-s,\eta,\xi,\alpha,\beta)+f(\lambda,\mu,s,\theta,\eta,\gamma+\xi-s,\alpha,\beta)+f(\lambda,\mu,s,\theta,\eta,\xi,\alpha,\gamma+\beta-s)+$$
$$+f(\lambda,\mu,\gamma,s,\theta+\eta-s,\xi,\alpha,\beta)+f(\lambda,\mu,\gamma,s,\eta,\xi,\theta+\alpha-s,\beta)+f(\lambda,\mu,\gamma,\theta,s,\eta+\xi-s,\alpha,\beta)+$$
$$+f(\lambda,\mu,\gamma,\theta,s,\xi,\alpha,\eta+\beta-s)+f(\lambda,\mu,\gamma,\theta,\eta,s,\xi+\alpha-s,\beta)+f(\lambda,\mu,\gamma,\theta,\eta,\xi,s,\alpha+\beta-s)]ds.$$

Where

$$h(\lambda,\mu,\gamma,\theta,\eta,\xi,\alpha,\beta)=8A+2B\sum_{k}^{\nu}[\cos\lambda_k+\cos\mu_k+\cos\gamma_k+\cos\theta_k+\cos\eta_k+\cos\xi_k+\cos\alpha_k+\cos\beta_k].$$

Since all parameters $\lambda,\mu,\gamma,\theta,\eta,\xi,\alpha,\beta$ and s a changed in the ν-dimensional torus T^{ν}, therefore, you can write as (4)

$$^2\widetilde{H}_s^0=\widetilde{H}_1^1\bigotimes I\bigotimes I\bigotimes I+I\bigotimes\widetilde{H}_1^2\bigotimes I\bigotimes I+I\bigotimes I\bigotimes\widetilde{H}_1^3\bigotimes I+I\bigotimes I\bigotimes I\bigotimes\widetilde{H}_1^4.$$

Here

$$(\widetilde{H}_1^1f)(\lambda,\mu)=\{2A+2B\sum_{k=1}^{\nu}[\cos\lambda_k+\cos\mu_k]\}f(\lambda,\mu)+4U\int_{T^{\nu}}f(s,\lambda+\mu-s)ds,$$

$$(\widetilde{H}_1^2f)(\gamma,\theta)=\{2A+2B\sum_{k=1}^{\nu}[\cos\gamma_k+\cos\theta_k]\}f(\gamma,\theta)+4U\int_{T^{\nu}}f(s,\gamma+\theta-s)ds,$$

$$(\widetilde{H}_1^3f)(\eta,\xi)=\{2A+2B\sum_{k=1}^{\nu}[\cos\eta_k+\cos\xi_k]\}f(\eta,\xi)+4U\int_{T^{\nu}}f(s,\eta+\xi-s)ds,$$

$$(\widetilde{H}_1^4f)(\alpha,\beta)=\{2A+2B\sum_{k=1}^{\nu}[\cos\alpha_k+\cos\beta_k]\}f(\alpha,\beta)+4U\int_{T^{\nu}}f(s,\alpha+\beta-s)ds.$$

Let the total quasimomentum of the two-electron systems $\lambda+\mu=\Lambda_1,\quad \gamma+\theta=\Lambda_2,\quad \eta+\xi=\Lambda_3,\quad \alpha+\beta=\Lambda_4$ be fixed. Denote by $L_2(\Gamma_{\Lambda_1})$ the space of functions that are square integrable over the manifold $\Gamma_{\Lambda_1}=\{(\lambda,\mu):\lambda+\mu=\Lambda_1\}$. Note that in this case the operator $\widetilde{H}_1^1$ and space $\mathcal{H}_1^1=L_2((T^{\nu})^2)$ can be expanded into the direct integral $\widetilde{H}_1^1=\bigoplus\int_{T^{\nu}}\widetilde{H}_{1\Lambda_1}^1d\Lambda_1$, $\mathcal{H}_1^1=\bigoplus\int_{T^{\nu}}\mathcal{H}_{1\Lambda_1}^1d\Lambda_1$.

It is clear that $\sigma(A\bigotimes I+I\bigotimes B)=\{\lambda+\mu:\lambda\in\sigma(A),\quad \mu\in\sigma(B)\}$. Therefore, we must investigate the spectrum of the operators $\widetilde{H}_{1\Lambda_1}^1$, $\widetilde{H}_{1\Lambda_2}^2$, $\widetilde{H}_{1\Lambda_3}^3$ and $\widetilde{H}_{1\Lambda_4}^4$.

It is known that the continuous spectrum of operator $\widetilde{H}_{1\Lambda_1}^1$ is independent of the parameter U and consists of the intervals

$$\sigma_{cont}(\widetilde{H}_{1\Lambda_1}^1)=G_{\Lambda_1}^{\nu}=[m_{\Lambda_1}^{\nu},M_{\Lambda_1}^{\nu}]=[2A-4B\sum_{k=1}^{\nu}\cos\frac{\Lambda_1^k}{2},2A+4B\sum_{k=1}^{\nu}\cos\frac{\Lambda_1^k}{2}].$$

Definition 1. *The eigenfunction $\phi_{\Lambda_1}\in L_2((T^{\nu})^2)$ of the $\widetilde{H}_{1\Lambda_1}^1$ corresponding to an eigenvalue $z_{\Lambda_1}\notin G_{\Lambda_1}^{\nu}$ is called a bound state (antibound state) of $\widetilde{H}_{1\Lambda_1}^1$ with the quasi momentum Λ_1, and the quantity z_{Λ_1} is called the energy of this state.*

We consider the one-dimensional case.

Theorem 3. Let ν=1 and $U<0$, then for all values of parameters of the Hamiltonian, the operator $\widetilde{H}_{1\Lambda_1}^1$ has a unique eigenvalue $z_1=2A-4\sqrt{U^2+B^2\cos^2\frac{\Lambda_1}{2}}$ that is below the continuous spectrum of the operator $\widetilde{H}_{1\Lambda_1}^1$, i.e., $z_1<m_{\Lambda_1}^1$.

Theorem 4. Let ν=1 and $U > 0$, then for all values of parameters of the Hamiltonian, the operator $\widetilde{H}^1_{1\Lambda_1}$ has a unique eigenvalue $\widetilde{z}_1 = 2A+4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_1}{2}}$ that is above the continuous spectrum of the operator $\widetilde{H}^1_{1\Lambda_1}$, i.e., $\widetilde{z}_1 > m^1_{\Lambda_1}$.

Now, using the above results and representation (4), we describe the structure of the essential spectrum and the discrete spectrum of the energy operator of eight-electron systems in the Hubbard model in the second singlet state of the system.

Theorem 5. Let $\nu = 1$, and $U < 0$, then the essential spectrum of operator ${}^2\widetilde{H}^0_s$ is the union of fifteen segments:

$$\sigma_{ess}({}^2\widetilde{H}^0_s) = [a+c+e+m, b+d+f+n]\cup[a+c+e+z_4, b+d+f+z_4]\cup[a+c+m+z_3, b+d+n+z_3]\cup[a+e+m+$$

$$+z_2, b+f+n+z_2]\cup[c+e+m+z_1, d+f+n+z_1]\cup[a+c+z_3+z_4, b+d+z_3+z_4]\cup[a+e+z_2+z_4, b+f+z_2+z_4]\cup$$

$$\cup[c+e+z_1+z_4, b+n+z_1+z_4]\cup[a+m+z_2+z_3, b+n+z_2+z_3]\cup[c+m+z_1+z_3, d+n+z_1+z_3]\cup$$

$$\cup[e+m+z_1+z_2, f+n+z_1+z_2]\cup[a+z_2+z_3+z_4, b+z_2+z_3+z_4]\cup[c+z_1+z_3+z_4, d+z_1+z_3+z_4]\cup$$

$$\cup[e+z_1+z_2+z_4, f+z_1+z_2+z_4]\cup[m+z_1+z_2+z_3, n+z_1+z_2+z_3],$$

and discrete spectrum of operator ${}^2\widetilde{H}^0_s$ is consists of a unique eigenvalue: $\sigma_{disc}({}^2\widetilde{H}^0_s) = \{z_1+z_2+z_3+z_4\}$, what lies to the below than the left edge of the essential spectrum of operator ${}^2\widetilde{H}^0_s$. Here

$$a = 2A-4B\cos\frac{\Lambda_1}{2}, b = 2A+4B\cos\frac{\Lambda_1}{2}, c = 2A-4B\cos\frac{\Lambda_2}{2}, d = 2A+4B\cos\frac{\Lambda_2}{2}, e = 2A-4B\cos\frac{\Lambda_3}{2},$$

$$f = 2A + 4B\cos\frac{\Lambda_3}{2}, m = 2A - 4B\cos\frac{\Lambda_4}{2}, n = 2A + 4B\cos\frac{\Lambda_4}{2}, z_1 = 2A - 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_1}{2}},$$

$$z_2 = 2A - 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_2}{2}}, z_3 = 2A - 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_3}{2}}, z_4 = 2A - 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_4}{2}},$$

$$\widetilde{z}_1 = 2A + 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_1}{2}}, \widetilde{z}_2 = 2A + 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_2}{2}}, \widetilde{z}_3 = 2A + 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_3}{2}},$$

$$\widetilde{z}_4 = 2A + 4\sqrt{U^2 + B^2\cos^2\frac{\Lambda_4}{2}}.$$

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Restoration of Convex Surfaces with Singularities

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It is well known [1] that the points of a convex surface are classified into three types: regular points, edge points, and conic points. In any polyhedron, all three types of these points are present: the interior points of the polygons constituting the polyhedron are regular points, the points along the edges correspond to edges, and the vertices represent conic points. However, besides polyhedra, there exist other surfaces characterized by the same three types of points. For instance, let us consider the surface defined by the following vector equation:

$$\mathbf{r} = \mathbf{r}(u, v) = (\cos 2u \cdot \cos u \cdot \cos v,\ \sin v,\ \cos 2u \cdot \sin u \cdot \cos v),$$

where $u \in \left[\frac{\pi}{4},\ \frac{3\pi}{4}\right]$ and $v \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$.

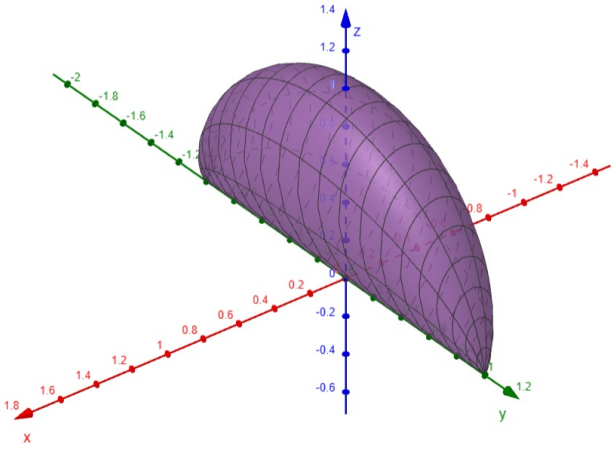


Рис. 4: A convex surface with two vertices and an edge

This surface primarily consists of regular points, featuring two conic points and a single edge connecting them (See Fig. 1).

Definition. Such surfaces are referred to as *convex surfaces with singularities.*

We consider the following problem:

Let G be a convex polygon in the plane, represented as the union of convex polygons $G_1, G_2, \ldots, G_k$. Some of G_i may share a common vertex, but the side edges of G_i belong to at most two polygons. Suppose we are given a set function $\mu(M)$ defined on Borel sets $M \subset G_i$. In space, a spatial network composed of lines L_i is given such that this network projects one-to-one onto the convex polygons G_i and G, with the vertices of L_i projecting bijectively onto the vertices of G_i and G.

Theorem. If $\mu(M) > 0$ is a positive additive function satisfying $0 < \mu(G_i) < \pi$, then there exists a convex surface with singularities having edges formed by the lines L_i and projecting bijectively onto the domain G.

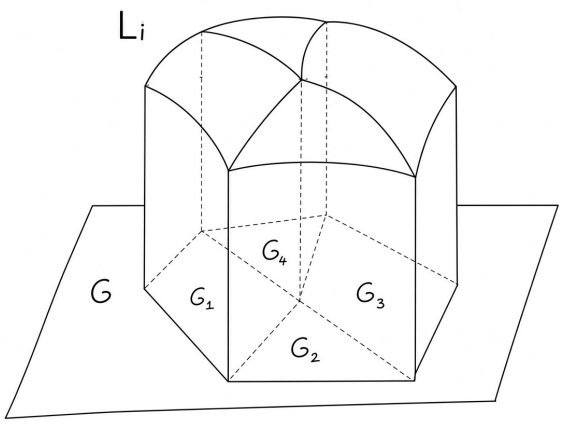


Рис. 5: A convex surfaces with singularities

The general appearance of the convex surface with singularities that satisfies the conditions of Theorem 1 is illustrated in Fig. 2.

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Kompakt operatorlar Banax ideallarining abstrakt tasnifi

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Bizga $(\mathcal{H}, (\cdot,\cdot))$ cheksiz o'lchovli $\mathbb{C}$ maydon ustida Hilbert fazosi berilgan bo'lsin va $(\mathcal{B}(\mathcal{H}), \|\cdot\|_\infty)$ juftlik $\mathcal{H}$ da barcha chiziqli chegaralangan operatorlarning $C^\star$-algebrasi bo'lsin. $\mathcal{K}(\mathcal{H})$(mos ravishda, $\mathcal{F}(\mathcal{H})$) orqali $\mathcal{B}(\mathcal{H})$ da kompakt (mos ravishda, chekli o'lchamli) operatorlar ikki tomonlama idealini belgilaymiz. Bizga ma'lumki, ixtiyoriy ikki tomonlama xosmas ideal $\mathcal{I} \subset \mathcal{B}(\mathcal{H})$ uchun $\mathcal{F}(\mathcal{H}) \subset \mathcal{I}$ o'rinli bo'ladi. Agar $\mathcal{H}$ separabel fazo bo'lsa, u holda $\mathcal{I} \subset \mathcal{K}(\mathcal{H})$ ekani ham ma'lum ([2]).

Quyidagi $\mathcal{B}_h(\mathcal{H}) = \{x \in \mathcal{B}(\mathcal{H}) : x = x^*\}$, $\mathcal{B}_+(\mathcal{H}) = \{x \in \mathcal{B}_\langle(\mathcal{H}) : x \geq 0\}$ belgilashlarni kiritaylik. Aytaylik $\tau : \mathcal{B}_+(\mathcal{H}) \to [0;\infty]$ $\mathcal{B}(\mathcal{H})$ da aniqlangan kanonik iz funksionali bo'lsin, ya'ni

$$\tau(x) = \sum_{j\in J} (x\varphi_j, \varphi_j), \quad x \in \mathcal{B}_+(\mathcal{H}),$$

bu yerda $\{\varphi_j\}_{j\in J}$ berilgan $\mathcal{H}$ fazoda ortonormal bazis ([3]).

Bizga $x \in \mathcal{B}(\mathcal{H})$ va x ning absolyut qiymati $|x| = (x^*x)^{\frac{1}{2}}$ uchun hosil qilingan ortogonal proyeksiyalarning $\{e_\lambda(|x|)\}_{\lambda\leq 0}$ spektral oilasi berilgan bo'lsin Agar $t > 0$ bo'lsa, u holda x ning t-umumlashgan xos soni deb quyidagi

$$\mu_t(x) = \inf\{\lambda > 0 : \tau(e_\lambda(|x|)^\perp) \leq t\}$$

songa aytiladi.

Bizga $\mathcal{B}(\mathcal{H})$ da Banax normasiga ega bo'lgan $(X, \|\cdot\|_X)$ qism fazo berilgan bo'lsin. Agar $x \in X$, $y \in \mathcal{B}(\mathcal{H})$ va barcha $t > 0$ lar uchun $\mu_t(y) \le \mu_t(x)$ ekanligidan (mos ravishda, $x \in X$, $y \in \mathcal{B}(\mathcal{H})$, barcha $s > 0$ uchun $\int_0^s \lambda_t(y)dt \le \int_0^s \lambda_t(x)dt$ (yozilishi $y \prec\prec x$)) ekanligidan) $y \in Y$ hamda $\|y\|_X \le \|x\|_X$ ekani kelib chiqsa, u holda $(X, \|\cdot\|_X)$ qism fazoga simmetrik (mos ravishda, to'la simmetrik) fazo deb ataladi.

$(\mathcal{B}(\mathcal{H}), \|\cdot\|)_\infty$ va $(\mathcal{K}(\mathcal{H}), \|\cdot\|)_\infty$ fazolari to'la simmetrik fazolariga misol bo'ladi.

Shuni ta'kidlashimiz lozimki, simmetrik fazo $(X, \|\cdot\|_X) \subset \mathcal{B}(\mathcal{H})$ uchun va barcha $x \in X$, $a, b \in \mathcal{B}(\mathcal{H})$ lar uchun quyidagi shartlar o'rinli

$$\|x\|_X = \||x|\|_X = \|x^*\|_X, \quad axb \in X, \quad va \quad \|axb\|_X \le \|a\|_\infty \|b\|_\infty \|x\|_X.$$

Bizga $(X, \|\cdot\|_X) \subset \mathcal{K}(\mathcal{H})$ simmetrik fazo berilgan bo'lsin. $\mathcal{H}$ da $\{\varphi_j\}_{j\in J}$ ortonormal bazisni fiksirlaymiz va $\{\varphi_{j_n}\}_{n=1}^\infty$ bo'lgan sanoqli qism to'plamni olamiz. p_n berilgan $\mathbb{C} \cdot \varphi_{j_n} \subset \mathcal{H}$ bir o'lchovli qism fazoning ortogonal proyeksiyasi bo'lsin.

Ravshanki,

$$E(X) = \{\xi = \{\xi_n\}_{n=1}^\infty \in c_o : x_\xi = \sum_{n=1}^\infty \xi_n p_n \in X\}$$

$\|\xi\|_{E(X)} = \|x_\xi\|_X$ normaga nisbatan simmetrik ketma-ketlik fazosi bo'ladi.

Boshqa tomondan, har bir $(E, \|\cdot\|) \subset c_o$ simmetrik ketma-ketlik fazosi mos ravishda $(C_E, \|\cdot\|_{C_E}) \subset \mathcal{K}(\mathcal{H})$ simmetrik fazoni quyidagi shartga ko'ra hosil qiladi ([1]):

$$\mathcal{C}_E = \{x \in \mathcal{K}(\mathcal{H}) : \{s_n(x)\} \in E\}, \quad \|x\|_{C_E} = \|\{s_n(x)\}\|_E.$$

Shu bilan birga,

$$E(\mathcal{C}_E) = E, \quad \|\cdot\|_{E(\mathcal{C}_E)} = \|\cdot\|_E, \quad \mathcal{C}_{E(\mathcal{C}_E)} = \mathcal{C}_E, \quad \|\cdot\|_{\mathcal{C}_{E(\mathcal{C}_E)}} = \|\cdot\|_{\mathcal{C}_E}.$$

Ushbu $(\mathcal{C}_E, \|\cdot\|_{\mathcal{C}_E}$ juftlikni biz kompakt operatorlarning Banax ideali deb ataymiz. Bizga ma'lumki, barcha $1 \le p < \infty$ lar uchun $(\mathcal{C}_p, \|\cdot\|_p) = (\mathcal{C}_{\ell^p}, \|\cdot\|_{\mathcal{C}_{\ell^p}})$, hamda $(\mathcal{K}(\mathcal{H}), \|\cdot\|_\infty) = (\mathcal{C}_{c_0}, \|\cdot\|_{\mathcal{C}_{c_0}})$ o'rinli.

$X \subset \mathcal{K}(\mathcal{H})$ da ikki tomonlama kompakt operatorlar ideali bo'lsin. Agar ixtiyoriy $\{T_n\}_{n=1}^\infty \subset \mathcal{B}_h(\mathcal{H})$ fundamental ketma-ketlik uchun $T_n \cdot T_m = T_m \cdot T_n$ $\forall n, m \in N$, shartlardan $\{T_n\}$ ketma-ketlikning $\mathcal{B}(\mathcal{H})$ fazosi normasi bo'yicha yaqinlashishi kelib chiqsa, u holda X idealga kommutativ to'la ideal deyiladi.

Endi biz Banax ideallari abstrak tasnifini beramiz.

Teorema. Agar $(X, \|\cdot\|_X)$ kommutativ ideal bo'lsa, u holda $(X, \|\cdot\|_X)$ Banax ideali bo'ladi.

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Translation-Invariant Symmetric Quantum Markov Chains for a Potts Model

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The purpose of the present paper is to study a quantum analogue of the Potts model on the semi-infinite Cayley tree of order two in the framework of forward quantum Markov chains. To each vertex of the tree we associate the algebra $M_d(\mathbb{C})$, where $d \geq 2$ is the number of spin states. The nearest-neighbour interaction considered here consists of two parts: the classical Potts interaction with coupling constant $J \in \mathbb{R}$ and a quantum exchange interaction governed by a parameter $\lambda \in \mathbb{R}$. Thus, for an edge $\langle x, y \rangle$, the local Hamiltonian is given by

$$H_{\langle x,y \rangle} = J \sum_{i=1}^{d} E_{ii}^{(x)} \otimes E_{ii}^{(y)} + \lambda \sum_{i \neq j} E_{ij}^{(x)} \otimes E_{ji}^{(y)}, \tag{1}$$

where $J, \lambda \in \mathbb{R}$ and $\{E_{ij}\}_{i,j=1}^{d}$ are the standard matrix units in $M_d(\mathbb{C})$.

When $\lambda = 0$, the exchange term disappears and the model reduces to the classical Potts model. Since this case is already related to the known theory of splitting Gibbs measures on Cayley trees, we focus on the genuinely quantum case $\lambda \neq 0$.

We consider translation-invariant symmetric boundary conditions of the form

$$h = a \sum_{i=1}^{d} E_{ii} + u \sum_{i \neq j} E_{ij}, \tag{2}$$

where $a, u \in \mathbb{R}$ and $h > 0$. For this class of boundary conditions, the compatibility condition for the finite-volume states is reduced to a fixed point equation $\Phi(h) = h$, where Φ is the nonlinear map obtained by taking the normalized partial trace over the two direct successors of a given vertex.

We prove that this map preserves the symmetric form of h.Consequently, the compatibility condition becomes a two-dimensional system for the parameters a and u. Introducing the ratio $r = \frac{u}{a}$, we further reduce this system to the one-dimensional fixed point equation $r = f_d(r)$. The main part of the paper is devoted to the explicit analysis of this fixed point equation in the case $d = 2$. The compatibility equation for translation-invariant symmetric boundary conditions was reduced to the one-dimensional fixed point equation

$$r = f_d(r),$$

where

$$f_d(r) = \frac{L_d(M_d + N_d r)r}{P_d + Q_d r^2}, \tag{3}$$

The admissible values of r are determined by the strict positivity condition $-\frac{1}{d-1} < r < 1$.

Due to $\beta > 0$ and $\lambda \neq 0$ we have $\gamma \neq 0$. For $d = 2$, the coefficients are

$$\begin{aligned} P_2 &= (\theta^2 + \alpha^2 + \gamma^2)^2, \\ Q_2 &= 4\theta\alpha\gamma^2, \\ L_2 &= 2\theta\gamma, \\ M_2 &= \theta^2 + 2\theta\alpha + \alpha^2 + \gamma^2, \\ N_2 &= 0. \end{aligned}$$

Since $\theta > 0$, $\alpha > 0$, and $\gamma \neq 0$, we have $Q_2 > 0$. The fixed point equation $f_2(r) = r$ is equivalent to

$$r\left(r^2 - \frac{L_2 M_2 - P_2}{Q_2}\right) = 0, \tag{4}$$

Put $D_2 := L_2M_2 - P_2$. Then non-zero fixed points exist if and only if $D_2 > 0$.

First assume that $\lambda < 0$. Then, since $\beta > 0$, we obtain $\gamma = \sinh(\beta\lambda) < 0$. Since $\theta > 0$, we get $L_2 = 2\theta\gamma < 0$. Therefore, $D_2 = L_2M_2 - P_2 < 0$, because $M_2 > 0$ and $P_2 > 0$. Hence equation (4) has only the trivial solution $r_0 = 0$.

Now assume that $\lambda > 0$ and put $x = \beta\lambda$. Then $x > 0$ implies $\alpha = \cosh(x) > 1$ and $\gamma = \sinh(x) > 0$, whereas $\theta = e^{\beta J}$ remains an independent positive parameter.

Using $\alpha^2 + \gamma^2 = \cosh(2x)$, we obtain

$$D_2(\theta, x) = -\left(\theta^2 - \eta_+(x)\theta + \xi(x)\right)\left(\theta^2 - \eta_-(x)\theta + \xi(x)\right),$$

where $\xi(x) = \cosh(2x)$ and $\eta_\pm(x) = \sinh(x) \pm \sqrt{\sinh^2(x) + 2\sinh(2x)}$.

Since $x > 0$, we have $\eta_-(x) < 0$. Hence $\theta^2 - \eta_-(x)\theta + \xi(x) > 0$ for all $\theta > 0$. Therefore,

$$D_2(\theta, x) > 0 \quad \Longleftrightarrow \quad \theta^2 - \eta_+(x)\theta + \xi(x) < 0.$$

The latter inequality can hold only when $\eta_+^2(x) > 4\xi(x)$. A direct calculation shows that this condition is equivalent to

$$e^{6x} - 5e^{4x} + 3e^{2x} - 7 > 0.$$

Putting $y = e^{2x}$, we obtain $y^3 - 5y^2 + 3y - 7 > 0$.

The cubic equation $y^3 - 5y^2 + 3y - 7 = 0$ has a unique real root. Indeed, let

$$p(y) = y^3 - 5y^2 + 3y - 7.$$

Then $p'(y) = 3y^2 - 10y + 3$, and hence the only critical points are $y = \frac{1}{3}$ and $y = 3$. Moreover,

$$p\left(\frac{1}{3}\right) = -\frac{176}{27} < 0, \qquad p(3) = -16 < 0.$$

Thus both the local maximum and the local minimum of p are negative. Since $p(y) \to -\infty$ as $y \to -\infty$, $\quad p(y) \to +\infty$ as $y \to +\infty$, the polynomial p has exactly one real root y_*. Numerically, $y_* \approx 4.6786$. Moreover,

$$y^3 - 5y^2 + 3y - 7 > 0 \quad \Longleftrightarrow \quad y > y_*.$$

Hence $x_* = \frac{\ln y_*}{2} \approx 0.7715$.

Thus, if $0 < \beta\lambda \le x_*$, then no non-zero fixed points exist. If $\beta\lambda > x_*$, then non-zero fixed points exist precisely when $\theta_-(x) < e^{\beta J} < \theta_+(x)$, where

$$\theta_\pm(x) = \frac{\eta_+(x) \pm \sqrt{\eta_+^2(x) - 4\xi(x)}}{2}.$$

In this case, the two non-zero fixed points are

$$r_\pm = \pm\sqrt{\frac{D_2(e^{\beta J}, \beta\lambda)}{4e^{\beta J}\cosh(\beta\lambda)\sinh^2(\beta\lambda)}}. \tag{5}$$

Proposition. *Assume that $\lambda \neq 0$, and let f_2 be the function defined by (3). Then*

$$\mathrm{Fix}(f_2) \cap (-1, 1) = \begin{cases} \{0, r_-, r_+\}, & \text{if } \beta\lambda > x_* \text{ and } \ln\left(\theta_-(\beta\lambda)\right) < \beta J < \ln\left(\theta_+(\beta\lambda)\right), \\ \{0\}, & \text{otherwise.} \end{cases}$$

For each admissible fixed point $r \in \left(-\frac{1}{d-1}, 1\right)$ of f_d, put

$$a_r = \frac{d^2}{P_d + Q_d r^2}, \qquad u_r = a_r r,$$

and define

$$h_r = a_r \sum_{i=1}^{d} E_{ii} + u_r \sum_{i \neq j} E_{ij}.$$

Then h_r is strictly positive and satisfies $\Phi(h_r) = h_r$. Hence it generates a translation-invariant symmetric forward quantum Markov chain, which will be denoted by φ_r.

We denote by $\mathcal{QMC}_d$ the set of all translation-invariant symmetric forward quantum Markov chains corresponding to the d-state model within the considered class of boundary conditions.

Theorem. *Let $d = 2$ and $\lambda \neq 0$. Then, within the class of translation-invariant symmetric boundary conditions, the following assertions hold.*

(*i*) *If $\lambda < 0$, then $\mathcal{QMC}_2 = \{\varphi_0\}$. Here φ_0 is generated by the trivial boundary condition corresponding to $r_0 = 0$.*

(*ii*) *Assume that $\lambda > 0$. Let $y_* \approx 4.6786$ be the unique real root of*

$$y^3 - 5y^2 + 3y - 7 = 0.$$

For $\beta\lambda > \frac{1}{2}\ln y_*$, put

$$\eta_+(\beta\lambda) = \sinh(\beta\lambda) + \sqrt{\sinh^2(\beta\lambda) + 2\sinh(2\beta\lambda)},$$

$$\theta_\pm(\beta\lambda) = \frac{\eta_+(\beta\lambda) \pm \sqrt{\eta_+^2(\beta\lambda) - 4\cosh(2\beta\lambda)}}{2}.$$

Then

$$\mathcal{QMC}_2 = \begin{cases} \{\varphi_0, \varphi_{r_-}, \varphi_{r_+}\}, & \text{if } \beta\lambda > \frac{1}{2}\ln y_*,\ \ln\theta_-(\beta\lambda) < \beta J < \ln\theta_+(\beta\lambda), \\ \{\varphi_0\}, & \text{otherwise.} \end{cases}$$

Here $r_\pm$ are given by (5).

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σ-matching and interchangeable structures on Quaternion algebras

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Algebraic structures equipped with multiple compatible operations, such as σ-matching and interchangeable products, play an important role in modern algebra. In this work, we study σ-matching, interchangeable, and totally compatible structures on the classical quaternion algebra $\mathbb{H}$. We show that interchangeable and (12)-matching products on $\mathbb{H}$ are naturally described in terms of mutations of the standard quaternion multiplication.

Definition 1.[3] Two associative products $\cdot$ and $\star$ on V are said to be

1. *id-matching*, if $(a \cdot b) \star c = a \cdot (b \star c)$ and $(a \star b) \cdot c = a \star (b \cdot c)$;
2. (12)*-matching*, if $(a \cdot b) \star c = a \star (b \cdot c)$ and $(a \star b) \cdot c = a \cdot (b \star c)$;
3. *totally compatible*, if $(a \cdot b) \star c = (a \star b) \cdot c = a \cdot (b \star c) = a \star (b \cdot c)$,

for all $a, b, c \in V$.

We say that two (not necessarily associative) products $\cdot$ and $\star$ on V are *interchangeable* if $(a \cdot b) \star c = (a \star b) \cdot c$ and $a \cdot (b \star c) = a \star (b \cdot c)$.

Note that any totally compatible products are interchangeable and σ-matching for every $\sigma \in S_2$. However, the converse does not hold in general.

For any associative algebra $(A, \cdot)$ and fixed $x \in A$, one defines the *mutation of* $\cdot$ *by* x (see [1]) to be the product $\cdot_x$ on A given by $a \cdot_x b = a \cdot x \cdot b$ for all $a, b \in A$.

In [2] it is proved that for any associative algebra $(A, \cdot)$ and for any $x \in A$ the product $\cdot_x$ is associative and id-matching with $\cdot$. Moreover, if $(A, \cdot)$ is a unital associative algebra, then the *id*-matching structures on A are exactly the mutations of $\cdot$.

Definition 2. The quaternion algebra, denoted by $(\mathbb{H}, \cdot)$, is a 4-dimensional associative division algebra over $\mathbb{R}$ spanned by the basis $\{1, i, j, k\}$. The multiplication on $\mathbb{H}$ is uniquely determined by the Hamilton's relations: $i^2 = j^2 = k^2 = ijk = -1$.

Proposition 1. Let $\cdot$ and $\star$ be associative products on the algebra $(\mathbb{H}, \cdot)$ that are interchangeable structures, then $\star$ multiplication is a mutation of $\cdot$, moreover $\cdot$ and $\star$ are *id*-matching.

Proposition 2. Let $\cdot$ and $\star$ be associative products on the algebra $(\mathbb{H}, \cdot)$, that are $(12)-$matching. Then the product $\star$ has the following form of scalar multiplication, where $\alpha \in \mathbb{R}$:

$$a \star b = \alpha(a \cdot b).$$

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UDK 511.356

BUTUN VA RATSIANAL KOEFFITSIYENTLI KO‘PHADLARNING BO‘LINISH ALOMATLARI HAQIDA

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Kommutativ algebraning markaziy ob’ektlaridan biri bo‘lgan ko‘phadlar halqasida elementlarning arifmetik xossalari, qoldiqli bo‘lish qonuniyatlari hamda ko‘phadlarning umumiy bo‘luvchilarini topish masalalari muhim nazariy va amaliy ahamiyatga egadir [1, 2]. Ixtiyoriy K sonlar maydoni (xususan, ratsional sonlar maydoni $\mathbb{Q}$ yoki haqiqiy sonlar maydoni $\mathbb{R}$) ustida bir o‘zgaruvchili ko‘phadlar halqasi $K[x]$ yevklid halqasi hisoblanadi. Shuning uchun ixtiyoriy $f(x) \in K[x]$ va nolga teng bo‘lmagan $g(x) \in K[x]$ ko‘phadlar juftligi uchun

$$f(x) = g(x)q(x) + r(x) \tag{1}$$

tenglikni qanoatlantiruvchi yagona $q(x)$ (to‘liqsiz bo‘linma) va $r(x)$ (qoldiq) ko‘phadlar mavjud bo‘lib, bunda yo $\deg r(x) < \deg g(x)$, yoki $r(x) = 0$ bo‘ladi.

Agar (1) yoyilmada $r(x) = 0$ bo‘lsa, $f(x)$ ko‘phad $g(x)$ ga qoldiqsiz bo‘linadi deyiladi va $g(x) \mid f(x)$ kabi belgilanadi. Xususan, $g(x) = x - c$ chiziqli ikkihadga bo‘lish masalasi Bezu teoremasiga asoslanadi: $f(x)$ ni $x-c$ ga bo‘lgandagi qoldiq $r = f(c)$ son qiymatiga teng. Bundan $x - c \mid f(x)$ bo‘lishi uchun c soni $f(x)$ ko‘phadning ildizi, ya’ni $f(c) = 0$ bo‘lishi zarur va yetarlidir [1]. Agar c soni $f(x)$ ning k-karrali ildizi bo‘lsa, $f(x)$ ko‘phad $(x - c)^k$ ga bo‘linadi, ammo $(x - c)^{k+1}$ ga bo‘linmaydi.

Ushbu ishda ko‘p uchraydigan $f_n(x) = x^n - a^n$ va $F_n(x) = x^n + a^n$ $(n \in \mathbb{N})$ ko‘rinishidagi ikkihadlarning $x - a$ va $x + a$ chiziqli ko‘phadlarga, shuningdek, yuqoriroq darajali $x^m \pm a^m$ ikkihadlarga qoldiqsiz bo‘linish qonuniyatlari hamda ularning eng katta umumiy bo‘luvchilarini (EKUB) analitik topish algoritmlari tadqiq etiladi.

1-lemma. *Ixtiyoriy $n \in \mathbb{N}$ natural soni va $a \in K$ elementi uchun quyidagi bo‘linish munosabatlari o‘rinli bo‘ladi:*

1) *Har qanday natural n uchun $x - a \mid x^n - a^n$;*

2) *Juft $n = 2k$ sonlar uchun $x + a \mid x^n - a^n$;*

3) *Toq $n = 2k + 1$ sonlar uchun $x + a \mid x^n + a^n$;*

4) *Juft $n = 2k$ sonlar uchun $x^n + a^n$ ko‘phad $x + a$ ga qoldiqsiz bo‘linmaydi (bu holda qoldiq $r = 2a^n \neq 0$ ga teng).*

Isbot. 1-bandning isboti bevosita Bezu teoremasidan kelib chiqadi: $f(x) = x^n - a^n$ desak, $f(a) = a^n - a^n = 0$. Bundan tashqari, elementar algebradagi

$$x^n - a^n = (x - a)(x^{n-1} + x^{n-2}a + \cdots + xa^{n-2} + a^{n-1})$$

ayniyati har qanday n da qoldiqsiz bo‘linishni ko‘rsatadi. 2-bandda $n = 2k$ juft bo‘lgani uchun $f(-a) = (-a)^{2k} - a^{2k} = a^{2k} - a^{2k} = 0$, demak $x + a \mid x^n - a^n$. 3-bandda $n = 2k + 1$ toq bo‘lgani bois $F_n(-a) = (-a)^{2k+1} + a^{2k+1} = -a^{2k+1} + a^{2k+1} = 0$, ya’ni $x + a \mid x^n + a^n$. Nihoyat, 4-bandda $n = 2k$ bo‘lganda $F_n(-a) = (-a)^{2k} + a^{2k} = 2a^{2k} = 2a^n \neq 0$ bo‘lib, $x^n + a^n$ ko‘phad $x + a$ ga bo‘linmaydi. Lemma to‘liq isbotlandi.

Ushbu xossalar umumiy $x^n - 1$ va $x^m - 1$ ko‘phadlarning umumiy ko‘paytuvchilarini aniqlashda fundamental tayanch hisoblanadi.

Teorema. *Ixtiyoriy natural n va m sonlari uchun $x^m - 1 \mid x^n - 1$ bo‘lishi uchun $m \mid n$ bo‘lishi zarur va yetarlidir. Umumiy holda quyidagi algebraik tenglik o‘rinlidir:*

$$\gcd(x^n - 1,\, x^m - 1) = x^{\gcd(n,m)} - 1. \tag{2}$$

Isbot. Dastlab teoremadagi bo‘linish shartining zarur va yetarliligini ko‘rsatamiz. Agar $m \mid n$ bo‘lsa, $n = mq$ deb yozish mumkin. U holda $y = x^m$ deb belgilasak, 1-lemmaga binoan $y - 1 \mid y^q - 1$ munosabat o‘rinlidir. Bu esa $x^m - 1 \mid (x^m)^q - 1 = x^n - 1$ ekanligini anglatadi, ya’ni shart yetarlidir.

Endi umumiy holni, ya’ni (2) formulani isbotlaymiz. n va m natural sonlariga Yevklid algoritmini tatbiq qilamiz. n ni m ga qoldiqli bo‘lamiz:

$$n = mq + r, \quad 0 \le r < m.$$

U holda quyidagi ayniyat o‘rinli bo‘ladi:

$$x^n - 1 = x^{mq+r} - 1 = x^r(x^{mq} - 1) + (x^r - 1). \tag{3}$$

Yuqorida ko‘rsatilganidek, $x^m - 1 \mid x^{mq} - 1$ munosabat doimo bajariladi. Demak, $x^{mq} - 1 = (x^m - 1)Q(x)$ shaklida yozilishi mumkin. Buni (3) tenglikka qo‘ysak:

$$x^n - 1 = (x^m - 1) \cdot [x^r Q(x)] + (x^r - 1).$$

Bu yerda $\deg(x^r - 1) = r < m = \deg(x^m - 1)$ bo‘lganligi uchun $x^n - 1$ ko‘phadni $x^m - 1$ ga bo‘lgandagi to‘liq qoldiq aynan $x^r - 1$ ga teng bo‘ladi.

Ko‘phadlar halqasida Yevklid algoritmining xossasiga ko‘ra, ixtiyoriy $f(x) = g(x)q(x) + r(x)$ munosabat uchun $\gcd(f(x), g(x)) = \gcd(g(x), r(x))$ tenglik o‘rinlidir. Shunday ekan,

$$\gcd(x^n - 1,\, x^m - 1) = \gcd(x^m - 1,\, x^r - 1).$$

Ko‘rinib turibdiki, ko‘phadlar darajalari ustida borayotgan ushbu jarayon butun sonlar ustidagi Yevklid algoritmining qadamlarini aynan takrorlaydi:

$$n = mq_1 + r_1 \implies \gcd(x^n - 1, x^m - 1) = \gcd(x^m - 1, x^{r_1} - 1),$$

$$m = r_1 q_2 + r_2 \implies \gcd(x^m - 1, x^{r_1} - 1) = \gcd(x^{r_1} - 1, x^{r_2} - 1), \ldots$$

Algoritmning oxirgi noldan farqli qoldig‘i $d = \gcd(n, m)$ bo‘lganligi sababli, oxirgi qadamda qoldiq ko‘phad $x^d - 1$ hosil bo‘ladi va u qoldiqsiz bo‘linadi. Bundan $\gcd(x^n - 1, x^m - 1) = x^{\gcd(n,m)} - 1$ ekanligi to‘la isbotlanadi.

Xususan, agar $\gcd(n, m) = 1$ bo‘lsa, $x^n - 1$ va $x^m - 1$ ko‘phadlar o‘zaro tub bo‘lib, ularning umumiy bo‘luvchisi faqatgina $x - 1$ chiziqli ko‘phaddan iborat bo‘ladi.

Mazkur natija algebraik sonlar nazariyasida doiraviy bo‘linish ko‘phadlari $\Phi_n(x)$ ning xossalarini o‘rganishda, chekli maydonlar $\mathbb{F}_q$ ustida keltirilmaydigan ko‘phadlarni klassifikatsiya qilishda hamda hisoblash algebrasi algoritmlarida muhim ahamiyat kasb etadi [3, 4].

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FRIDRIXS MODELLARI OILASIGA MOS FREDGOLM DETERMINANTINING ASIMPTOTIK YOYILMASI: BIR O‘LCHAMLI HOL

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Bugungi kunda zamonaviy matematik fizikaning dolzarb yo‘nalishlaridan biri uch zarrachali kvant sistemalarining matematik modellarini o‘rganishdir. Bunday sistemalar uchun panjaradagi model operatorlarining spektral xususiyatlarini o‘rganish, nafaqat nazariy, balki amaliy jihatdan ham alohida ahamiyat kasb etadi. Shu sababli, Fridrixs modellari oilasining muhim va diskret spektrlarini, shuningdek, xos qiymatlarning soni, joylashuvi va mavjudlik shartlarini o‘rganish dolzarb hisoblanadi [1,2,3]. Ushbu ishda ham bir o‘lchamli panjaradagi uchta zarrachalar sistemasiga mos model operatorlarning spektral xossalarini o‘rganishda muhim bo‘lgan chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma Fridrixs modellari oilasi qaralgan.

$\mathbb{T}-$ bir o‘lchamli tor, $\mathbb{T}$ to‘plamda aniqlangan kvadrati bilan integrallanuvchi funksiyalardan tashkil topgan $L_2(\mathbb{T})$ Hilbert fazosida chiziqli, chegaralangan va o‘z-o‘ziga qo‘shma bo‘lgan quyidagi Fridrixs modellari oilasini qaraymiz:

$$h_\mu^{(\gamma)}(k) := h_0^{(\gamma)}(k) - \mu v, k \in \mathbb{T},$$

bunda $\mu, \gamma > 0$ fiksirlangan haqiqiy sonlar, $h_0^{(\gamma)}(k)$ operator $E_\gamma(k, \cdot)$ funksiyaga ko‘paytirish operatori:

$$\left(h_0^{(\gamma)}(k) f\right)(x) = E_\gamma(k, x) f(x), \qquad f \in L_2(\mathbb{T}),$$

$$E_\gamma(k, x) := \varepsilon(k) + \varepsilon(x) + \gamma\varepsilon(k + x), \qquad \varepsilon(x) := 1 - \cos x.$$

v operator esa lokal bo‘lmagan potensial operator:

$$(vf)(x) = \sin(x) \int_{\mathbb{T}} \sin(t) f(t)\, dt.$$

Chekli o‘lchamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi Veyl teoremasiga ko‘ra $h_\mu^{(\gamma)}(k)$ operatorning muhim spektri $h_0^{(\gamma)}(k)$ operatorning muhim spektri bilan ustma-ust tushadi va quyidagi

$$\sigma_{\text{ess}}\left(h_\mu^{(\gamma)}(k)\right) = [m_\gamma(k); M_\gamma(k)]$$

tenglik o‘rinlidir, bunda

$$m_\gamma(k) := \min_{x \in \mathbb{T}} E_\gamma(k, x), \qquad M_\gamma(k) := \max_{x \in \mathbb{T}} E_\gamma(k, x).$$

Har bir fiksirlangan $\mu, \gamma > 0$ sonlari va $k \in \mathbb{T}$ element uchun $\mathbb{C} \setminus [m_\gamma(k); M_\gamma(k)]$ sohada analitik bo‘lgan

$$\Delta_\mu^{(\gamma)}(k; z) := 1 - \mu \int_{\mathbb{T}} \frac{\sin^2(t)\, dt}{E_\gamma(k, t) - z}$$

yordamchi funksiyani kiritamiz.

Xususiyatiga ko‘ra $\Delta_\mu^{(\gamma)}(k; \cdot)$ funksiyaga $h_\mu^{(\gamma)}(k)$ operatorga mos Fredgolm determinanti deyiladi.

1-lemma. Har bir fiksirlangan $k \in \mathbb{T}$ element va $\mu, \gamma > 0$ sonlari uchun $z \in \mathbb{C} \setminus [m_\gamma(k); M_\gamma(k)]$ soni $h_\mu^{(\gamma)}(k)$ operatorning xos qiymati bo‘lishi uchun $\Delta_\mu^{(\gamma)}(k; z) = 0$ bo‘lishi zarur va yetarlidir.

Bayon qilingan 1-lemmadan quyidagi natija bevosita kelib chiqadi.

1-natija. $h_\mu^{(\gamma)}(k)$ operatorning diskret spektri uchun

$$\sigma_{\text{disc}}\left(h_\mu^{(\gamma)}(k)\right) = \left\{ z \in \mathbb{C} \setminus [m_\gamma(k); M_\gamma(k)] : \Delta_\mu^{(\gamma)}(k; z) = 0 \right\}$$

tenglik o‘rinlidir.

$h_\mu^{(\gamma)}(k)$ Fridrixs modellari oilasining xos qiymatlarini tahlil qilishda muhim bo‘lgan lemmalarni keltiramiz.

2-lemma. Har bir fiksirlangan $k \in \mathbb{T}$ element va $\mu, \gamma > 0$ sonlar uchun $\Delta_\mu^{(\gamma)}(k; \cdot)$ funksiya $(-\infty; m_\gamma(k))$ va $(M_\gamma(k); +\infty)$ oraliqlarda monoton kamayuvchi funksiyadir.

3-lemma. Har qanday o‘zgarmas $\mu, \gamma > 0$ va $k \in \mathbb{T}$ sonlari uchun $h_\mu^{(\gamma)}(k)$ Fridrixs modeli $m_\gamma(k)$ nuqtadan chapda ko‘pi bilan bitta oddiy xos qiymatga ega va $M_\gamma(k)$ nuqtadan o‘ngda xos qiymatga ega emas.

Har qanday $\gamma > 0$ soni uchun $E_\gamma(\cdot, \cdot)$ funksiya $(0, 0) \in \mathbb{T}^2$ nuqtada qiymati nolga teng yagona global minimumga erishadi. Shuning uchun quyidagi yoyilma o‘rinli bo‘ladi:

$$E_\gamma(x, y) = \frac{1}{2}\left(\frac{\partial^2 E_\gamma(0,0)}{\partial x^2} x^2 + 2\frac{\partial^2 E_\gamma(0,0)}{\partial x \partial y} xy + \frac{\partial^2 E_\gamma(0,0)}{\partial y^2} y^2 \right) + o(x^2) + o(y^2),$$

ya’ni $x, y \to 0$ bo‘lganda yuqoridagi tenglik bajariladi.

Shu sababli, shunday $C_1(\gamma), C_2(\gamma) > 0$ o‘zgarmas sonlar va $\delta_\gamma > 0$ son mavjud bo‘lib, quyidagilar munosabatlar bajariladi:

$$C_1(\gamma)(x^2 + y^2) \le E_\gamma(x, y) \le C_2(\gamma)(x^2 + y^2), \qquad (x, y) \in (-\delta_\gamma; \delta_\gamma) \times (-\delta_\gamma; \delta_\gamma). \tag{4}$$

$$E_\gamma(x, y) \ge C_1(\gamma)(x, y) \in (-\delta_\gamma; \delta_\gamma) \times (-\delta_\gamma; \delta_\gamma). \tag{5}$$

(4), (5) tengsizliklar va $x \to 0$ bo‘lganda $\sin x \sim x$ asimptotik munosabatga tayangan holda, quyidagi integral

$$\int_{\mathbb{T}} \frac{\sin^2(t)\, dt}{E_\gamma(k, t)}$$

har qanday $\gamma > 0$ va $k \in \mathbb{T}$ sonlar uchun musbat va chekli bo‘lishini hosil qilamiz.

Shuning uchun integral belgisi ostida limitga o‘tish haqidagi Lebeg teoremasiga ko‘ra

$$\Delta_\mu^{(\gamma)}(0; 0) = \lim_{k \to 0} \Delta_\mu^{(\gamma)}(k; 0)$$

tenglik bajariladi, ya’ni $\Delta_\mu^{(\gamma)}(\cdot; 0)$ funksiyasi $\mathbb{T}$ torda uzluksiz.

Quyidagicha belgilash kiritamiz:

$$\mu_0^\gamma = (1+\gamma)\left(\int_{\mathbb{T}} \frac{\sin^2(t)\,dt}{\varepsilon(t)}\right)^{-1}.$$

Aniqlanishicha, faqat $\mu = \mu_\gamma^0$ munosabat bajarilganda

$$\Delta_\mu^{(\gamma)}(0;0) = 0$$

tenglik o‘rinli bo‘ladi.

Endi Fridrixs modeli oilasining $k = 0$ bo‘lgandagi xos qiymatlari haqidagi teoremani keltiramiz.

1-teorema. Agar $0 < \mu \le \mu_\gamma^0$ bo‘lsa, u holda $h_\mu^{(\gamma)}(0)$ operator xos qiymatga ega emas, ixtiyoriy $\mu > \mu_\gamma^0$ soni uchun esa $h_\mu^{(\gamma)}(0)$ operator yagona manfiy xos qiymatga ega.

Endi ishning asosiy natijasini bayon qilamiz.

2-teorema. Faraz qilaylik, $\mu, \gamma > 0$ o‘zgarmas sonlar bo‘lsin. $x \to 0$ va $z \to 0$ bo‘lganda quyidagi yoyilma

$$\Delta_\mu^{(\gamma)}(k;z) = \Delta_\mu^{(\gamma)}(0;0) + \frac{2\mu\pi(1+2\gamma-\gamma^2)}{(1+\gamma)^2\sqrt{1+2\gamma}}\sqrt{k^2 - \frac{2(1+\gamma)}{1+2\gamma}z} + O(k^2) + O(|z|)$$

bajariladi.

Bayon qilingan 2-teoremadan quyidagi muhim natija kelib chiqadi.

2-natija. Shunday $C_1(\gamma), C_2(\gamma) > 0$ va $\delta_\gamma > 0$ sonlari topilib,

$$C_1(\gamma)|k| \le \left|\Delta_\mu^{(\gamma)}(k;0)\right| \le C_2(\gamma)|k|, \qquad k \in U_{\delta_\gamma}(0)$$

qo‘sh tengsizlik o‘rinli bo‘ladi.

Mazkur ishda keltirilgan natijalar bir o‘lchamli panjaradagi uchta zarrachalar sistemasiga mos model Hamiltonianning xos qiymatlari sonini o‘rganishda alohida ahamiyat kasb etadi.

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Brownian motion and boundary regularity for the Dirichlet problem

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Abstract. We relate the sample-path properties of Brownian motion to harmonic extension and boundary regularity. Using the cone-avoidance estimate in [1], we state a boundary modulus for the probabilistic solution with locally Hölder boundary data. A punctured ball illustrates why continuity of paths and recurrence do not by themselves ensure attainment of prescribed boundary values. The underlying probabilistic and capacity criteria are classical.

Keywords: Brownian motion, Dirichlet problem, boundary regularity, Hölder continuity, capacity.

1. From random paths to harmonic functions. Let $\{B(t) : t \geq 0\}$ be Brownian motion in $\mathbb{R}^d$, $d \geq 2$, started at x, and let $\mathbb{P}_x$ and $\mathbb{E}_x$ denote its law and expectation. Its increments are independent and Gaussian, with $B(t+h) - B(t) \sim N(0, hI_d)$. Almost surely, its paths are continuous, locally α-Hölder for every $0 < \alpha < 1/2$, and nowhere differentiable. Moreover, $a^{-1}(B(a^2t) - x)$ is standard Brownian motion for $a > 0$ [1, Chapter 1]. Here $\mathcal{B}(z, r)$ denotes the open Euclidean ball.

For a bounded domain $U \subset \mathbb{R}^d$, define the first exit time $\tau_U = \inf\{t > 0 : B(t) \notin U\}$. This time is finite almost surely for $x \in U$, and continuity gives $B(\tau_U) \in \partial U$. For $\varphi \in C(\partial U)$, set

$$u(x) = \mathbb{E}_x[\varphi(B(\tau_U))] = \int_{\partial U} \varphi(y)\, \omega_U(x, dy), \qquad x \in U, \tag{1}$$

where $\omega_U(x, \cdot)$ is the exit distribution, or harmonic measure. By [1, Theorem 3.8], u is harmonic in U. Indeed, the strong Markov property at the exit time from a ball compactly contained in U, together with rotational invariance, gives the mean-value property.

A point $z \in \partial U$ is *regular for* U^c when $\mathbb{P}_z\{\tau_U = 0\} = 1$, with the infimum taken over strictly positive times. Theorem 8.5 of [1] states that (1) attains the boundary value $\varphi(z)$ at every such point. If all boundary points are regular, it extends continuously to $\overline{U}$ and is the unique solution of $\Delta u = 0$ in U, $u = \varphi$ on ∂U.

The same stopped process describes the Green operator. If $p_U(t, x, y)$ is the transition subdensity of Brownian motion killed upon leaving U, then

$$G_U(x, y) = \int_0^\infty p_U(t, x, y)\, dt, \qquad \mathbb{E}_x \int_0^{\tau_U} g(B(t))\, dt = \int_U G_U(x, y) g(y)\, dy, \tag{2}$$

for nonnegative measurable g [1, Theorem 3.32]. Thus G_U is the density of the expected occupation measure. When g is bounded and Hölder continuous and every boundary point is regular, the function defined by (2) solves $-\frac{1}{2}\Delta v = g$ in U with $v = 0$ on ∂U [1, Remark 8.7]. The factor $1/2$ corresponds to the variance normalization $B(t+h) - B(t) \sim N(0, hI_d)$.

2. A quantitative consequence of the cone estimate. Suppose that U satisfies the exterior Poincaré cone condition at $z \in \partial U$: an open cone V with vertex z and positive aperture satisfies $V \cap \mathcal{B}(z, r_0) \subset U^c$ for some $r_0 > 0$. Brownian scaling and successive applications of the strong Markov property yield a number $q \in (0, 1)$ such that avoiding this cone across k dyadic scales has probability at most q^k [1, Lemma 3.11]. Write $\gamma = -\log_2 q > 0$.

Theorem 1 (a consequence of [1, Lemma 3.11]). *Assume that $\varphi \in C(\partial U)$ and, for some $L \geq 0$ and $0 < \beta \leq 1$,*

$$|\varphi(y) - \varphi(z)| \leq L|y - z|^{\beta}, \qquad y \in \partial U, \quad |y - z| \leq r_0.$$

Then the function in (1) satisfies, for $x \in U$ with $0 < |x - z| < r_0/2$,

$$|u(x) - \varphi(z)| \leq C\,\Psi_{\beta,\gamma}(|x - z|/r_0), \qquad \Psi_{\beta,\gamma}(h) = \begin{cases} h^{\beta}, & \beta < \gamma, \\ h^{\beta}\log(e/h), & \beta = \gamma, \\ h^{\gamma}, & \beta > \gamma, \end{cases} \tag{3}$$

where C is independent of x. The exponent γ depends only on the cone and the dimension and is not asserted to be optimal.

Thus the modulus of the harmonic extension is controlled jointly by the boundary data and the probability of avoiding the exterior cone. The exponent γ concerns spatial accessibility of the boundary, not the time Hölder exponent $1/2$ of Brownian paths.

In particular, when $\beta < \gamma$, the data exponent is preserved at z. At $\beta = \gamma$ the estimate contains a logarithmic factor, whereas for $\beta > \gamma$ it guarantees exponent γ. These are sufficient bounds, not an exact modulus for every domain. Each case in (3) tends to zero as $x \to z$ within U, giving a quantitative statement of boundary continuity at a cone point.

3. Recurrence, capacity, and a polar puncture. Brownian motion is neighbourhood recurrent in $d = 2$ and transient in $d \geq 3$ [1, Theorem 3.20]. Correspondingly, the unrestricted Green kernel $G(x, y) = \int_0^{\infty} p(t, x, y)\,dt$ is finite for $x \neq y$ in $d \geq 3$, whereas in the plane the occupation-potential construction requires killing, for example at τ_U [1, Section 3.3]. Global recurrence, however, does not imply that every boundary point is regular. Further treatments of Brownian motion and classical potential theory appear in [2,3].

Potential theory makes this distinction precise. If $K \subset \mathbb{R}^d$ is compact and nonpolar, $d \geq 3$, and $T_K = \inf\{t > 0 : B(t) \in K\}$, its equilibrium measure ν_K satisfies

$$\mathbb{P}_x\{T_K < \infty\} = \int_K G(x, y)\,d\nu_K(y), \qquad x \notin K,$$

by the last-exit formula [1, Theorem 8.8]. Thus the probability of hitting a set is represented as a Green potential. A capacity criterion then measures whether the complement is sufficiently large near a boundary point.

For $d \geq 3$, use the Newtonian capacity

$$C_{d-2}(K) = \left(\inf_{\mu \in \mathcal{P}(K)} \iint |\xi - \eta|^{2-d}\,d\mu(\xi)\,d\mu(\eta)\right)^{-1},$$

where $\mathcal{P}(K)$ denotes the probability measures on K and $C_{d-2}(\varnothing) = 0$. Put $K_k = U^c \cap \{y : 2^{-k-1} \leq |y - z| \leq 2^{-k}\}$. Wiener's test [1, Theorem 8.37] gives the exact local criterion

$$z \text{ is regular for } U^c \quad \Longleftrightarrow \quad \sum_{k=1}^{\infty} 2^{k(d-2)} C_{d-2}(K_k) = \infty. \tag{4}$$

This form is stated only for $d \geq 3$; the planar potential kernel is logarithmic [1, Section 8.3].

For a concrete contrast with Theorem 1, let $U = \mathcal{B}(0, 1) \setminus \{0\}$, $d \geq 2$, and prescribe $\varphi = 0$ on $\partial\mathcal{B}(0, 1)$, $\varphi(0) = 1$. These data are continuous on ∂U. Singletons are polar in every $d \geq 2$ [1, Section 8.3], so a path started at $x \in U$ almost surely exits through the outer sphere.

Hence (1) gives $u(x) = 0$ throughout U, and the value at the puncture is not attained. By [1, Theorem 8.5(a)], there is no solution continuous on $\overline{U}$ for these data. This is the punctured-domain obstruction of [1, Example 3.15]. For $d \geq 3$ it is also immediate from (4): all the sets K_k at $z = 0$ are empty. In $d = 2$, neighbourhood recurrence still does not force a Brownian path to hit the specified point.

Conclusion. Brownian motion connects random continuous paths with harmonic functions, but boundary continuity depends on the geometry and capacity of the complement. The cone estimate supplies the explicit modulus (3), while a polar puncture shows why a local regularity hypothesis cannot be omitted.

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IKKINCHI TUR FRIDGOLM INTEGRAL TENGLAMALARINI YECHISHNING TAQRIBIY USULLARI VA ULARNING TADBIQLARI

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Matematik fizika, mexanika va amaliy matematikaning ko'plab masalalari, xususan, differensial tenglamalar uchun chegaraviy masalalar integral tenglamalarga keltiriladi. Ushbu tezisda ikkinchi tur Fridgolm integral tenglamalarini yechishning asosiy analitik va taqribiy usullari hamda ularning tatbiqlari o'rganiladi.Ikkinchi tur Fridgolm integral tenglamasi umumiy holda quyidagicha yoziladi:

$$\varphi(x) - \lambda \int_a^b K(x,s)\varphi(s)ds = f(x) \tag{1}$$

Bu yerda $f(x)$ — $[a,b]$ kesmada berilgan uzluksiz funksiya, $K(x,s)$ — $a \leq x, s \leq b$ kvadratda aniqlangan uzluksiz funksiya (integral tenglamaning yadrosi), λ — parametr, $\varphi(x)$ — topilishi kerak bo'lgan noma'lum funksiya.Tenglama yechimining mavjudligi va yagonaligi Fridgolmning uchta asosiy teoremasi bilan to'liq tavsiflanadi.

Xususan, agar λ parametr yadroning xos qiymati bo'lmasa, u holda bir jinsli bo'lmagan tenglama ixtiyoriy $f(x)$ uzluksiz funksiya uchun yagona yechimga ega bo'ladi. Agar λ xos qiymat bo'lsa, yechim mavjud bo'lishi uchun ozod had qo'shma bir jinsli tenglamaning barcha chiziqli erkli yechimlariga ortogonal bo'lishi zarur va yetarlidir.Agar $K(x,s)$ yadro chegaralangan ($|K(x,s)| \leq M$) bo'lsa, u holda parametr λ quyidagi shartni qanoatlantirganda:

$$|\lambda| < \frac{1}{M(b-a)} \tag{2}$$

integral tenglama yagona uzluksiz yechimga ega bo'ladi. Yechim ketma-ket yaqinlashish iterativ jarayoni orqali topiladi:

$$\varphi_0(x) = f(x)$$

$$\varphi_n(x) = f(x) + \lambda \int_a^b K(x,s)\varphi_{n-1}(s)ds, \quad n = 1, 2, \dots$$

Bu ketma-ketlik $\varphi(x)$ yechimga tekis yaqinlashadi va yechim cheksiz Neyman qatori ko'rinishida ifodalanadi: $\varphi(x) = \sum_{n=0}^{\infty} \lambda^n \varphi_n(x)$.

Aynigan yadro usulida $K(x,s) = \sum_{i=1}^{n} a_i(x)b_i(s)$ ko'rinishdagi yadro uchun masala chiziqli algebraik tenglamalar sistemasiga keltiriladi.

Noma'lum $C_i = \int_a^b b_i(s)\varphi(s)ds$ koeffitsiyentlarni topish uchun ifoda integrallanib, n ta noma'lumli sistemaga ega bo'lamiz:

$$C_i - \lambda \sum_{k=1}^{n} \alpha_{ik} C_k = \beta_i, \quad i = 1, \dots, n \tag{3}$$

Bu yerda $\alpha_{ik} = \int_a^b b_i(s)a_k(s)ds$ va $\beta_i = \int_a^b b_i(s)f(s)ds$. Ixtiyoriy uzluksiz yadroni taqribiy ravishda aynigan yadro bilan almashtirish algoritmi uzluksiz yadroli Fridgolm tenglamasini yechishning samarali yo'lidir.

Xulosa qilib aytganda, Fridgolm integral tenglamalarini yechishning tahlil qilingan usullari amaliy masalalarni modellashtirishda keng qo'llaniladi. Aynigan yadro usuli masalani chekli o'lchamli fazoga proyeksiyalash orqali algebraik usullardan samarali foydalanish imkonini beradi va hisoblash matematikasida muhim vosita bo'lib xizmat qiladi.

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CHIZIQLI ALMASHTIRISHNING XOS SON VA XOS VEKTORLARINI TOPISHNING USULLARI (AN'ANAVIY VA SONLI YONDASHUVLAR)

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Bugingi kunda matematika jadal rivojlanib bormoqda. Tabiiyki, bu masalalarni yechish ham xilma-xildir. Algebrada uchraydigan kop masalalarni

$$y = Ax \tag{1}$$

shaklda yozish mumkin, bu yerda x va y berilgan W va V funksional fazolarning elementlari bolib, A operator yoki xususiy holda funksionaldir. Matritsaning xos sonlari va xos vektorlarini topish muammosi fizika, mexanika va astronomiyaning koplab amaliy muammolarini hal qilishda

zarur boladi. Bunda $Ax = \lambda x$ korinishdagi tenglamalarning notrivial yechimini aniqlash kerak boladi va bunday yechim mavjud bolgan λ son xos son deb ataladi [1].

Ushbu maqolada chiziqli almashtirishlarning xos soni va xos vektorlarini topishning turli usullari, jumladan, an'anaviy algebraik usullar (xarakteristik tenglama, Faddeev metodi) hamda sonli usullardan biri quvvat (stepen) usuli ko'rib chiqiladi.

Xos vektor chiziqli algebradagi tushuncha bolib, kvadrat matritsa uchun vektor sifatida belgilanadi, matritsani x vektorga kopaytirish natijasida bir xil vektor matritsaning xos son deb ataladigan shunday skalyar songa kopaytiriladi [2]. Shunday qilib, A matritsaning xos vektori shunday nolga teng bolmagan x vektor bolib, shunday λ son uchun

$$Ax = \lambda x \tag{2}$$

bajariladi. Oz navbatida, A matritsasining xos qiymati λ soni bolib, uning uchun xos vektor x mavjud, yani (2) tenglik orinli [3].

1-teorema. n ochovli R_n fazodagi ixtiyoriy A chiziqli operatorning xos vektori mavjud. Agar A operatorning matritsasi

$$A(e) = \begin{pmatrix} a_{11} & a_{21} & ... & a_{n1} \\ a_{12} & a_{22} & ... & a_{n2} \\ ... & ... & ... & ... \\ a_{1n} & a_{2n} & ... & a_{nn} \end{pmatrix}$$

dan iborat bolsa, u holda

$$D(\lambda) = \det(A - \lambda E) = \begin{pmatrix} a_{11} - \lambda & a_{21} & ... & a_{n1} \\ a_{12} & a_{22} - \lambda & ... & a_{n2} \\ ... & ... & ... & ... \\ a_{1n} & a_{2n} & ... & a_{nn} - \lambda \end{pmatrix} = 0 \tag{3}$$

tenglama λ ga nisbatan A operatorning xarakteristik tenglamasi deb ataladi. (3) tenglamaning chap tomoni

$$\det(A - \lambda E) = (-1)^n(\lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - ... - p_n) \tag{4}$$

$n-$ darajali kophad bolib, u A operatorning xarakteristik kophadi deyiladi. Ayrim hollarda (4) kophad ornida A operatorning xos kophadi deb ham ataladi.

$$P(\lambda) = \lambda^n - p_1\lambda^{n-1} - p_2\lambda^{n-2} - ... - p_n \tag{5}$$

Bu yerda A operatorni matritsa deb olsak. Matritsaning xos sonlari uning xos kophadlari ildizlari boladi. (5) kophad $n-$ darajali bolgani uchun n ta ildizga ega [3]. Bizga $\det(A - \lambda_i E)\overline{x} = \overline{0}$ tenglama berilgan bolsin. Algebradan malumki, umumiy holda, $p(\lambda)$ ning p_i koeffisienti A matritsaning $(-1)^{i-1}$ ishora bilan olingan $i-$ tartibli bosh minorlari yigindisiga teng. Viyet teoremasidan foydalanib, quyidagi tengliklarni yozishimiz mumkin:

$$\begin{aligned} &\lambda_1 + \lambda_2 + ... + \lambda_n = p_1, \\ &..... \\ &\lambda_1 \cdot \lambda_2 \cdot ... \cdot \lambda_n = (-1)^{n-1}p_n. \end{aligned}$$

Endi biz matritsalar nazariyasidagi yana bir muhim metodni taqdim etamiz, u A matritsaga (butunlay boshqa goyalar va usullardan foydalangan holda) toliq umumlashtiriladi.

1-tarif. Ixtiyoriy $A \in M_n(C)$ matritsaning bosh diagonal elementlari yigindisiga A matritsaning izi deb ataladi.

Bu tarifga kora A matritsaning izi quyidagicha belgilanadi. $a_{11} + a_{22} + ... + a_{nn} = Tr(A)$, endi biz bu tenglikdan foydalanib quyidagi kophadni hosil qilamiz.

$$\det(A - \lambda E) = \lambda^2 - Tr(A)\lambda + (\det A)\, E_2 = 0 \tag{6}$$

2-tarif. (6) xarakteristik kophadning λ_1, λ_2 kompleks ildizlari $A \in M_2(C)$ matritsaning xos sonlari deb ataladi.
Bu yerda $\lambda_1 + \lambda_2 = Tr(A)$, $\lambda_1 \cdot \lambda_2 = \det(A)$.

2-teorema. (Gamilton – Keli) Ixtiyoriy $A \in M_2(C)$ tenglik orinli

$$A^2 - Tr(A)A + \det(A)E_2 = O_2.$$

Shunday qilib, matritsaning barcha xos sonlar yigindisi uning iziga teng bolib, ularning kopaytmasi shu matritsaning determinantiga teng.

Quvvat (stepen) usuli. Yuqorida keltirilgan usullar asosan xarakteristik ko'phadni topishga asoslangan bo'lib, katta o'lchamli matritsalar uchun hisoblash jihatdan murakkabdir. Shuning uchun amaliyotda ko'pincha sonli usullar, jumladan, quvvat usuli qo'llaniladi. Bu usul matritsaning moduli bo'yicha eng katta xos soni va unga mos xos vektorni topishga imkon beradi.

Faraz qilaylik, A kvadrat matritsaning xos sonlari

$$|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_n|$$

shartni qanoatlantirsin, ya'ni λ_1 yagona va moduli bo'yicha eng katta bo'lsin. U holda ixtiyoriy boshlang'ich $x^{(0)}$ vektor (xos vektorlar bo'yicha yoyilmasi λ_1 ga mos komponenti nolga teng bo'lmagan) uchun quyidagi iteratsion jarayonni tashkil qilamiz:

$$x^{(k+1)} = Ax^{(k)}, \qquad k = 0, 1, 2, \ldots$$

Agar har bir qadamda vektorni normallash (masalan, eng katta komponentga bo'lish) amalga oshirilsa, u holda $x^{(k)}$ vektorlar ketma-ketligi λ_1 ga mos xos vektorga yaqinlashadi. Xos sonning o'zi esa

$$\lambda_1 \approx \frac{(Ax^{(k)})_i}{(x^{(k)})_i}$$

yoki Reley nisbati

$$\lambda_1 \approx \frac{(x^{(k)})^T Ax^{(k)}}{(x^{(k)})^T x^{(k)}}$$

orqali baholanadi [4]. Bu usulning yaqinlashish tezligi $|\lambda_2/\lambda_1|$ nisbatga bog'liq bo'lib, bu nisbat qanchalik kichik bo'lsa, shuncha tez yaqinlashadi.

Quvvat usuli matritsaning eng katta xos sonini hisoblashda juda samarali, ayniqsa, matritsa siyrak (ko'p nolli) bo'lgan hollarda. Bundan tashqari, bu usulni teskari matritsaga qo'llab, eng kichik xos sonni ham topish mumkin (teskari quvvat usuli). Shuningdek, spektral siljitish (shift) orqali boshqa xos sonlarni hisoblash ham mumkin.

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TO‘G‘RI CHIZIQ TENGLAMALARINING IQTISODIY TADBIQLARI

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Iqtisodiyot fanining fundamental muammosi — bu cheksiz ehtiyojlar va cheklangan resurslar o‘rtasidagi nomutanosiblikdir. Har bir korxona ishlab chiqarish jarayonida xomashyo zaxiralari, ishchi kuchi soatlari, dastgohlar quvvati va moliyaviy budjet kabi qat’iy chegaralarga duch keladi. Mana shu cheklangan resurslar sharoitida maksimal foyda keltiruvchi ishlab chiqarish rejasini tanlash masalasi matematikaning maxsus bo‘limi — "Chiziqli dasturlash"modellari orqali hal etiladi.

Chiziqli dasturlash masalasining barcha elementlari (maqsadli funksiya ham, chegaraviy shartlar ham) faqat birinchi darajali to‘g‘ri chiziq tenglamalari va tengsizliklaridan tashkil topadi. Agar korxona ikki turdagi mahsulot (X_1 va x_2) ishlab chiqarsa, uning umumiy iqtisodiy-matematik modeli quyidagicha ko‘rinishda shakllantiriladi. Birinchidan, foydani maksimallashtirishni ifodalovchi maqsadli funksiya to‘g‘ri chiziq tenglamasi tuziladi: $Z = c_1 \cdot X_1 + c_2 \cdot X_2 -> max$

Bu yerda c_1 va c_2 — mos ravishda birinchi va ikkinchi turdagi mahsulot birligidan olinadigan sof foyda miqdori. Ikkinchidan, ishlab chiqarishda ishlatiladigan har bir resurs turi bo‘yicha to‘g‘ri chiziqli chegaraviy tengsizliklar tizimi o‘rnatiladi:

$$\begin{cases} a_{11} \cdot X_1 + a_{12} \cdot X_2 \geq B_1 \text{ (1-resurs bo‘yicha chegara)} \\ a_{21} \cdot X_1 + a_{22} \cdot X_2 \geq B_2 \text{ (2-resurs bo‘yicha chegara)} \end{cases}$$

Bundan tashqari, iqtisodiy ma’noga ko‘ra ishlab chiqarish hajmi manfiy bo‘lishi mumkin emasligi sababli, har doim $X_1 \geq 0$, $X_2 \geq 0$ chiziqli shartlari ham qo‘shiladi. Ushbu tengsizliklarning har biri geometrik jihatdan tekislikda ma’lum bir to‘g‘ri chiziqdan pastda joylashgan yarim tekisliklar sohasini ifodalaydi.

Chiziqli dasturlash masalalarini yechishning eng ko‘rgazmali usuli — grafik usuldir. To‘g‘ri chiziqlar tizimi kesishib, tekislikda "Mumkin bo‘lgan yechimlar ko‘pburchagi"deb ataladigan qavariq sohani hosil qiladi. Chiziqli dasturlashning asosiy teoremalariga ko‘ra, maqsadli funksiyaning optimal qiymati doimo ushbu ko‘pburchakning burchak uchlarida (ya’ni to‘g‘ri chiziqlarning o‘zaro kesishish nuqtalarida) erishiladi.

Amaliy masala: Mebel fabrikasi stollar X_1 va stullar X_2 ishlab chiqaradi. Bitta stol sotishdan olinadigan foyda 50 dollar, bitta stuldan esa 30 dollarni tashkil etadi. Ishlab chiqarish resurslari cheklangan: yog’och zaxirasi haftasiga jami 36 m^2, ishchilar mehnat vaqti esa haftasiga jami 40 soat.

Bitta stol uchun 4 m^2 yog‘och va 5 soat mehnat talab etiladi. Bitta stul uchun esa 3 m^2 yog‘och va 2 soat mehnat sarflanadi. Fabrikaga maksimal foyda keltiruvchi haftalik ishlab chiqarish rejasini to‘g‘ri chiziqlar yordamida aniqlang.

Yechilishi: Masalaning chiziqli modelini shakllantiramiz:

Maqsad funksiyasi: $Z = 50X_1 + 30X_2 \to max$

Resurslar bo‘yicha chegaraviy tenglamalar tizimi:

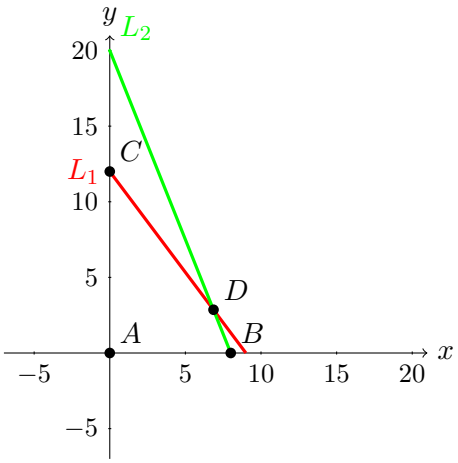
$$\begin{cases} 4X_1 + 3X_2 \leq 36 \text{ (Yog‘och chegarasi to‘g‘ri chizig‘i, L1.)} \\ 5X_1 + 2X_2 \leq 40 \text{ (Ish vaqti chegarasi to‘g‘ri chizig‘i, L2.)} \end{cases}$$

Ushbu to‘g‘ri chiziqlarni grafikda chizish uchun ularning o‘qlar bilan kesishish nuqtalarini topamiz:

• To‘g‘ri chiziq $L1$: Agar $X_1 = 0$ bo‘lsa, $X_2 = 12$. Agar $X_2 = 0$ bo‘lsa, $X_1 = 9$.

Nuqtalar: $(0;12)$ va $(9;0)$.

• To‘g‘ri chiziq $L2$: Agar $X_1 = 0$ bo‘lsa, $X_2 = 20$. Agar $X_2 = 0$ bo‘lsa, $X_1 = 8$.
Nuqtalar: (0;20) va (8;0).



Grafikda ushbu ikki to‘g‘ri chiziqni chizganimizda, ular bir-biri bilan kesishadi va mumkin bo‘lgan sohaning eng muhim ichki burilish burchak uchi nuqtasini hosil qiladi. Ushbu kesishish nuqtasisi koordinatalarini ikki noma’lumli tenglamalar tizimini yechish orqali aniqlaymiz: $4 \cdot X_1 + 3 \cdot X_2 = 36 \Rightarrow 5 \cdot X_1 + 2 \cdot X_2 = 40$

Birinchi tenglamani 2 ga, ikkinchisini esa 3 ga ko‘paytirib, o‘zgaruvchilarni tenglashtiramiz: $8 \cdot X_1 + 6 \cdot X_2 = 72, \Rightarrow 15 \cdot X_1 + 6 \cdot X_2 = 120$

Ikkinchi tenglamadan birinchisini ayirib tashlaymiz: $7 \cdot X_1 = 48 => X_1 = 48/7 = 6.86(Stollarsoni)$

Topilgan qiymatni o‘rniga qo‘yib, X_2 ni topamiz: $4 \cdot (48/7) + 3 \cdot X_2 = 36 => 192/7 + 3 \cdot X_2 = 36 => 3 \cdot X_2 = 36 - 27.43 = 8.57 => X_2 = 2.86$(Stullar soni).

Endi mumkin bo‘lgan soha burchak uchlaridagi barcha muqobil variantlarning foyda qiymatlarini hisoblaymiz va solishtiramiz:

• A nuqta $(0;0)$ — Ishlab chiqarish yo‘q: $Z = 50 \cdot 0 + 30 \cdot 0 = 0$ dollar.

• B nuqta $(8;0)$ — Faqat stol chiqarish: $Z = 50 \cdot 8 + 30 \cdot 0 = 400$ dollar.

• C nuqta $(0;12)$ — Faqat stul chiqarish: $Z = 50 \cdot 0 + 30 \cdot 12 = 360$ dollar.

• D nuqta $(6.86;2.86)$ — Aralash ishlab chiqarish: $Z = 50 \cdot 6.86 + 30 \cdot 2.86 = 343 + 85.8 = 428.8$dollar.

Iqtisodiy tahlil va qaror qabul qilish: Matematik solishtirish shuni ko‘rsatadiki, eng yuqori daromad va foyda D nuqtada, ya’ni aralash ishlab chiqarish rejasida erishiladi (Z = 428.8 dollar). Demak, fabrika haftasiga taxminan 7 ta stol va 3 ta stul kombinatsiyasini ishlab chiqarsa, o‘z resurslaridan eng samarali va optimal usulda foydalangan bo‘ladi. Bu to‘g‘ri chiziqlar tizimining biznes boshqaruvidagi beqiyos amaliy kuchini ko‘rsatadi.

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Some properties of the space of solutions of nonlinear Volterra-Fredholm integro-differential equations

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Let $J = [a, b],\ \ell = b - a > 0$, and consider

$$x'(t) = F(t, x(t), (\mathcal{V}x)(t), (\mathcal{W}x)(t)), \quad t \in J, \tag{1}$$

where

$$(\mathcal{V}x)(t) = \int_a^t K_V(t, s, x(s))\, ds, \tag{2}$$

and

$$(\mathcal{W}x)(t) = \int_a^b K_F(t, s, x(s))\, ds. \tag{3}$$

The main novelties of the present paper, compared with the previously studied ODE solution-space setting [1, 2] are the following. First, we isolate an abstract mechanism for equations $x' = \mathcal{G}(x)$: continuity of $\mathcal{G}$ gives closedness and Polishness, while boundedness on bounded sets yields compact bounded solution strata. This leads to local compactness, $\sigma-$compactness, hemicompactness, and a $k_\omega-$structure. Second, we prove complete continuity of the Volterra and Fredholm operators under the basic continuity assumptions on the kernels. Third, we strengthen the probability-measure theory by proving that every finite-support stratum $\mathcal{P}_n(\mathcal{S})$ is closed, Polish, and $\sigma-$compact. Fourth, we connect the topological theory with solvability: an invariant-ball condition yields existence, while a local contraction condition gives uniqueness, quantitative continuous dependence, and a bi-Lipschitz parametrization of a bounded solution branch by initial values.

The Volterra-Fredholm equation (1) fits this abstract scheme. Continuity of K_V, K_F, and F is sufficient. More precisely, the operators $\mathcal{V}$ and $\mathcal{W}$ are continuous and completely continuous on $C(J, \mathbb{R}^m)$, and the induced nonlinear operator

$$\mathcal{G}(x)(t) = F(t, x(t), (\mathcal{V}x)(t), (\mathcal{W}x)(t))$$

is continuous and bounded on bounded sets.

These conclusions do not require a global uniqueness assumption for the basic topology of the full solution space. Uniqueness is imposed only in the final section, where it is needed to obtain a single-valued parametrization by initial data.

Let $\mathcal{G} : X \to X$ be nonlinear and define

$$\mathcal{S}_\mathcal{G} = \{x \in C^1(J, \mathbb{R}^m) : x'(t) = (\mathcal{G}x)(t) \text{ for all } t \in J\}.$$

Theorem 1 [Closedness and Polishness]. If $\mathcal{G} : X \to X$ is continuous, then $\mathcal{S}_\mathcal{G}$ is closed in X. Consequently it is completely metrizable, separable, and Polish in the uniform topology.

For $R > 0$ put $\mathcal{S}_{\mathcal{G},R} = \{x \in \mathcal{S}_\mathcal{G} : \|x\|_\infty \le R\}$.

Theorem 2 [Compact bounded strata]. If $\mathcal{G}$ is continuous and bounded on bounded sets, then $\mathcal{S}_{\mathcal{G},R}$ is compact for every $R > 0$.

Theorem 3 [Strong global topology]. Under the hypotheses of Theorem 1, $\mathcal{S}_{\mathcal{G}}$ is Polish, locally compact, $\sigma-$compact, hemicompact, and a $k_\omega-$space. With $K_n = \mathcal{S}_{\mathcal{G},n}$,

$$K_1 \subset K_2 \subset \cdots, \quad \mathcal{S}_{\mathcal{G}} = \cup_{n=1}^{\infty} K_n,$$

each K_n is compact, and every compact subset of $\mathcal{S}_{\mathcal{G}}$ is contained in some K_n.

Let $D_V = \{(t,s) \in J^2 : a \le s \le t \le b\}$. Assume

$$K_V : D_V \times \mathbb{R}^m \to \mathbb{R}^m, \quad K_F : J^2 \times \mathbb{R}^m \to \mathbb{R}^m$$

and $F : J \times (\mathbb{R}^m)^3 \to \mathbb{R}^m$ are continuous.

Theorem 4 [Complete continuity]. The operators $\mathcal{V}$ and $\mathcal{W}$ are completely continuous: they map bounded subsets of X into relatively compact subsets of X.

Define

$$(\mathcal{G}x)(t) = F(t, x(t), (\mathcal{V}x)(t), (\mathcal{W}x)(t)).$$

Let

$$\mathcal{S} = \big\{x \in C^1(J, \mathbb{R}^m) : x'(t) = F(t, x(t), (\mathcal{V}x)(t), (\mathcal{W}x)(t))\big\}.$$

Theorem 5. Under the continuity assumptions on F, K_V, K_F, $\mathcal{S}$ is a closed Polish subspace of $C(J, \mathbb{R}^m)$. For every $R > 0$,

$$\mathcal{S}_R = \{x \in \mathcal{S} : \|x\|_\infty \le R\}$$

is compact.

Theorem 6. The space $\mathcal{S}$ is locally compact, $\sigma-$compact, hemicompact, and a $k_\omega-$space. The sequence $\mathcal{S}_n$ is a compact exhaustion and every compact subset of $\mathcal{S}$ is contained in some $\mathcal{S}_n$.

Theorem 7. The solution space $\mathcal{S}$, and each compact stratum $\mathcal{S}_R$, has a countable network, $cs-$network, cs^*-network, $cn-$network, and $ck-$network.

Theorem 8. Assume $\mathcal{S} \neq \varnothing$. Then $\mathcal{P}(\mathcal{S})$ is Polish in the weak topology and therefore has all five countable network-type properties above.

Theorem 9 [Finite-support strata]. For every $n \in \mathbb{N}$,

$$\mathcal{P}_n(\mathcal{S}) = \big\{\mu \in \mathcal{P}(\mathcal{S}) : |\operatorname{supp}\mu| \le n\big\}$$

is closed in $\mathcal{P}(\mathcal{S})$. Hence it is Polish. Moreover it is $\sigma-$compact.

Theorem 10. Let $R > 0$ and assume $\mathcal{S}_R \neq \varnothing$. Then $\mathcal{I}(\mathcal{S}_R)$, endowed with pointwise convergence on $C(\mathcal{S}_R)$, is compact metrizable.

Corollary. For every $n \in \mathbb{N}$, both $\mathcal{I}(\mathcal{S}_R)$ and its subspace

$$\mathcal{I}_n(\mathcal{S}_R) = \big\{\mu \in \mathcal{I}(\mathcal{S}_R) : |\operatorname{supp}\mu| \le n\big\}$$

have countable network, $cs-$network, cs^*-network, $cn-$network, and $ck-$network.

For $\xi \in \mathbb{R}^m$, define $(\mathcal{T}_\xi x)(t) = \xi + \int_a^t (\mathcal{G}x)(s)\,ds$. Fixed points are precisely solutions of (1) satisfying $x(a) = \xi$. Retain the constant M_R.

Theorem 11 [Existence by Schauder]. If

$$\|\xi\| + \ell M_R \le R, \tag{4}$$

then (1) has at least one solution $x \in \mathcal{S}_R$ with $x(a) = \xi$.

Assume now that for $\|u\|, \|\bar{u}\| \le R$,

$$\|K_V(t,s,u) - K_V(t,s,\bar{u})\| \le \kappa_{V,R}\|u - \bar{u}\|, \quad \|K_F(t,s,u) - K_F(t,s,\bar{u})\| \le \kappa_{F,R}\|u - \bar{u}\|,$$

and on $\mathcal{C}_R$,

$$\|F(t,u,v,w) - F(t,\bar{u},\bar{v},\bar{w})\| \le L_{0,R}\|u-\bar{u}\| + L_{V,R}\|v-\bar{v}\| + L_{F,R}\|w-\bar{w}\|.$$

Set

$$\Lambda_R = L_{0,R} + \ell L_{V,R}\kappa_{V,R} + \ell L_{F,R}\kappa_{F,R}, \quad q_R = \ell\Lambda_R.$$

Lemma. For $x, y \in B_R$,

$$\|\mathcal{G}x - \mathcal{G}y\|_\infty \le \Lambda_R\|x-y\|_\infty, \quad \|\mathcal{T}_\xi x - \mathcal{T}_\xi y\|_\infty \le q_R\|x-y\|_\infty.$$

Theorem 12 [Uniqueness and continuous dependence]. Assume (4) and $q_R < 1$. Then the solution in B_R with initial value ξ is unique. If ξ, η satisfy the same invariant-ball condition and x_ξ, x_η are their solutions, then

$$\|x_\xi - x_\eta\|_\infty \le \frac{1}{1-q_R}\|\xi-\eta\|.$$

Theorem 13 [Homeomorphic solution branch]. Let $0 \le R_0 < R$ and assume $R_0 + \ell M_R \le R, \quad q_R < 1$. Define

$$\Sigma_{R_0,R} = \{x \in \mathcal{S} : \|x\|_\infty \le R,\ \|x(a)\| \le R_0\}.$$

Then the solution map

$$\Psi : \overline{B}_{R_0}(0) \to \Sigma_{R_0,R}, \quad \Psi(\xi) = x_\xi,$$

is a bi-Lipschitz homeomorphism and

$$\|\xi-\eta\| \le \|x_\xi - x_\eta\|_\infty \le \frac{1}{1-q_R}\|\xi-\eta\|.$$

Consequently $\Sigma_{R_0,R}$ is compact, contractible, locally connected, and path connected.

Let $\mathcal{S}$ and $\mathcal{T}$ be nonempty solution spaces and $h : \mathcal{S} \to \mathcal{T}$ continuous. Define the push-forward $\mathcal{P}(h)(\mu) = \mu \circ h^{-1}$.

Proposition 1. $\mathcal{P}(h)$ is weakly continuous and satisfies

$$\mathcal{P}(\mathrm{id}) = \mathrm{id}, \qquad \mathcal{P}(g \circ h) = \mathcal{P}(g) \circ \mathcal{P}(h).$$

If $h(\mathcal{S}_R) \subset \mathcal{T}_Q$ for nonempty compact strata, define $\mathcal{I}(h)(\mu)(\varphi) = \mu(\varphi \circ h)$.

Proposition 2. $\mathcal{I}(h)$ is continuous in the pointwise topology and satisfies the analogous identity and composition laws.

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Quasicontinuous image of compact sets

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Quasicontinuous functions have key applications in mathematical analysis, topology, and differential equations, specifically for studying set-valued mappings, general dynamics, and viscosity solutions. In topology, set-valued analysis they use to analyze minimal upper semi-continuous with compact convex values maps and selection theorems. In calculus, differential equations this type functions apply in extended differential frameworks and finding viscosity solutions for Hamiltonian equations. In dynamical systems, quasicontinuous maps help to model the behavior of discontinuous or real-valued maps like doubling functions.

Assume X, Y are Hausdorff topological spaces. Let $\mathbb{R}$ be the space of real numbers with the usual metric, $\mathbb{N}$ the set of positive integers, $\mathbb{Z}$ the set of all integers, $\mathbb{Q}$ the set of all rational numbers, considered as subspaces of $\mathbb{R}$. The symbol $\bar{A}$ will stand for the closure of the set A in a topological space.

A map $f : X \to Y$ is quasicontinuous [1, 2] at $x \in X$ if for every open set $V \subset Y$, $f(x) \in V$ and every open set $U \subset X$, $x \in U$ there is a nonempty open set $W \subset U$ such that $f(W) \subset V$. If f is quasicontinuous at every point of X, we say that f is quasicontinuous.

Remind that a subset A of X is quasi-open (or semi-open) [2] if it is contained in the closure of its interior, i. e. $A \subset \overline{\operatorname{int} A}$. Each open set is quasi-open. Sets of the view $[a, b)$, $(a, b]$, or $[a, b]$ in $\mathbb{R}$ are examples of quasi-open but not open sets. Here we assume that $a < b$.

Remark 1. *A family consisting of the interior of elements of a quasi-open cover of a set need not cover this set.*

Really, a system

$$\sigma = \left\{ \left[0, \tfrac{1}{3}\right), \left[\tfrac{1}{4}, \tfrac{1}{2}\right), \left[\tfrac{1}{2}, \tfrac{2}{3}\right), \left[\tfrac{2}{3}, 1\right] \right\}$$

is a quasi-open cover of $[0, 1]$. But the system

$$\operatorname{int} \sigma = \left\{ \left(0, \tfrac{1}{3}\right), \left(\tfrac{1}{4}, \tfrac{1}{2}\right), \left(\tfrac{1}{2}, \tfrac{2}{3}\right), \left(\tfrac{2}{3}, 1\right) \right\}$$

does not cover the segment $[0, 1]$.

A map $f : X \to Y$ is quasicontinuous if and only if $f^{-1}(V)$ is quasi-open in X for every open set $V \subset Y$.

Any continuous map $g : X \to Y$ is quasicontinuous.

Example 1. Consider a segment $[0, 1] \subset \mathbb{R}$ with the natural topology, a map $f : [0, 1] \to [0, 1]$ defined as

$$f(x) = \begin{cases} x^2, & 0 \le x < \frac{1}{4}, \\ \sqrt{x}, & \frac{1}{4} \le x \le 1, \end{cases}$$

tolerates discontinuity at the point $x = \frac{1}{4}$. But it is quasicontinuous.

We have $f([0,1]) = \left[0, \frac{1}{16}\right) \cup \left[\frac{1}{4}, 1\right]$. It is enough to check that a preimage of the view $f^{-1}((a,b))$ is a quasi-open set for any open $(a,b) \subset [0,1]$. The following cases are possible:

Case 1. $0 < a < \frac{1}{16} < b \leq \frac{1}{4}$. Then for open set (a,b) the preimage

$$f^{-1}((a,b)) = \left(\sqrt{a}, \tfrac{1}{4}\right)$$

is open;

Case 2. $0 < a < \frac{1}{16}$, $\frac{1}{4} < b < 1$. Then for open (a,b) its preimage

$$f^{-1}(a,b) = \left(\sqrt{a}, \tfrac{1}{4}\right) \cup \left[\tfrac{1}{4}, b^2\right) = (\sqrt{a}, b^2)$$

is open;

Case 3. $\frac{1}{16} \leq a < \frac{1}{4} < b < 1$. Then for open (a,b) the preimage

$$f^{-1}((a,b)) = \left[\tfrac{1}{4}, b^2\right)$$

is quasi-open.

Example 2. a) Function

$$g(x) = \begin{cases} x^2, & 0 \leq x < \frac{1}{4}, \\ \frac{1}{8}, & x = \frac{1}{4}, \\ \sqrt{x}, & \frac{1}{4} < x \leq 1 \end{cases}$$

is not quasicontinuous at $x = \frac{1}{4}$. Indeed, a preimage of the open neighborhood $\left(\frac{3}{32}, \frac{5}{32}\right)$ of $\frac{1}{8}$ in $[0,1]$ is not quasi-open: $g^{-1}\left(\left(\frac{3}{32}, \frac{5}{32}\right)\right) = \left\{\frac{1}{4}\right\}$. Note, since $g([0,1]) = \left[0, \frac{1}{16}\right) \cup \left\{\frac{1}{8}\right\} \cup \left(\frac{1}{4}, 1\right]$, one can consider this set with the induced topology. Then the singleton $\left\{\frac{1}{8}\right\}$ is open in $g([0,1])$ but its preimage $g^{-1}\left(\left\{\frac{1}{8}\right\}\right) = \left\{\frac{1}{4}\right\}$ is not quasi-open in $[0,1]$. So, g is not a quasicontinuous map.

b) The Dirichlet function $D : \mathbb{R} \to \{0,1\}$,

$$D(x) = \begin{cases} 0, & x \in \mathbb{Q}, \\ 1, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

is not quasicontinuous. Here the set $\{0,1\}$ is equipped with the discreet topology.

Example 3. a) Function $h : \mathbb{R} \to \mathbb{R}$,

$$h(x) = \begin{cases} \frac{1}{x}, & x \neq 0, \\ 0, & x = 0 \end{cases}$$

is not quasicontinuous. Really, $h^{-1}((-1,1)) = (-\infty, -1) \cup \{0\} \cup (1, +\infty)$. Clearly, $\operatorname{int}(h^{-1}(-1,1)) = (-\infty,-1) \cup (1,+\infty)$, and $h^{-1}(-1,1) \not\subset \overline{\operatorname{int}(h^{-1}(-1,1))}$.

b) Function $h_1 : \mathbb{R} \to \mathbb{R}$,

$$h_1(x) = \begin{cases} \frac{1}{x}, & x \neq 0, \\ 1, & x = 0 \end{cases}$$

is also not quasicontinuous. Really, $h_1^{-1}\left(\frac{1}{2}, \frac{3}{2}\right) = \left(\frac{2}{3}, 2\right) \cup \{0\}$. Clearly, $\operatorname{int}\left(h_1^{-1}\left(\frac{1}{2}, \frac{3}{2}\right)\right) = \operatorname{int}\left(\frac{2}{3}, 2\right) \cup \{0\} = \left(\frac{2}{3}, 2\right)$, and $\left(\frac{2}{3}, 2\right) \cup \{0\} \not\subset \left[\frac{2}{3}, 2\right] = \overline{\left(\frac{2}{3}, 2\right)} = \overline{\operatorname{int}\left(\left(\frac{2}{3}, 2\right) \cup \{0\}\right)}$.

Example 4. a) The floor function $\lfloor \cdot \rfloor : \mathbb{R} \to \mathbb{R}$ defined by

$$\lfloor x \rfloor = \max\{n \in \mathbb{Z} : n \leq x\}$$

is quasicontinuous.

b) The fractional part function $\{x\} = x - \lfloor x \rfloor$ is quasicontinuous.

c) The ceiling function $\lceil \cdot \rceil : \mathbb{R} \to \mathbb{R}$ defined by

$$\lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$$

is quasicontinuous.

Example 5. a) The function $\lfloor \cdot \rfloor \circ (\cdot)^{-1} : (0, 1] \to \mathbb{R}$ defined by

$$\lfloor \cdot \rfloor \circ (\cdot)^{-1}(x) = \lfloor x^{-1} \rfloor = \max\left\{n \in \mathbb{N} : n \leq \tfrac{1}{x}\right\}$$

is quasicontinuous.

b) The function $\lfloor \cdot \rfloor \circ (\cdot)^{-1} : [0, 1] \to \mathbb{R}$ defined by

$$\lfloor \cdot \rfloor \circ (\cdot)^{-1}(x) = \begin{cases} \lfloor x^{-1} \rfloor, & x \neq 0, \\ a, & x = 0 \end{cases}$$

is not quasicontinuous for any $a \in \mathbb{R}$.

It is well known a continuous image of a compact set is also a compact set.

Remark 2. *Quasicontinuous maps do not preserve the compactness of a set.* Example 1 shows that the compactness of sets is not preserved under the influence of quasicontinuous maps. Really, one has

$$f\left(\left[\tfrac{1}{8}, \tfrac{3}{8}\right]\right) = \left[\tfrac{1}{64}, \tfrac{1}{16}\right) \cup \left[\tfrac{1}{2}, \tfrac{\sqrt{6}}{4}\right].$$

Nevertheless, the following result takes place.

Theorem 1. The closure of the quasicontinuous image of a compact set is a compact set.

Proof. For Hausdorff topological spaces X, Y, let $f : X \to Y$ be a quasicontinuous map, and $K \subset X$ any compact subset. Take any open cover γ of $\overline{f(K)}$ in Y. Since $f(K) \subset \bigcup \gamma$, for each $x \in K$ there exists $V \in \gamma$ such that $x \in f^{-1}(V)$. Inasmuch as $f^{-1}(V)$ is quasi-open, a family $\{f^{-1}(V) : V \in \gamma\}$ is a quasi-open cover of K. Put

$$D = \{x \in K : x \in f^{-1}(V) \setminus \operatorname{int} f^{-1}(V)\}.$$

Note that $O_x \cap \operatorname{int} f^{-1}(V) \neq \emptyset$ for any open neighborhood O_x of $x \in f^{-1}(V) \setminus \operatorname{int} f^{-1}(V)$. Let $f(x) \in V'$ for some $V' \in \gamma$, $x \in D$. There exists $O'_x \subset O_x$ such that $f(O'_x) \subset V'$. Now construct a system

$$\Gamma = \{\operatorname{int} f^{-1}(V) : V \in \gamma\} \cup \{O'_x : x \in D\}.$$

It is an open cover of K. One can pick out a finite subcover

$$\Gamma = \{\operatorname{int} f^{-1}(V_i) : i = 1, \ldots, m\} \cup \{O'_{x_j} : j = 1, \ldots, l\},$$

$m, l \in \mathbb{N}$, of K. Then

$$\gamma_1 = \{V_1, \ldots, V_m\} \cup \{V'_1, \ldots, V'_l\}$$

is an open cover of $f(K)$ which is a finite subcover of γ. Theorem 1 is proved.

It is worth noting one more time that the closure of the image is essential (see Remark 2).

An important application of theorem 1 is boundedness of quasicontinuous functions.

Corollary 1. The image of a compact set under a quasicontinuous real-valued function is always bounded.

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Topologik fazolar kategoriyasida kovariant funktorlar

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$C(X,Y)$ bilan X topologik fazoni Y topologik fazoga akslantiruvchi barcha uzluksiz akslantirishlar to'plamini belgilaymiz. $\mathscr{F} : Comp \to Comp$ kovariant funktor deyiladi, agar ixtiyoriy $f,\ g\ \in C(X,Y)$ va id_X akslantirishlar uchun

1. $F(f \circ g)$=$F(f) \circ F(g)$;
2. $F(id_x)$=$id_{F(X)}$ shartlar o'rinli bo'lsa.[1]

Biz exp-eksponenta funktori, Π^n-fazoni n marta o'zini o'ziga ko'paytirish funktori, P-ehtimol o'lchovli funktor, λ - zanjirlangan oilalar fazosi, O - kuchsiz additiv funksionallar fazosini bilamiz. λ, O_z, OH, OS - funktorlar O funktorning funktor ostisi bo'ladi.

$O : Comp \to Comp$ funktorini 1998-yilda T.Radul kiritgan. Bu funktor exp, P, λ funktorlarini o'z ichiga oladi. T.Radul O fuktorini kuchsiz normal funktor ekanligini isbotlagan, ya'ni bu funktor uzluksiz akslantirishning proobrazini saqlamasligiga misol keltirgan [2].

Professor V.V.Fedorchuk tomonidan quyidagi masala qo'yilgan:

Agar X fazo P xossaga ega bo'lsa, u holda $F(X)$ fazo ham P xossaga ega bo'ladimi?

va aksincha,

agar $F(X) \in P$ xossaga ega bo'lsa, u holda $X \in P$ fazo ham xossaga ega bo'ladimi?

(bunda F kovariant (kontravariant) funktor) [3].

Funktorlardan exp funktorini ko'rib chiqamiz.

$X \in T_1$ fazo berilgan bo'lsin. X to'plamning bo'sh bo'lmagan barcha yopiq to'plamlar to'plamini $\exp X$ bilan belgilaymiz. $\exp X$ da topologiya bazasi quyidagicha kiritiladi:

$$O\langle U_1, U_2, ..., U_n\rangle = \left\{ F \in \exp X : F \subset \bigcup_{i=1}^{n} U_i, F \bigcap U_i \neq \varnothing, i = 1, 2, ..., n \right\}$$

bunda $U_1, U_2, ..., U_n \subset X$ dagi ochiq to'plamlar. Hosil bo'lgan topologiya Viyetoris topologiyasi, $\exp X$ to'plam Viyetoris topologiyasi bilan eksponensial yoki giperfazo deyiladi.

$X, Y \in Comp$-kompakt fazolar va $f : X \to Y$-uzluksiz akslantirish bo'lsin. $\exp f : \exp X \to \exp Y$ akslantirish quyidagi formula bilan topiladi:

$$\exp(f)(F) = f(F)$$

bunda $F \in \exp X$ [4].

Bizga quyidagi teoremalar ma'lum. **teorema**.*Kompakt fazoning uzluksiz obrazi kompakt fazo bo'ladi [5].*

teorema.*Xausdorf fazosida ixtiyoriy kompakt to'plam yopiq to'plam bo'ladi [6].*

Agar X va Y fazolar kompakt bo'lsalar, u holda yuqoridagi 1-teorema va 2-teoremadan X fazoning ixtiyoriy yopiq $F \subset X$ to'plam ostisi uchun $f(F)$ ham yopiq to'plam bo'lishi kelib chiqadi.

Bu tasdiq X kompakt bo'lmagan fazolar uchun o'rinli emas.

Misol uchun, $\mathbb{R}^2$ tekislikda $y = \frac{1}{x}$ funktsiya grafigini qaraylik. Funksiya grafigi tekislikda yopiq to'plam bo'ladi. $Gr(y)$ funksiya grafigi tekislikda yopiq to'plam bo'ladi.$\pi : \mathbb{R}^2 \to \mathbb{R}$ akslantirish

$\pi(x,y)=x$ proyeksiya uzluksiz ochiq akslantirish bo'ladi. Funksiya grafigi obrazi $\pi(Gr(y))=(-\infty,0)\bigcup(0,\infty)$ to'plam $\mathbb{R}$ to'g'ri chiziqda ochiq to'plam bo'ladi. Bundan $\exp(\pi)(Gr(y))=\pi(Gr(y))=(-\infty,0)\bigcup(0,\infty)\notin\exp\mathbb{R}$. Bundan quyidagi xulosani olamiz. **teorema**.. *X va Y topologik fazolar berilgan bo'lsin. U holda topologik fazolar kategoriyasida harakatlanuvchi* exp *funktori uchun* $(\exp f)(\exp X)\not\subset\exp Y$ *munosabat o'rinli bo'lishi mumkin.*

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FUNCTIONAL CHARACTERIZATION OF STRUCTURAL A-AUTOMORPHISMS AND FINITENESS OF LEMNISCATE SYMMETRIES

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Let $\Omega\subset\mathbb{C}$ be a simply connected hyperbolic domain equipped with the Beltrami A-structure

$$f_{\bar z}=\mu_A(z)f_z,\qquad \mu_A\in L^\infty(\Omega),\qquad \|\mu_A\|_\infty<1. \tag{1}$$

Let $\mathcal{O}_A(\Omega)$ denote the class of ACL mappings $f\in W^{1,2}_{\mathrm{loc}}(\Omega)$ satisfying (1) almost everywhere. By the measurable Riemann mapping theorem there exists an orientation-preserving quasiconformal homeomorphism $\Phi:\Omega\to\mathbb{D}$ such that

$$\Phi_{\bar z}=\mu_A(z)\Phi_z. \tag{2}$$

The Stoilow factorization principle yields

$$f\in\mathcal{O}_A(\Omega)\quad\Longleftrightarrow\quad f=h\circ\Phi,\qquad h\in\mathcal{O}(\mathbb{D}). \tag{3}$$

Since Beltrami coefficients transform nonlinearly under composition, structural symmetry must preserve the deformed complex structure itself. We call an orientation-preserving quasiconformal homeomorphism $F:\Omega\to\Omega$ a *structural A-automorphism* if

$$H=\Phi\circ F\circ\Phi^{-1}\in\mathrm{Aut}(\mathbb{D}). \tag{4}$$

The corresponding group is denoted by $\mathrm{Aut}^{\mathrm{str}}_A(\Omega)$.

Theorem 1. Let $F:\Omega\to\Omega$ be an orientation-preserving quasiconformal homeomorphism. Then

$$F\in\mathrm{Aut}^{\mathrm{str}}_A(\Omega)\quad\Longleftrightarrow\quad P_F\big(\mathcal{O}_A(\Omega)\big)=\mathcal{O}_A(\Omega),\qquad P_F(f)=f\circ F. \tag{5}$$

Proof. If $F \in \mathrm{Aut}_A^{\mathrm{str}}(\Omega)$, then $H = \Phi F \Phi^{-1} \in \mathrm{Aut}(\mathbb{D})$. For $f = h \circ \Phi \in \mathcal{O}_A(\Omega)$ we have

$$f \circ F = h \circ H \circ \Phi \in \mathcal{O}_A(\Omega).$$

Applying the same argument to F^{-1} gives equality in (5). Conversely, assume $P_F(\mathcal{O}_A(\Omega)) = \mathcal{O}_A(\Omega)$. Since $\Phi \in \mathcal{O}_A(\Omega)$, the function $\Phi \circ F$ is A-analytic. By (3), $\Phi \circ F = H \circ \Phi$ for some $H \in \mathcal{O}(\mathbb{D})$. Because $H = \Phi F \Phi^{-1}$ is a bijective self-map of $\mathbb{D}$, it is a biholomorphic automorphism; hence $H \in \mathrm{Aut}(\mathbb{D})$ and $F \in \mathrm{Aut}_A^{\mathrm{str}}(\Omega)$. □

Thus the structural group is recovered intrinsically from its action on $\mathcal{O}_A(\Omega)$, independently of the normalization of Φ.

The same structural symmetry has a direct metric interpretation. Let $\rho_{\mathbb{D}}$ denote the classical Poincaré distance on $\mathbb{D}$ and define the induced A-Poincaré distance on Ω by

$$\rho_A(z_1, z_2) = \rho_{\mathbb{D}}\big(\Phi(z_1), \Phi(z_2)\big). \tag{6}$$

This definition is independent of the normalization of the uniformizing map: if $\widetilde{\Phi} = M \circ \Phi$ with $M \in \mathrm{Aut}(\mathbb{D})$, then

$$\rho_{\mathbb{D}}\big(\widetilde{\Phi}(z_1), \widetilde{\Phi}(z_2)\big) = \rho_{\mathbb{D}}\big(\Phi(z_1), \Phi(z_2)\big),$$

because disk automorphisms are Poincaré isometries.

Corollary 1. The structural A-automorphism group coincides with the group of orientation-preserving surjective isometries of the induced A-Poincaré distance:

$$\mathrm{Aut}_A^{\mathrm{str}}(\Omega) = \mathrm{Isom}^+(\Omega, \rho_A). \tag{7}$$

Proof. If $F \in \mathrm{Aut}_A^{\mathrm{str}}(\Omega)$, then $H = \Phi F \Phi^{-1} \in \mathrm{Aut}(\mathbb{D})$. Hence, for $z_1, z_2 \in \Omega$,

$$\begin{aligned} \rho_A(F(z_1), F(z_2)) &= \rho_{\mathbb{D}}\big(H(\Phi(z_1)), H(\Phi(z_2))\big) \\ &= \rho_{\mathbb{D}}\big(\Phi(z_1), \Phi(z_2)\big) = \rho_A(z_1, z_2). \end{aligned}$$

Thus F is an orientation-preserving surjective ρ_A-isometry. Conversely, if F is such an isometry, then $H = \Phi F \Phi^{-1}$ is an orientation-preserving surjective Poincaré isometry of $\mathbb{D}$. The classical identity $\mathrm{Isom}^+(\mathbb{D}, \rho_{\mathbb{D}}) = \mathrm{Aut}(\mathbb{D})$ yields $H \in \mathrm{Aut}(\mathbb{D})$, and hence $F \in \mathrm{Aut}_A^{\mathrm{str}}(\Omega)$. □

Now let $L \in \mathcal{O}_A(\Omega)$ be nonconstant. Write

$$L = h \circ \Phi, \qquad h \in \mathcal{O}(\mathbb{D}), \tag{8}$$

and define the modulus-preserving structural subgroup

$$\Gamma_L = \{F \in \mathrm{Aut}_A^{\mathrm{str}}(\Omega) : |L(F(z))| = |L(z)| \text{ for all } z \in \Omega\}. \tag{9}$$

Hence every $F \in \Gamma_L$ preserves each level lemniscate $\{z \in \Omega : |L(z)| = c\}$, $c \geq 0$, so Γ_L is the common structural symmetry group of this lemniscate family.

In disk coordinates this group is conjugate to

$$\widetilde{\Gamma}_h = \{M \in \mathrm{Aut}(\mathbb{D}) : |h(M(w))| = |h(w)| \text{ for all } w \in \mathbb{D}\}, \tag{10}$$

namely $\Gamma_L = \Phi^{-1} \widetilde{\Gamma}_h \Phi$. For $M \in \widetilde{\Gamma}_h$, let $Q = (h \circ M)/h$ off $Z(h)$. The identity $|h \circ M| = |h|$ forces $h \circ M$ and h to have the same vanishing order at every zero; hence all singularities of Q are removable. The extension satisfies $|Q| = 1$ and is therefore constant, so

$$h \circ M = c_M h, \qquad |c_M| = 1. \tag{11}$$

Thus M preserves $Z(h)$ with multiplicities.

Theorem 2. Suppose that h has finitely many and at least two distinct zeros in $\mathbb{D}$,

$$Z(h) = \{a_1, \dots, a_N\}, \qquad N \geq 2.$$

Then Γ_L is a finite cyclic group. If n_m denotes the number of distinct zeros of multiplicity m, then

$$|\Gamma_L| \leq \prod_m n_m! \leq N!. \tag{12}$$

Moreover, if $q = |\Gamma_L|$, then there exist $\psi \in \mathrm{Aut}(\mathbb{D})$, $r \in \{0, \dots, q-1\}$ and $g \in \mathcal{O}(\mathbb{D})$ such that

$$h \circ \psi^{-1}(w) = w^r g(w^q). \tag{13}$$

Proof. Every $M \in \widetilde{\Gamma}_h$ permutes the finite set $Z(h)$ and, by (11), preserves multiplicities. Hence there is a homomorphism

$$\Pi : \widetilde{\Gamma}_h \longrightarrow \prod_m S_{n_m}. \tag{14}$$

If $M \in \ker \Pi$, then M fixes all zeros of h. Since $N \geq 2$, it fixes two distinct interior points of $\mathbb{D}$, and therefore $M = \mathrm{id}$. Thus Π is injective and (12) follows. Consequently $\widetilde{\Gamma}_h$ is a finite subgroup of $\mathrm{Aut}(\mathbb{D})$ and hence is conjugate to a finite rotation group. Thus, for some $\psi \in \mathrm{Aut}(\mathbb{D})$,

$$\psi \widetilde{\Gamma}_h \psi^{-1} = \langle R \rangle, \qquad R(w) = \omega w, \qquad \omega = e^{2\pi i/q}. \tag{15}$$

Putting $\widehat{h} = h \circ \psi^{-1}$ and using (11), we obtain $\widehat{h}(\omega w) = c\widehat{h}(w)$ with $c^q = 1$. Hence $c = \omega^r$ for some $r \in \{0, \dots, q-1\}$. Comparing coefficients in the power series of $\widehat{h}$ shows that only exponents congruent to r modulo q occur, which gives (13). $\square$

The condition $N \geq 2$ is essential in general: for $h(w) = w^m$, every rotation $R_\alpha(w) = e^{i\alpha} w$ preserves $|h|$, so the modulus-preserving group contains the full circle group and is infinite.

Thus structural A-automorphisms are characterized intrinsically through $\mathcal{O}_A(\Omega)$. If the uniformized function has finitely many and at least two distinct zeros, the common symmetry group of the level lemniscates is finite cyclic, with normal form (13).

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The Classical Case in the Theory of $A(z)$-Analytic Automorphisms

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The theory of automorphisms occupies an important place in complex analysis and geometric function theory because it describes the symmetries of analytic structures. In the classical setting, automorphisms of the unit disk are completely characterized by Mobius transformations, and finite subgroups of these automorphisms are generated by rotations. These mappings preserve the geometry of the unit disk and play a fundamental role in the study of holomorphic functions and invariant metrics.

Vekua [1] established the theory of generalized analytic functions, while Bers [2] developed pseudoanalytic function theory for generalized complex structures. Ahlfors [3] provided the foundations of quasiconformal mapping theory, including the Beltrami equation and distortion properties. Together, these works form the theoretical basis for studying $A(z)$-analytic automorphisms and their rigidity.

The theory of $A(z)$-analytic functions extend classical complex analysis by replacing the Cauchy-Riemann equation with a generalized Beltrami-type equation. This extension makes it possible to investigate analytic structures in anisotropic media and more general geometric settings. The corresponding automorphisms are described by generalized coordinate transformations that depend on the coefficient $A(z)$.

The most fundamental special case in the theory of $A(z)$-analytic functions arises when the deformation coefficient vanishes identically, $A(z) \equiv 0$. This condition has an important mathematical meaning because it completely eliminates the anisotropic deformation introduced by the generalized Beltrami equation. Consequently, the $A(z)$-analytic structure coincides with the classical complex analytic structure. In other words, the generalized theory reduces to the well-known theory of holomorphic functions, providing a natural consistency check for every theorem established in the broader framework. Recall that an $A(z)$-analytic function satisfies the generalized Beltrami equation

$$\frac{\partial f}{\partial \bar{z}} = A(z)\frac{\partial f}{\partial z} \tag{1}$$

When $A(z) \equiv 0$, this equation immediately becomes $\frac{\partial f}{\partial \bar{z}} = 0$. This is exactly the classical Cauchy-Riemann equation in complex notation. Hence every $A(z)$-analytic function becomes an ordinary holomorphic function. No additional deformation remains, and the geometry coincides with the standard Euclidean geometry of the complex plane. An important consequence concerns the canonical coordinate transformation ψ. In the general $A(z)$-analytic theory, ψ transforms the deformed geometry into the classical complex plane. It compensates for the distortion created by the coefficient $A(z)$. However, if $A(z) \equiv 0$, no compensation is necessary, and therefore $\psi(z) = z, \quad \psi^{-1}(z) = z$. Substituting these identities into the general automorphism formula $f(z) = \psi^{-1}\left(e^{\frac{2\pi ik}{n}}\psi(z)\right)$ gives the classical expression $f(z) = \psi^{-1}\left(e^{\frac{2\pi ik}{n}}z\right)$. Thus every admissible automorphism is simply a rotation of the unit disk. The modulus of every point remains unchanged because

$$|f(z)| = \left|e^{\frac{2\pi ik}{n}}\right| |z| = |z|, \tag{2}$$

where $\left|e^{\frac{2\pi ik}{n}}\right| = 1$.

Therefore, every circle centered at the origin is invariant. The mapping changes only the argument of each point while preserving its distance from the origin. Geometrically, it is a pure rotation without stretching, shrinking, or shearing.

Example 1. Let $m = 4$. Then $f(z) = iz$. Starting from $z = 1$ we obtain $1 \to i \to -1 \to -i \to 1$. These four points are the vertices of a square on the unit circle. After four iterations, $f^4(z) = z$, showing that the automorphism has finite order four.

Example 2. Let $m = 6$. Then $f(z) = e^{\frac{\pi i}{3}} z$. Every iteration rotates each point by 60 degrees. Starting from $z = 1$ we obtain six equally spaced points on the unit circle, namely the vertices of a regular hexagon. After six iterations, the original point is recovered.

Consequently, the classical case $A(z) \equiv 0$ serves as the simplest model for understanding automorphism groups. Every theorem proved in the general $A(z)$-analytic framework should reproduce these classical rotational automorphisms when the deformation coefficient vanishes. This limiting case therefore provides both a geometric interpretation and an important verification of the generalized theory.

Example 3. The automorphism $f(z) = e^{\frac{\pi i}{2}} z = iz$ corresponds to the case $m = 4$. Let $z = \frac{3}{5} + \frac{3}{5}i$. Its modulus is $|z| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \frac{\sqrt{13}}{5} < 1$. Therefore, the point belongs to the unit disk. Applying the automorphism,

$$f(z) = i\left(\frac{3}{5} + \frac{2}{5}i\right) = -\frac{2}{5} + \frac{3}{5}i.$$

The modulus of the image is

$$|f(z)| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \frac{\sqrt{13}}{5} = |z|.$$

Hence $|f(z)| = |z| < 1$, which proves that the automorphism maps the unit disk onto itself while preserving the Euclidean distance from the origin. Geometrically, the point is rotated through an angle of 90 degrees without changing its modulus. This example illustrates one of the fundamental properties of classical automorphisms: they preserve both the unit disk and its circular geometry.

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Об одновременном представление чисел ввиде линейных форм с простыми переменными

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В этой статье я поделюсь результатами, которых я достиг за последние годы. Эти результаты можно условно разделить на три группы:

З1). Об одновременном представление чисел в виде линейных форм с простыми переменными;

З2). О представление данного числа как сумма квадратов четырех и пяти простых чисел;

З3). О представление данного числа как сумма двух простых и квадрата третьего простого числа.

Сначала я вкратце буду останавливаться о результатах полученные нами относительно З1).

Известно, что задача о разрешимости системы

$$b_i = a_{i1}p_1 + a_{i2}p_2 + \ldots + a_{im}p_m \ ; \ i = 1, 2, 3, \ldots, s \tag{1}$$

была поставлена Хуа-Ло-Кеном [1]. Разрешимость системы (1) связана следующими двумя условиями:

а) для любого простого p существуют такие целые числа $l_1, \ldots, l_m$ с условиями $1 \leq l_1, \ldots, l_m \leq p-1$, которые удовлетворяют систему линейных сравнений:

$$a_{i1}l_1 + a_{i2}l_2 + \ldots + a_{im}l_m \equiv b_i \, (mod p) \, , \ \ (i = 1, \ldots, s) \tag{2}$$

b) существуют такие действительные положительные числа $y_1, y_2, \ldots, y_m$ для которых выполняются равенства $a_{i1}y_1 + \ldots + a_{im}y_m = b_i$, $(i = 1, 2, ..., s)$.

Обозначим через $W_{s,m}(X)-$ множество векторов $\vec{b} = (b_1, b_2, \ldots, b_s)$, $1 \leq b_1, b_2, \ldots, b_s \leq X$, которые удовлетворяют условиям а), b) и пусть $B = max\{3|a_{ij}|\}$, $U_{s,m}(X)-$ множество наборов $(b_1, b_2, \ldots, b_s)$, $1 \leq b_1, b_2, \ldots, b_s \leq X$, для которых система (1) неразрешима в простых числах, т.е.

$$U_{s,m}(X) = \{ (b_1, \ldots, b_s), \ \ 1 \leq b_1, \ldots, b_s \leq X, \, b_i \neq a_{i1}p_1 + a_{i2}p_2 + \cdots + a_{im}p_m \}$$

и пусть $E_{s,m}(X) = card U_{s,m}(X)-$ количество элементов в множестве $U_{s,m}(X)$.

В первые Ву Фанг[2] рассматривая системы (1) при выполнение условии а), в) и $m \geq 2s+1$ получил асимптотическая формула для $J_{m,s}\left(\vec{b}\right)$ – числа решений системы (1).

Что будет если $s \leq m \leq 2s$? Этот вопрос оставался открытым до 1989 года. В 1989 году была опубликовано статья M.C.Liu, K.M. Tsang[3] в нем рассматривается система (1) при $s = 2, \ m = 3$ и доказано

1) если для некоторых действительных положительных чисел y_1, y_2, y_3 выполняется неравенство $\sum\limits_{j=1}^{3} a_{ij}y_j > 0, \ \ i = 1, 2$, то для достаточно больших X имеем $card W_{2,3}(X) \gg X^2(B^2 lnlnB)^{-2}$;

2) $E_{2,3}(X) < X^{2-\delta}$, при $X > B^A$, где A (достаточно большое) и $\delta > 0$ (достаточно малое) абсолютные эффективно вычисляемые постоянные.

Пусть a_1, a_2- взаимно простые целые числа, которые одновременно не являются отрицательными и

$$U_1(X) = \{1 \leq n \leq X, \ \ (n, a_1a_2) = 1, \ n = a_1 + \ a_2 \, (\mathrm{mod} \ 2) \, , \ n \neq a_1p_1 + a_2p_2\} \, .$$

Авторы [3] из утверждение 2) получили оценку $card \ U_1(X) < X^{1-\delta}$. При $a_1 = a_2 = 1$ отсюда получается оценка для исключительного множества бинарной проблемы Гольдбаха.

Хочу отметить, что я используя методику работы Liu M.C., Tsang K.M., вычислил численное значение $\delta > 0$, но у меня получилась результат хуже чем полученные мною используя методику работы Montgomery H.L., Vaughan R.C. Поэтому мы начли исследовать системы вида (1).

Мы сначала уточнили результаты [3], оценку $card W_{2,3}(X) \gg X^2(B^2 lnlnB)^{-2}$ написали без символа $\ll$ и вычислили численное значение $\delta > 0$ в оценке $E_{2,3}(X) < X^{2-\delta}$.

В [4,5] обобщили результаты работы [3] для системы (1) при $s = n$ и $m = n+1$. Тем самым доказали справедливость одно из предположении Хуа-Ло-Кена [1] для всех наборов

$(b_1, b_2, \ldots, b_n)$, $1 \le b_1, b_2, \ldots, b_n \le X$, за исключением не более чем $E_{n,n+1}(X) < X^{n-\delta}$ наборов из них.

Затем мы с учениками А.Ш.Сафаров [6], Б.Х.Абраев [7] и Б.Х.Эрдоновым [8] полностью исследовали систему (1) в оставщиеся значении s и m из интервала $s \le m \le 2s$. Для всех случаях получили соответствующую оценки для $E_{s,m}(X)-$ исключительного множества и оценка $J_{m,s}\left(\vec{b}\right)$ – снизу для числа представлению данных чисел в указанном виде.

Далее, приводится результаты полученные относительно задач З.2) и З.3). Полученные результаты являются существенным шагом относительно решений задач поставленные Хуа-Ло-Кен'ом в [1].

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О топологической эквивалентности пар и троек бесконечномерных многообразий в пространстве вероятностных мер

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Аннотация. *В работе исследуются топологические и геометрические свойства подпространств пространства всех вероятностных мер $P(X)$, определенных на бесконечном компакте X. Получены новые результаты о топологической эквивалентности пар и троек бесконечномерных многообразий, образованных специальными подпространствами пространств мер. Изучены гомотопическая плотность и граничные свойства подмножеств в $P(X)$, а также описаны гомеоморфизмы между различными бесконечномерными структурами.*

Оказалось вероятностные меры $P(X)$ не только объект, но и метод исследования, с помощью которого А.Пелчинский заложил основы теории пространств Дугунджи и пространств Милютина [1-3]. Рассматриваем и геометрические и топологические свойства компактных и $\sigma-$компактных подмножеств пространства $P(X)$ связанное расположенностью друг в друге [2-3], геометрические и топологические свойства граничных, гомотопически плотных подмножеств пространства $P(X)$[2], для подмножеств E, M, N, $P(X) \supset E \supset M \supset N$ пространства $P(X)$ топологической эквивалентности тройки $(P(X), E, N)$, (E, M, N) к тройкам $(Q, S, \sum)$, $(Q, \sum, \sigma)$, $(S, \sum, \sigma)$.

Для бесконечного ANR компакта X изучается топологической эквивалентности пар многообразий $(P_\infty(X), P_\omega(X))$ к парам многообразий (R^∞, σ) и $(Q^\infty, \sum)$.

Напомним, что топологическое пространство Y называется абсолютным (окрестностным) ретрактом в классе K (записывается) $Y \in A(N)R(K)$ [3], если $Y \in K$ и для всякого гомеоморфизма h, отображающего Y на замкнутое подмножество $h(Y)$ пространства X из класса K, множество $h(Y)$ является ретрактом (окрестностным) пространства X.

Напомним, что топологическое пространство X называется многообразием, моделированным на пространстве Y, или $Y-$многообразием [3], если всякая точка пространства X имеет окрестность, гомеоморфную открытому подмножеству пространства X.

Обычно Q-многообразием называют сепарабельное метрическое пространство, локально гомеоморфное гильбертовому кубу Q, где $Q = \prod_{i=1}^{\infty}[-1,1]_i$ гильбертов куб [1], $W_i^{\pm} = \{(g_j) \in Q | g_i = \pm 1\}$ $i-$ая грань гильбертова куба Q, $BdQ = \bigcup_{i=1}^{\infty} W_i^{\pm}-$называется псевдограницей куба Q, а $S = Q \backslash BdQ-$псевдовнутренностью куба Q. Известно, что $S \cong \prod_{i=1}^{\infty}(-1,1)_i$[2].

В теории многообразий бесконечной размерности важную роль играют следующие объекты: гильбертов куб Q, сепарабельное гильбертово пространство ℓ_2, $\sum-$линейная оболочка стандартного кирпича $Q' = \prod_{n=1}^{\infty}[0, \frac{1}{2^n}]$ в гильбертовом пространстве ℓ_2. Через ℓ_2^f или σ обозначается линейное подпространство гильбертова пространство ℓ_2, состоящее из всех точек лишь конечное число координат которых отлично от нуля, а Q^f-подпространство гильбертова куба Q, состоящее из всех точек лишь конечное число координат которых отлично от нуля.

По теореме Андерсона-Кадеца [3] гильбертово пространство ℓ_2 гомеоморфно S. Из результатов Бессаги- Пелчинского следует, что $\sum$гомеоморфно $\mathrm{rint}Q$ и $\sigma \simeq \ell_2^f \simeq Q^f$[1]. Здесь через $\mathrm{rint}Q$ обозначается множество $\{x = (x_n) \in Q | \, |x_n| < t < 1$ для всех $n \in N\}$. Далее, очевидно, что $\mathrm{rint}Q \approx BdQ$, значит верно: $BdQ \approx \sum$.

Известно, что пространства Q,$\sum$ и ℓ_2 сильно бесконечномерны, а пространства ℓ_2^f, σ и Q_f слабо бесконечномерны и все эти пространства однородны.

Определение. Замкнутое множество A пространства X называется Z-множеством в X, если тождественное отображение id_X пространства Xможет быть сколь угодно близко аппроксимировано отображениям $f : X \to X \backslash A$.

Счетное объединение Z-множеств в X называется $\sigma - Z-$множеством в X.

Следуя по [3], $\sigma - Z-$множество, B гильбертова куба Q называют граничным множеством в Q (обозначается через $B(Q)$), если $Q \backslash B \approx \ell_2$. Более общим образом, граничным множеством в Q-многообразии называют $\sigma - Z-$множество, дополнение до которого является ℓ_2-многообразием.

Из приведенного выше следует, что псевдограница BdQ, гильбертова куба Q -является его граничным множеством.

Пусть X топологическое пространство.

Множество $A \subset X$ называется гомотопически плотным в X[2], если существует гомотопия $h(x,t) : X \times [0,1] \to X$ такая, что $h(x,0) = id_X$ и $h(X \times (0,1]) \subset A$.

A множество $A \subset X$ гомотопически пренебрежимо в X, если $X \backslash A$ гомотопически плотно в X.

Вложение $e : Y \to X$ гомотопически плотным (соответственно, гомотопически пренебрежимо), если $e(Y)$ гомотопически плотное множество (соотственно гомотопически пренебрежимо) в X[2].

Для компактов X имеется простая топологическая классификация пространств $P(X)$ всех вероятностных мер. В случае конечного $n-$точечного пространства $X = \{n\}$ точки μ пространства $P(n) = P_n(n)$ являются выпуклыми линейными комбинациями мер Дирака:

$$\mu = m_0\delta(0) + m_1\delta(1) + ... + m_{n-1}\delta(n-1)$$

Поэтому они естественно отождествляются с точками $(n-1)$ мерного симплекса σ^{n-1}. При этом Дирака $\delta(i)$ образуют вершины симплекса, а массы m_i, помещенные в точки i, являются барицентрическими координатами меры μ. Таким, образом, компакт $P(n)$ аффинно гомеоморфен симплексу σ^{n-1}.

В случае бесконечного компакта X пространство $P(X)$ также является компактом [1]. Далее оно, содержа симплексы сколь угодно большого числа измерений, бесконечномерны. По теореме Кэли выпуклый компакт $P(X) \subset R^{C(X)}$ аффинно вкладывается в ℓ_2. Следовательно, по теореме Келлера компакт $P(X)$, как бесконечномерный выпуклый компакт, лежащий в ℓ_2, гомеоморфен гильбертовому кубу $Q = I^{\chi_0}$. С другой стороны пространство $P(X)$ всех вероятностных мер на компакте X называется множеством всех регулярных борелевских вероятностных мер на X, снабженное слабейшей из топологий, для которых непрерывен каждый функционал $f_u : C(X) \to R$, переводящей меру μ в $\mu(U)$ (U открытое в X множество).

Определение [3]. Битопологическим пространством называется пара (X_1, X_2) определенное в одном множестве $X(X_1 = X_2)$ или пространства (X, τ_1, τ_2) с двумя топологиями τ_1 и τ_2 обозначаемое $X_i = (X, \tau_i)$, $i = \overline{1,2}$.

Теорема 1. Пусть X бесконечный компакт на пространстве $P_{\nabla\omega}(X) = P(X)\backslash P_\omega(X)$ существует такие плотные $\Sigma_\omega(\omega)$ и $\ell_2^f(\omega)$ подмножества, что $\Sigma(\omega) \simeq \Sigma$, $\ell_2(\omega) \simeq \ell_2$ и Тройка $(P_{\nabla\omega}(X), \Sigma(\omega), \ell_2^f(\omega)) \simeq (S, \Sigma, \sigma)$.

Предложение 1. Пусть X бесконечный компакт. Для любого непустого подмножества $A \subset X$ отличного от X, подпространство $P(X)\backslash P(A)$ гомотопически плотно в $P(X)$.

Предложение 2. Для любого бесконечного компакта X и любого $n \in N$ подпространство $P(X)\backslash P_n(X)$ гомотопически плотно в $P(X)$.

Предложение 3. Для любого бесконечного компакта X подпространство $P(X)\backslash P_f(X)$ гомотопически плотно в $P(X)$.

Нами было замечено, что для бесконечного компакта X и его всюду плотного подмножества A отличного от X, подпространства $P_\omega(X)$ и $P_\omega(A)$ является граничным множеством компакта $P(X)$. т.е. $P(X)\backslash P_\omega(X)$ и $P(X)\backslash P_\omega(A)$ гомеоморфны ℓ_2. В этом случае можно утверждать следующие:

а) для любого $n \in N$ пространство $P_\omega(X)\backslash P_n(X)$ гомеоморфно $P_\omega(X)$;

б) для любого $n \in N$ пространство $P_\omega(A)\backslash P_n(A)$ гомеоморфно $P_\omega(A)$;

Известно, что $P_f(X)$ компактно и является счетным объединением компактов $P_{f,n}(X)$ которые является подмножеством компакта $P_n(X)$. Следовательно, компакты $P_{f,n}(X)$ есть $Z-$множество в $P_\omega(X)$. В этом случае можно утверждать:

в) для любого бесконечного компакта Xпространство $P_\omega(X)\backslash P_f(X)$ гомеоморфно пространству $P_\omega(X)$.

В работе [1] нами было доказано, что для любого бесконечного компакта X и его любого замкнутого подмножества A отличного от X пространство $S_P(A)$ гомеоморфно гильбертовому пространству ℓ_2.

Если B есть компакт в $S_P(A)$ то B является $Z-$множеством в $S_P(A)$. То по теореме [3] пространство $S_P(A)\backslash B$ гомеоморфно ℓ_2. В частности, $S_P(A)\backslash P(A)$ гомеоморфно ℓ_2.

Теорема 2. Для любого бесконечного метрического компакта X и любого всюду плотного, не открытого, подмножества Y отличного от X подпространство $P(X)\backslash P(Y)$ гомеоморфно пространству $\sum$.

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О некоторых свойствах пространства τ-замкнутых подмножеств

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Пусть X - топологическое T_1-пространство и τ - некоторое бесконечное кардинальное число. Множество всех замкнутых подмножеств пространства X обозначим через $\exp X$. Базой топологии Виеториса, определённой на множестве $\exp X$, является семейство множеств вида:

$$O\langle U_1, U_2, ..., U_n\rangle = \{F : F \in \exp X,\ F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \neq \varnothing, \text{ для } i = 1, ..., n\}, \qquad (1)$$

где U_1, U_2, ..., U_n - открытые подмножества пространства X [1].

В 1980 году I.Juhasz в книге [2] ввёл определение τ-замкнутого множества.

Определение 1 [2] Подмножество $F \subset X$ называется τ-замкнутым в топологическом пространстве X, если для каждого подмножества $B \subset F$ такого, что мощность подмножества B не превосходит бесконечного кардинального числа τ, замыкание множества B в X лежит в F.

В 2016 году О.Г.Окуневым было введено следующее понятие τ-замыкания подмножества.

Определение 2 [3]. τ-замыканием подмножества A топологического пространства X называется множество вида

$$[A]_\tau = \bigcup \left\{\overline{B} : B \subset A, |B| \leq \tau\right\} \qquad (2)$$

Множество называется τ-плотным, если его τ-замыкание совпадает со всем пространством X.

Через $\exp^\tau X$ обозначим семейство всех τ-замкнутых подмножеств пространства X.

Теорема 1. Семейства множеств вида

$$O\langle U_1,...,U_n\rangle = \{F : F \in \exp^\tau X,\ F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \neq \varnothing,\ \text{для всякого } i = \overline{1.n}\} \qquad (3)$$

образуют базу некоторой топологии на множестве $\exp^\tau X$.

Теорема 2. Пусть X есть T_1-пространство и $X_0 \subset X$. Тогда множество $\{F : F \in \exp^\tau X,\ X_0 \subset F\}$ замкнуто в пространстве $\exp^\tau X$.

Утверждение 1. Если $\exp^\tau Y$ всюду плотно лежит в пространстве $\exp^\tau X$, то пространство Y всюду τ-плотно в X.

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О топологической эквивалентности и гомеоморфизме подпространств бесконечномерных многообразий пространств вероятностных мер

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Аннотация. В работе исследованы топологические и $A(N)R$-геометрические свойства пространств вероятностных мер $P(X)$, определенных на бесконечных компактах X, а также их всюду плотных подмножеств $B \subset P(X)$ и подпространства $P_\omega(X)$ мер с конечными носителями. Установлены критерии гомеоморфизма и топологической эквивалентности пар и троек бесконечномерных многообразий моделям (Q^∞, Σ), (R^∞, σ) и (S, Σ, σ). В частности, описано строение многообразий вида $P_\omega(X) \setminus P_\omega(A)$ и $P(A) \setminus P_\omega(A)$, гомеоморфных произведению $\Sigma \times S$.

Для бесконечного ANR компакта X изучается топологической эквивалентности пар многообразий $(P_\infty(X), P_\omega(X))$ к парам многообразий (R^∞, σ) и $(Q^\infty, \sum)$.

Напомним, что топологическое пространство Y называется абсолютным (окрестностным) ретрактом в классе K (записывается) $Y \in A(N)R(K)$ [1], если $Y \in K$ и для всякого гомеоморфизма h, отображающего Y на замкнутое подмножество $h(Y)$ пространства X из класса K, множество $h(Y)$ является ретрактом (окрестностным) пространства X.

Напомним, что топологическое пространство X называется многообразием, моделированным на пространстве Y, или $Y-$многообразием [2], если всякая точка пространства X имеет окрестность, гомеоморфную открытому подмножеству пространства X.

Обычно Q-многообразием называют сепарабельное метрическое пространство, локально гомеоморфное гильбертовому кубу Q, где $Q = \prod_{i=1}^{\infty}[-1,1]_i$ гильбертов куб [3], $W_i^{\pm} =$

$\{(g_j) \in Q | g_i = \pm 1\}$ $i-$ая грань гильбертова куба Q, $BdQ = \bigcup_{i=1}^{\infty} W_i^{\pm}-$называется псевдограницей куба Q, а $S = Q \backslash BdQ-$псевдовнутренностью куба Q. Известно, что $S \cong \prod_{i=1}^{\infty}(-1,1)_i$[3].

В теории многообразий бесконечной размерности важную роль играют следующие объекты: гильбертов куб Q, сепарабельное гильбертово пространство ℓ_2, $\sum -$линейная оболочка стандартного кирпича $Q' = \prod_{n=1}^{\infty}[0, \frac{1}{2^n}]$ в гильбертовом пространстве ℓ_2. Через ℓ_2^f или σ обозначается линейное подпространство гильбертова пространство ℓ_2, состоящее из всех точек лишь конечное число координат которых отлично от нуля, а Q^f-подпространство гильбертова куба Q, состоящее из всех точек лишь конечное число координат которых отлично от нуля.

Определение [1]. Замкнутое множество A пространства X называется Z-множеством в X, если тождественное отображение id_X пространства Xможет быть сколь угодно близко аппроксимировано отображениям $f : X \to X \backslash A$.

Определение [2]. Битопологическом пространством называется пара (X_1, X_2) определенное в одном множестве $X(X_1 = X_2)$ или пространства (X, τ_1, τ_2) с двумя топологиями τ_1 и τ_2 обозначаемое $X_i = (X, \tau_i)$, $i = \overline{1,2}$.

Результаты работы [1-2] показывает, что для бесконечного компакта X битопологическая пара $(P_\infty(X), P_\omega(X))$ является $bi - LEC$ парой (т.е. удовлетворяет условию теоремы 5[1]) в следующих случаях:

а) X конечномерно:

б) $A \subset X$, A есть $F_\sigma-$множество или $A - \sigma-$компактно и $\bar{A} = X$;

в) $U \subset X$, $U-$открыто и $\bar{U} = X$;

г) A-счетно и $\bar{A} = X$;

т.е. а) $(P_\infty(X), P_\omega(A)) \simeq (Q^\infty, \sigma))$;

б) $(P_\infty(X), P_\omega(A)) \simeq (R^\infty, \sigma)$;

в) $(P_\infty(X), P_\omega(U)) \simeq (R^\infty, \sigma)$;

г) $(P_\infty(X), P_\omega(A)) \simeq (R^\infty, \sigma)$.

В следующих случаях, пара $(P_\infty(X), P_\omega(A))$ гомеоморфно пара (Q^∞, Σ), если X есть ANR компакт

а) $P_\omega(X)$ содержит гильбертов куб;

б) X есть содержит гильбертов куб Q;

В частном, случае $(P_\infty(Q), P_\omega(Q)) \simeq (Q^\infty, \Sigma)$.

Мы заметили, что во многих случаях для подмножеств $A \subset X$ бесконечного компакта X подпространство $P_\omega(A)$ гомеоморфно σ. Существует различные бесконечные компакты X и Y для которых $P_\omega(X)$ гомеоморфно $P_\omega(Y)$, но $P_\infty(X) \neq P_\infty(Y)$[1]. т.е. имеет место следующая

Предложение 1 [2]. Для различных компактов X и Y из $P_\infty(X) \simeq P_\infty(Y)$ вытекает $P_\omega(X) \simeq P_\omega(Y)$.

В работе [2] имеется следующая

Теорема 1 [1]. $\Sigma \backslash \sigma \simeq \Sigma \times S$.

Нами было замечено что для бесконечного компакта X, разные всюду плотные подпространства $Y \subset P(X)$ пространства гильбертова куба $P(X)$ является его граничным множеством. Следовательно, $P_\omega(X)$ всегда является граничным множеством компакта $P(X)$. Но подпространства $P_\omega(X)$ бывает гомеоморфным Σ и σ. т.е. для некоторых бесконечных компактов X подпространство $P_\omega(X)$ сохраняет гильбертов куб Q, то $P_\omega(X) \simeq \Sigma$. Но существует всюду плотное подмножества $A \subset X$ отличное от X, что $P_\omega(A)$ есть граничное множество. Но в этом случае, подпространство $P_\omega(X)$ гомеоморфно σ. В этих случаях, подпространство $P_\omega(X) \backslash P_\omega(A)$ является $\Sigma \times S$ многообразием. т.е. имеет место следующая

Теорема 2. Пусть X такой бесконечный компакт, что $P_\omega(X)$ содержит гильбертов куб Q, и $A \subset X$, $A \neq X$, $\bar{A} = X$ и $|A| = \chi_0$. Тогда $P_\omega(X)\backslash P_\omega(A) \simeq \Sigma \times S$.

Следствие 1. Пусть X бесконечный содержащей гильбертов куб Q компакт и $A \subset X$, $A \neq X$, $\bar{A} = X$ и $|A| = \chi_0$. Тогда $P_\omega(X)\backslash P_\omega(A) \simeq \Sigma \times S$.

Следствие 2. $A \subset Q$, A счетно и $\bar{A} = X$ тогда $P_\omega(Q)\backslash P_\omega(A) \simeq \Sigma \times S$.

В работах [2,3] было показано, что для всюду плотного подмножества $A \subset X$ компакта X подпространства $P(A)$, $P_\omega(A)$ является граничным множеством компакта $P(X)$ и $P(A) \cong \Sigma$ и $P_\omega(A) \simeq \sigma$. С другой стороны имеет место: $P_\omega(A) \subset P(A)$. Значит, верно следующая

Теорема 3. Пусть X бесконечный компакт $A_1 \subseteq A_2 \subseteq ... \subseteq A_n \subseteq ...$ замкнутых подмножеств, что $A = \bigcup_{i=1}^{\infty} A_i$ всюду плотно в X. а) $P(A) \simeq \Sigma$, если $P(A)$ содержит гильбертов куб Q; Тогда б) $P(A) \simeq \ell_2^f$, если $P(A)$ не содержит гильбертов куб Q.

Теорема 4. Пусть X бесконечный компакт $A_1 \subset A_2 \subset ... \subset A_n \subset ...$ последовательность замкнутых множеств, что $A \neq X$ и $A = \bigcup_{n=1}^{\infty} A_i$ всюду плотно в X.

Тогда а) $P_\omega(A)$ гомеоморфно ℓ_2^f ; если $P_\omega(A)$ не содержит гильбертов куб Q;
б) $P_\omega(A) \simeq \Sigma$, если $P_\omega(A)$ содержит гильбертов куб Q.

Следствие 3. В условиях теоремы 2. Пусть N счетное всюду плотное подмножество. Тогда $P(A)\backslash P(N) \simeq \Sigma \times S$, если $P(A)$ содержит гильбертов куб Q;

Следствие 4. В условиях теоремы 3. Пусть N счетное всюду плотное подмножество. Тогда $P_\omega(A)\backslash P_\omega(N) \simeq \Sigma \times S$, если $P_\omega(A)$ содержит гильбертов куб Q;

Теорема 5. Пусть X конечномерный компакт, а $A_1 \subset A_2 \subset ... \subset A_n \subset ...$ последовательность его замкнутых подмножеств, что $A \neq X$, $\bar{A} = X$. Тогда $P(A)\backslash P(N) \simeq \sum \times S$, если существует A_{i_0}−бесконечно, где N счетное подмножество в X.

Следствие 5. Пусть X конечномерный компакт, а $A_1 \subset A_2 \subset ... \subset A_n \subset ...$, его такие замкнутые подмножества, что $A = \bigcup_{n=1}^{\infty} A_i$ всюду плотно и A отлично от X.

Тогда $P(A)\backslash P_\omega(A) \simeq \Sigma \times S$ и $P(A)\backslash P_\omega(X) \simeq \Sigma \times S$, если существует индекс i_0, что A_{i_o} - бесконечно;

Предложение 2. Для любого бесконечного компакта X пространства а) $P_\omega(X)$, $S_P(A)$, $P(X)\backslash P(A)$ и $P(X\backslash A)$ гомотопически плотно в $P(X)$, где $A \subset X$, $A \neq \emptyset$, $A \neq X$;
б) Для любого $n \in N$ пространство $P_\omega(X)\backslash P_n(X)$ гомотопически плотно в $P_\omega(X)$; в) Для любого $n \in N$ пространство $P_\omega(X\backslash A)$ гомотопически плотно в $P_\omega(X)$ где $A \subset X$, $A \neq \emptyset$, $A \neq X$;

Теорема 6. Для любого бесконечного компакта X и его замкнутого непустого бесконечного подмножества A отличного от самого компакта X в пространство $S_P(A)$ существует такие плотные $\Sigma_P(A)$ и $\sigma_P(A)$ подмножество, что $\Sigma_P(A) \cong \Sigma$ и $\sigma_P(A) \cong \sigma$, Тройка $(S_P(A), \Sigma_P(A), \sigma_P(A)) \simeq (S, \Sigma, \sigma)$.

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Эргодические теоремы Блума-Хансона для положительных сжатий в пространствах Орлича-Канторовича

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Пусть B произвольная полная булевая алгебра, и пусть $Q(B)$ обозначает ее пространство представлений Стоуна, которое является экстремально несвязным компактом. Пусть $C_\infty(Q(B))$ алгебра всех непрерывных функций $x : Q(B) \to \overline{\mathbb{R}} = [-\infty, +\infty]$, принимающих значения $\pm\infty$ лишь на нигде не плотных множествах из $Q(B)$. Для каждого элемента $x \in C_\infty(Q(B))$ его носитель определяется формулой $s(x) := \sup\limits_{n\geq 1}\{|x| > n^{-1}\}$. Здесь для каждого $\alpha \in \mathbb{R}$ символ $\{|x| > \alpha\}$ означает единичный элемент в B, соответствующий открыто-замкнутому множеству $E_\alpha \subseteq Q(B)$, являющемуся замыканием множества $\{t \in Q(B) : |x(t)| > \alpha\}$.

Пусть $L^0(\Omega) := L^0(\Omega, \Sigma, \mu)$ алгебра всех классов равных почти всюду действительных измеримых функций на измеримом пространстве (Ω, Σ, μ) с σ-конечной мерой μ, и пусть $B(\Omega)$ полная булева алгебр всех идемпотентов из $L^0(\Omega)$.

Пусть $m : B \to L^0(\Omega)$ – строго положительная $L^0(\Omega)$-значная мера на B, обладающая свойством Магарам, т.е. для любых $e \in B,\ f \in L^0(\Omega),\ 0 \leq f \leq m(e)$, существует такое $q \in B,\ q \leq e$, что $m(q) = f$. В этом случае [1], существует единственный инъективный вполне аддитивный булев гомоморфизм $\varphi : B(\Omega) \to B$ такой, что $\nabla(m) = \varphi(B(\Omega))$ есть правильная булева подалгебра в B, и $m(\varphi(q)e) = qm(e)$ для всех $q \in B(\Omega)$, $e \in B$. Кроме того, алгебра $L^0(\Omega)$ отождествляется с подалгеброй $C_\infty(Q(\nabla(m)))$ в алгебре $C_\infty(Q(B))$.

Пусть $1 \leq p < \infty$ и $L^p(B, m)$ пространство всех функций из $C_\infty(Q(B))$, для которых p-я степень абсолютного значения $|x|^p$ является m-интегрируемой. Для каждого $x \in L^p(B, m)$ формула $\|x\|_{p,m} := \left[\int |x|^p dm\right]^{\frac{1}{p}}$ определяет $L^0(\Omega)$-значную норму на $L^p(B, m)$. Поскольку, m – мера Магарам, то пара $(L^p(B, m), \|x\|_{p,m})$ является пространством Банаха-Канторовича над $L^0(\Omega)$ (см. [2]).

Пусть $\Phi : [0, \infty) \to [0, \infty)$ функция Орлича, т.е. возрастающая выпуклая функция удовлетворяющая условиям $\lim\limits_{t\to\infty} \frac{\Phi(t)}{t} = \infty$ и $\lim\limits_{t\to 0} \frac{\Phi(t)}{t} = 0$. Говорят, что функция Орлича Φ удовлетворяет (Δ_2)-условию, если $\sup\limits_{0<t<\infty} \frac{\Phi(2t)}{\Phi(t)} < \infty$.

Для каждой функции $x \in C_\infty(Q(B))$ множество $G = \{t \in Q(B) : -\infty < x(t) < +\infty\}$ всюду плотно и открыто в $Q(B)$. Следовательно, для непрерывной функции $\Phi(x(t)), t \in G$, существует единственное непрерывное продолжение $y(t)$ на $Q(B)$, т.е. $\Phi(x) := y \in C_\infty(Q(B))$. Пусть $\mathcal{P}(\nabla(m))$ – множество всех положительных элементов $\lambda \in C_\infty(Q(\nabla(m)))$ для которых носитель $s(\lambda) = \mathbf{1}$, где $\mathbf{1}$ - единица в булевой алгебре $B(\Omega)$. Очевидно, что для любого $\lambda \in \mathcal{P}(\nabla(m))$ существует $\lambda^{-1} \in \mathcal{P}(\nabla(m))$ такое, что $\lambda \cdot \lambda^{-1} = \mathbf{1}$.

Определим $L^0(\Omega)$-пространство Орлича

$$L_\Phi(B, m) = \{x \in C_\infty(Q(B)) : \Phi(\lambda^{-1}|x|) \in L^1(B, m) \text{ для некоторого } \lambda \in \mathcal{P}(\nabla(m))\}$$

и снабжаем $L_\Phi(B, m)$ одной из "естественных" норм, так называемой $L^0(\Omega)$-значной нормой Люксембурга:

$$\|x\|_\Phi := \inf\{\lambda \in \mathcal{P}(\nabla(m)) : \int \Phi(\lambda^{-1}|x|)dm \leq \mathbf{1},\ x \in L_\Phi(B, m)\}.$$

Известно, что $(L_\Phi(B,m),\|\cdot\|_\Phi)$ является пространством Банаха-Канторовича над $L^0(\Omega)$ и называется пространством Орлича-Канторовича над $L^0(\Omega)$ (см.[3]). Отметим также, что в случае функции Орлича $\Phi(t)=\frac{t^p}{p},\ t\geq 0,\ 1\leq p<\infty$ выполняется равенство $L_\Phi(B,m)=L^p(B,m)$.

Пусть $(X,\|\cdot\|_X)$ и $(Y,\|\cdot\|_Y)$ решеточно нормированные пространства над $L^0(\Omega)$. Линейный оператор $T:X\to Y$ называется $L^0(\Omega)$-*ограниченным,* если существует такой элемент $\mathbf{0}\leq\alpha\in L^0(\Omega)$, что $\|T(x)\|_Y\leq\alpha\|x\|_X$ для всех $x\in X$. Пусть $L_b(X,Y)$ векторное пространство всех $L^0(\Omega)$-ограниченных операторов из X в Y. Каждый оператор $T\in L_b(X,Y)$ имеет $L^0(\Omega)$-значную норму, которая определяется по формуле $\|T\|=\sup\{\|T(x)\|_Y:x\in B_X\}$, где $B_X=\{x\in X:\|x\|_X\leq\mathbf{1}\}$ – единичный шар в $(X,\|\cdot\|_X)$. Если $(X,\|\cdot\|_X)$ разложимое решеточно нормированное пространство, а $(Y,\|\cdot\|_Y)$ пространство Банаха-Канторовича над $L^0(\Omega)$, тогда $(L_b(X,Y),\|\cdot\|)$ также является пространством Банаха-Канторовича над $L^0(\Omega)$ (см. [2, 4.2.6]).

Если $Y=L^0(\Omega)$, тогда $X^*:=L_b(X,L^0(\Omega))$ называется $L^0(\Omega)$-*сопряженным пространством* к решеточно нормированному пространству $(X,\|\cdot\|_X)$. Будем говорить, что последовательность $\{x_n\}$ из X $\sigma(X,X^*)$-*сходится к* $x\in X$, если последовательность $\{T(x_n)\}$ (o)-сходится к $T(x)$ для всех $T\in X^*$.

В случае, когда $Y=X$, через $BL(X):=L_b(X,X)$ обозначим пространство всех $L^0(\Omega)$-ограниченных линейных операторов в $(X,\|\cdot\|_X)$. Если $T\in BL(X)$ и $\|T\|\leq\mathbf{1}$, то T называется *сжатием.*

Определение 1. *(Свойство Блума-Хансона для оператора). Пусть* $(E,\|\cdot\|_E)$ *– пространство Банаха-Канторович над* $L^0(\Omega)$*. Будем говорить, что оператор* $T\in BL(E)$ *обладает свойством Блума-Хансона, если последовательность* $\|\frac{1}{n}\sum_{i=1}^{n}T^{k_i}x\|_E$ (o)*-сходится в решетке* $L^0(\Omega)$ *для каждого* $x\in E$ *и для каждой строго возрастающей последовательности* $(k_i)_{i=1}^{\infty}$ *натуральных чисел.*

Определение 2. *(Эквивалентность Блума-Хансона). Пусть* $(E,\|\cdot\|_E)$ *– пространство Банаха-Канторович над* $L^0(\Omega)$ *и* $(E^*,\|\cdot\|_{E^*})$ *его* $L^0(\Omega)$*-сопряженное пространство. Говорят, что оператор* $T\in BL(E)$ *удовлетворяет эквивалентности Блума-Хансона, если для каждого* $x\in E$ *следующие два условия эквивалентны:*

(A) *Последовательность* $(T^nx)_{n\geq1}$ $\sigma(E,E^*)$*-сходится в* E*.*

(B) *Оператор* T *обладает свойством Блума-Хансона.*

При доказательстве эквивалентности Блума-Хансона импликация $(B)\Rightarrow(A)$ является фактически верным утверждением для всех пространств Банаха-Канторовича над $L^0(\Omega)$.

Предложение 1. *Пусть* T *– оператор в пространстве Банаха-Канторовича* $(E,\|\cdot\|_E)$ *над* $L^0(\Omega)$*. Если* T *обладает свойством Блума-Хансона, то последовательность* $(T^nx)_{n\geq1}$ $\sigma(E,E^*)$*-сходится в* E *для любого* $x\in E$*.*

Напротив, импликация $(A)\Rightarrow(B)$ в общем случае далеко не простая. Мы рассматриваем эту задачу для некоторых конкретных пространств.

Следующая теорема является векторнозначным вариантом теоремы Блума-Хансона [4, Предложение 5.1. (2)] для положительных сжатий, действующих на пространстве Орлича-Канторовича $(L_\Phi(B,m),\|\cdot\|_\Phi)$.

Теорема 1. *Если функция Орлича* Φ *является* C^1*-гладкой и удовлетворяет* (Δ_2)*-условию, то всякое положительное сжатие* T *в* $(L_\Phi(B,m),\|\cdot\|_\Phi)$ *удовлетворяет эквивалентности Блума-Хансона.*

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О решении некоторых аддитивных задач типа Варинга–Гольдбаха с простыми числами из арифметической прогрессии

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На мировом уровне значительная часть научно-практических исследований в области теории чисел посвящена задачам, связанным с простыми числами. В частности, задача представления натуральных чисел в виде суммы квадратов простых чисел из арифметической прогрессии, является одной из актуальных и сложных задач теории чисел. Теоретические основы данного направления восходят к круговому методу, разработанному Г. Харди и Дж. Литтлвудом, а также к усовершенствованному И. М. Виноградовым методу тригонометрических сумм. Эти методы сыграли решающую роль в решении задач голдбаховского типа, в частности, связанных с представлением заданного числа в виде суммы простого числа и сумм квадратов простых чисел.В последние годы интерес к задачам распределения простых чисел в различных арифметических прогрессиях, к представлению заданного числа в виде суммы простого числа и степеней простых чисел из арифметической прогрессии, а также к оценкам количества таких представлений значительно возрос.

В данной статье представлены некоторые результаты, полученные мной в последние годы совместно с моим научным руководителем, профессором Исмаилом Аллаковым, по ряду аддитивных задач типа Варинга–Гольдбаха.

Пользуясь случаем, выражаю глубокую благодарность моему научному руководителю, профессору Исмаилу Аллакову, за оказанную помощь и поддержку при получении данных результатов.

Классический результат Hua Loo-Keng'a [1] о диофантовых квадратных уравнениях гласит, что каждое большое целое число, сравнимое с 5 по модулю 24, представимо в виде суммы 5 простых квадратов. Впоследствии Yonghui Wang [2] доказал, что для простых чисел, принадлежащих арифметической прогрессии

$$\begin{cases} b = p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2, \\ p_i \equiv l_i \,(\mathrm{mod}\, D)\,, \ i = 1, \ldots, 5. \end{cases} \tag{1}$$

диофантово уравнение имеет решение при условиях $D \leq N^{\delta},\ b \equiv 5\,(\mathrm{mod}24)$. Для числа решений уравнения (1), обозначаемого через $r(N)$, впервые была получена следующая оценка [3].

Теорема 1. Если N- достаточно большое число и $Q = N^{21\delta}$, существует эффективная положительная константа δ для $D \leq N^{\delta}$ справедлива оценка

$$r(N) \geq N^{\frac{3}{2}} Q^{-\frac{2}{5}} (\ln N)^{-5}.$$

Расширенным видом уравнения (1) является уравнение

$$\begin{cases} b = a_1p_1^2 + a_2p_2^2 + a_3p_3^2 + a_4p_4^2 + a_5p_5^2 \\ p_j \equiv l_j(\mathrm{mod}D),\ \ 1 \le j \le 5 \end{cases} \tag{2}$$

Для числа решений уравнения (2), обозначаемого через $r(N)$, нами получена следующая оценка [4],[5].

Теорема 2. Пусть $P := N^{\delta}$, где δ достаточно малое положительное число.Тогда существует эффективно вычисляемая положительная постоянная $c > 0$ такая , что для всех достаточно больших $N \ge B$ справедлива следующая оценка:

$$r(N) \gg \frac{N^{\frac{3}{2}}D}{\varphi^5(D)P^{\frac{5}{39}}|a_1\cdots a_5|^{\frac{9}{2}}\ln^5 N}.$$

Здесь $a_1,\ \ldots\ , a_5$ целые числа, удовлетворяющие условиям $a_1\cdots a_5 \neq 0$ и $(a_1, ..., a_5) = 1$ а такжеа $B := \max\{3, D, |a_1|, ..., |a_5|\}$.

Известно, что после доказательства теоремы Ж. Л. Лагранжа, о представлении заданного целого числа в виде суммы квадратов четырех целых чисел первым, кто обратил внимание на задачу представления данного целого числа в виде суммы квадратов четырех простых чисел $p_1, ..., p_4$ был Л. К.Хуа.

$$n = p_1^2 + p_2^2 + p_3^2 + p_4^2 \tag{3}$$

Пусть N-достаточно большое натуральное число и $U(N) = \{n \mid 1 < n \le N,$
$n \equiv 4(\mathrm{mod}24), n \neq p_1^2 + p_2^2 + p_3^2 + p_4^2\}$,
$E(N) = cardU(N)$. Л. К.Хуа доказал, что $E(N) \ll N\log^{-A}N$, где $A > 0-$ некоторая постоянная, $\ll -$ символ Виноградова.

В 2000 году J. Y. Liu и M. C. Liu [3] доказал, что если $N \ge 2$ va $U(N)$ множество положительных целых чисел $n \equiv 4(mod24)$, $n \le N$ которое нельзя представит в виде (3). $E(N) = cardU(N)$ тогда справедлива оценка $E(N) \ll N^{\theta}$, где $\theta > 13/15$. Ранее Хуа Ло-Кен [6] и В. Шварц [7] доказали, что, $E(N) \ll N\log^{-A}N$ для любого $A > 0$. Отметим, также П. Шилдс [8], В. А. Плаксин [9] и Е. Б. Ковальчик [10] получили асимптотическую формулу в этой задаче, которая справедлива для всех $n \equiv 4(mod24)$, $n \le N$, за исключением не более чем $E(N) < N\exp\left(-c_1\sqrt{\ln N}\right)$ значения из них, где c_1-абсолютная постоянная.

Мы исследуем существования решений уравнения (3) в простых числах из арифметической прогрессии. Для удобства введем следующие обозначения:

$$U(N, D) = \left\{\bar{b} \in \mathbb{N}^4 :\ 1 \le b_i \le d,\ (b_i, D) = 1,\ \ b_1^2 + \cdots + b_4^2 \equiv n\,(\mathrm{mod}\sigma(D)D)\right\}.$$

В дальнейшем будем рассматривать $p_i \equiv b_i(\mathrm{mod}\ \ D)$, $i = 1, ..., 4$ и $\bar{b} = (b_1, \ldots, b_4) \in U(N, D)$. Здесь $\sigma(D) = 1, 4, 2$ соответственно означает $2 \nmid D,\ 2 \| D$ и $4|D$. Пусть $S_D(n) -$ количество решений уравнения (3) в простых числах $p_i \equiv b_i(\mathrm{mod}\ \ D)$, $i = 1, ..., 4$; а $E_D(n) -$ количество n $(2 < n \le N)$, которые не представимо в виде суммы четырёх квадратов простых чисел из арифметической прогрессии $p_i \equiv b_i(\mathrm{mod}\ D)$, $i = 1, ..., 4$. Тогда имеют место следующие теоремы [11,12].

Теорема 4. Если $n \equiv 4\,(\mathrm{mod}24)$, $2 \le D \le N^{\delta}$, тогда справедлива оценка $E_D(N) \ll N(Q^{15/14}\,D)^{-1}$.

Метод, используемый в доказательстве теоремы, позволяет получить оценку для $S_d(n)$ при $N/50 < n \le N$.

Теорема 5. Для всех n $(N/50 < n \leq N)$, удовлетворяющие условию $n \equiv 4\,(\mathrm{mod}24)$, за исключением не более чем $N\big(Q^{15/14}\,D\big)^{-1}$ значений, справедлива оценка
$S_D\,(n) \gg n^{1-7\delta}(D^{1/2}\log^4 n)^{-1}$.

Рассмотрим уравнение

$$a_1p_1 + a_2p_2 + a_3p_3^2 = b, \tag{4}$$

где a_1, a_2, a_3, b -целые числа, а p_1, p_2, p_3 простые числа. Если (4) положить $a_1 = a_2 = a_3 = 1$ то получим классическую задачу, поставленную Хуа Ло-Кеном в 1938 г.

Не ограничивая общности, можно считать, что $a_i \neq 0$ при $i = 1, 2, 3$ и $EKUB(a_1, a_2, a_3) = 1$. Пусть $B := \max\{2, |a_1|, |a_2|, |a_3|\}$. Тогда для решения уравнения (4) в простых числах p_i из арифметической прогрессии ($p_i \equiv l_i(\mathrm{mod}D)$,$i = 1, 2, 3$, $D \leq N^{\delta}$) справедлива следующая теорема [13].

Теорема 6. Если a_1, a_2, a_3, b удовлетворяют вышеуказанным условиям. Тогда существует эффективное вычисляемое постоянное число $A > 0$ такое, что справедливы следующие утверждения:

a) если все a_1, a_2, a_3 положительные и $b \geq B^A$, то уравнение (1.1) имеет решение в простых числах p_1, p_2, p_3 из арифметической прогрессии $p_i \equiv l_i(\mathrm{mod}D)$,$i = 1, 2, 3$, $D \leq N^{\delta}$;

b) если не все a_1, a_2, a_3 имеют одинаковой знак, то уравнение (1.1) имеет решение в простых числах p_1, p_2, p_3 из арифметической прогрессии $p_i \equiv l_i(\mathrm{mod}D)$,$i = 1, 2, 3$, $D \leq N^{\delta}$, которые не превосходит $3\,|b| + B^A$.

Теорема 7. Если N достаточно велико, то количество решений уравнения (1.1) в простых числах $NB^{-1} < p_1, p_2, p_3 \leq N$, $p_i \equiv l_i(\mathrm{mod}D)$, $i = 1, 2, 3$, $D \leq N^{\delta}$ не меньше чем $c_1N^{3/2}\big(BQ^{83/420}D\ln^3 N\big)^{-1}$, где c_1 -положительная постоянная, $Q = N^{21\delta}$.

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СВЕДЕНИЕ НЕКОТОРЫХ СЛОЖНЫХ ИНТЕГРАЛОВ К ИНТЕГРАЛАМ ЭЙЛЕРА I И II РОДА

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В математическом анализе встречается большое количество определённых интегралов, непосредственное вычисление которых с помощью элементарных функций оказывается невозможным или крайне затруднительным. Для исследования таких интегралов широко используются специальные функции, среди которых особое место занимают интегралы Эйлера первого и второго рода, известные соответственно как бета- и гамма-функции.

Интегралы Эйлера имеют большое значение не только в теоретическом анализе, но и в теории вероятностей, математической физике, механике и других прикладных областях. Многие интегралы, содержащие степенные, показательные и тригонометрические функции, с помощью подходящих замен переменной могут быть приведены к стандартному виду бета- или гамма-функции. Такой подход позволяет заменить непосредственное сложное интегрирование использованием известных свойств специальных функций.

Цель данной работы — рассмотреть основные виды интегралов, сводимых к интегралам Эйлера первого и второго рода, показать соответствующие замены переменной и продемонстрировать их применение на конкретных примерах.

Интеграл Эйлера I рода (бета-функция) записывается в виде:

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx, \quad (a>0,\, b>0). \tag{1}$$

Интеграл Эйлера II рода (гамма-функция) записывается в виде:

$$\Gamma(a) = \int_0^{+\infty} x^{a-1}e^{-x}\,dx, \quad (a>0). \tag{2}$$

Сведение сложных интегралов к интегралам Эйлера (гамма- и бета-функциям) осуществляется заменой переменных, приводящих их к видам (1) или (2). Это позволяет вычислять интегралы, не выражающиеся в элементарных функциях. Кроме того, часто используется связь между ними:

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

Например, рассмотрим интеграл Пуассона (Гаусса): $I = \int_0^{+\infty} e^{-x^2}\,dx$. Замена переменной $x^2 = t$ ($x = t^{1/2}$, $dx = \frac{1}{2}t^{-1/2}dt$) приводит к следующему результату:

$$\int_0^{+\infty} e^{-x^2}\,dx = \frac{1}{2}\int_0^{+\infty} t^{-\frac{1}{2}}e^{-t}\,dt = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}.$$

Или при вычислении интеграла $J = \int_0^1 \frac{dx}{\sqrt{1-x^4}}$ выполним замену $x^4 = t$, откуда $x = t^{1/4}$, $dx = \frac{1}{4}t^{-3/4}dt$. В итоге получим:

$$\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{1}{4}\int_0^1 t^{-\frac{3}{4}}(1-t)^{-\frac{1}{2}}\,dt = \frac{1}{4}B\left(\frac{1}{4},\,\frac{1}{2}\right) = \frac{1}{4}\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{4}\right)}.$$

Целью данной статьи является рассмотрение способов нахождения замен в более сложных интегралах, для того чтобы студент мог самостоятельно решать подобные задачи. Покажем это на примерах.

Пример. Вычислить интеграл: $K = \int_0^1 x^3(1-x^2)^4\,dx$. Очевидно, что данный интеграл можно свести к бета-функции. Выполним замену $x^2 = t$ ($x = t^{1/2}$, $xdx = \frac{1}{2}dt$):

$$\int_0^1 x^3(1-x^2)^4\,dx = \frac{1}{2}\int_0^1 t(1-t)^4\,dt = \frac{1}{2}B(2,5) = \frac{1}{2}\frac{\Gamma(2)\Gamma(5)}{\Gamma(7)} = \frac{1}{2}\frac{1!\cdot 4!}{6!} = \frac{1}{60}.$$

Понятно, что данный пример не вызывает никаких сложностей. Однако на начальном этапе полезно начать именно с таких «очевидных» примеров, а только после этого можно переходить к более сложным интегралам.

Заключение. Интегралы Эйлера первого и второго рода являются эффективным инструментом для вычисления широкого класса определённых интегралов. Бета-функция позволяет сводить к стандартному виду интегралы, содержащие произведения степенных функций и множители типа $(1-x^k)^p$, а также многие тригонометрические интегралы. Гамма-функция, в свою очередь, особенно удобна для интегралов на бесконечном промежутке с экспоненциальным множителем.

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Средние значение квадратного корня от функции делителей

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Пусть $k \geq 2$ — фиксированное целое число. Обозначим через

$$d_k(n) = \sum_{\substack{n_1\cdots n_k = n \\ n_j \geq 1}} 1$$

обобщённую функцию делителей порядка k. Рассмотрим сумматорную функцию

$$D_k(x) := \sum_{n\le x} d_k(n).$$

Задача получения асимптотической формулы для $D_k(x)$ при $k = 2$ называется проблемой делителей Дирихле, а при $k \ge 3$ — многомерной проблемой делителей Дирихле. Для $k \ge 2$ эта асимптотическая формула имеет вид

$$D_k(x) = xP_{k-1}(\ln x) + \Delta_k(x), \tag{1}$$

где $P_{k-1}(t)$ — многочлен степени $k-1$, а $\Delta_k(x)$ — остаточный член.

В настоящей работе рассматривается одно из обобщений многомерной проблемы делителей. А именно, получена асимптотическая формула с остаточным членом для среднего значения квадратного корня из функции делителей четвёртого порядка. Это представление позволяет применить метод комплексного интегрирования, а также современные оценки дзета-функции Римана.

Лемма 1. Пусть ряд (1) для $f(s)$ абсолютно сходится при $\sigma > 1$, $|a_n| \le A(n)$, где $A(n) > 0$ — монотонно возрастающая функция n, и при $\sigma \to 1+0$

$$\sum_{n=1}^{\infty} |a_n| n^{-\sigma} = O\big((\sigma-1)^{-\alpha}\big), \quad \alpha > 0.$$

Тогда при любых $b_0 \ge b \ge 1$, $T \ge 1$, $x = N + \frac{1}{2}$, имеет место формула

$$\Phi(x) = \sum_{n\le x} a_n = \frac{1}{2\pi i}\int_{b-iT}^{b+iT} f(s)\frac{x^s}{s}\,ds + O\left(\frac{x^b}{T(b-1)^{\alpha}}\right) + O\left(\frac{xA(2x)\log x}{T}\right),$$

где постоянная в символе O зависит только от b_0.

Доказательство см.[1]

Лемма 2. Пусть $T > T_0$, тогда при $0.5 \le \sigma \le 2$ равномерно имеет место соотношение

$$\int_1^T |\zeta(\sigma+it)|^2\,dt \le cT\min\left(\ln T, \frac{1}{\sigma-0.5}\right).$$

Доказательство см.[2]

Лемма 3. Справедливо соотношение

$$f(s) = \sum_{n=1}^{\infty} \frac{\sqrt{d_4(n)}}{n^s} = \zeta^2(s)\Phi(s),$$

где $\zeta(s)$ — дзета-функция Римана, $\Phi(s)$ — регулярная функция в области $\operatorname{Re} s > 0.5$ и

$$\Phi(s) = \prod_p \left(1 - \frac{3}{p^{2s}} + \frac{2}{p^{3s}} + \left(1-\frac{1}{p^s}\right)^2 \sum_{k=2}^{\infty} \sqrt{\binom{k+3}{3}}\, p^{-ks}\right).$$

Доказательство см.[3]

Теорема. Справедлива следующая асимптотическая формула:

$$\sum_{n\le x} \sqrt{d_4(n)} = x\big(A\ln x + B\big) + O\Big(x^{\frac{1}{2}}\ln^2 x\Big),$$

где

$$A = \Phi(1), \qquad B = 2\Phi(1)\int_1^{\infty} \frac{\rho(u)}{u^2}\,du + \Phi'(1).$$

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О КОЛИЧЕСТВЕ НЕПОДВИЖНЫХ ТОЧЕК КВАРТИЧЕСКОГО СТОХАСТИЧЕСКОГО ОПЕРАТОРА

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В данной работе исследуется строго положительный стохастический оператор, действующий на одномерном симплексе. Данный оператор находит важное применение в популяционной генетике, где он описывает эволюцию частот аллелей в последовательных поколениях [1–3].

В m-мерном вещественном векторном пространстве $\mathbb{R}^m$ определим множество

$$S^{(m-1)}=\left\{(x_1,x_2,...,x_m)\in\mathbb{R}^m:\sum_{i=1}^{m}x_i=1,x_i\geq 0,i=1,\dots,m\right\},$$

где $S^{(m-1)}-$ это $(m-1)$-мерный базисный симплекс в $\mathbb{R}^m$.

Рассмотрим количество неподвижных точек стохастического оператора $V_4:S^1\to S^1$ на одномерном симплексе в случае, когда все коэффициенты оператора строго положительные, т.е.

$$V_4(\mathbf{x})=\left(\sum_{k=0}^{4}\binom{4}{k}p_{k(1,2),1}x_1^{4-k}x_2^k,\sum_{k=0}^{4}\binom{4}{k}p_{k(1,2),2}x_1^{4-k}x_2^k\right),\quad \mathbf{x}=(x_1,x_2)\in S^1$$

где $\binom{4}{k}=\frac{4!}{(4-k)!k!}$, $p_{k(1,2),1}>0$, $p_{k(1,2),2}>0$, $p_{k(1,2),1}+p_{k(1,2),2}=1$, $k(1,2)=\underbrace{11...1}_{4-k}\underbrace{22...2}_{k}$, $k=\overline{0,4}$.

С биологической точки зрения, неподвижные точки соответствуют стабильным генетическим состояниям популяции, а их количество определяет возможное число устойчивых генетических конфигураций.

Поскольку в литературе полином четвёртой степени часто называют квартикой, естественно соответствующий оператор называть квартическим.

Определим полином четвертой степени

$$P_4(x)=b_0x^4+b_1x^3+b_2x^2+b_3x+b_4,$$

где

$$b_0=p_{1111,1}-4p_{1112,1}+6p_{1122,1}-4p_{1222,1}+p_{2222,1},\quad b_1=4p_{1112,1}-12p_{1122,1}+12p_{1222,1}-4p_{2222,1},$$

$$b_2=6p_{1122,1}-12p_{1222,1}+6p_{2222,1},\quad b_3=4p_{1222,1}-4p_{2222,1}-1,\quad b_4=p_{2222,1}.$$

Введем следующие обозначения, предполагая $b_0 \neq 0$:

$$p = \frac{8b_0b_2 - 3b_1^2}{16b_0^2}, \quad q = \frac{b_1^3 - 4b_0b_1b_2 + 8b_0^2b_3}{32b_0^3},$$

$$Q = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2.$$

Теорема 1. *Если $Q \geq 0$, то строго положительный квартический стохастический оператор V_4 имеет единственную неподвижную точку в S^1.*

В случае $Q < 0$ определим

$$\theta_l = 2\sqrt{-\frac{p}{3}}\cos\left(\frac{\alpha + 2\pi(l-2)}{3}\right), \quad l = \overline{1,2,3},$$

$$\cos\alpha = -\frac{q}{2}\left(-\frac{3}{p}\right)^{\frac{3}{2}}, \quad \alpha \in [0,\pi].$$

Обозначим

$$\gamma_1 = \theta_3 - \frac{b_1}{4b_0}, \quad \gamma_2 = \theta_1 - \frac{b_1}{4b_0}, \quad \gamma_3 = \theta_2 - \frac{b_1}{4b_0}.$$

Множество неподвижных точек оператора V_4 в S^1 обозначим через $\mathrm{Fix}(V_4)$, т.е.

$$\mathrm{Fix}(V_4) = \left\{\mathbf{x} \in S^1 : V_4(\mathbf{x}) = \mathbf{x}\right\}.$$

Теорема 2. *Пусть выполнены условия $Q < 0$, $\gamma_1 > 0$ и $\gamma_2 < 1$. Тогда количество неподвижных точек $|\mathrm{Fix}(V_4)|$ строго положительного квартического стохастического оператора V_4 в S^1 равно:*

$$|\mathrm{Fix}(V_4)| = \begin{cases} 1, & \text{если} \quad b_0\prod\limits_{i=1}^{3} P_4(\gamma_i) < 0; \\ 2, & \text{если} \quad \prod\limits_{i=1}^{3} P_4(\gamma_i) = 0; \\ 3, & \text{если} \quad b_0\prod\limits_{i=1}^{3} P_4(\gamma_i) > 0. \end{cases}$$

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АСИМПТОТИКА СОБСТВЕННЫХ ЗНАЧЕНИЙ ВОЗМУЩЕННОГО ОДНОМЕРНОГО ДИСКРЕТНОГО БИЛАПЛАСИАНА

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Рассмотрим семейство самосопряжённых ограниченных операторов

$$\widehat{H}_\mu = \widehat{\Delta}\widehat{\Delta} - \mu\widehat{V}, \qquad \mu > 0,$$

действующих в гильбертовом пространстве $\ell^2(\mathbb{Z})$, где $\widehat{\Delta}$ — дискретный лапласиан, а $\widehat{V}$ — конечномерное возмущение ранга три.

Исследование спектральных свойств дискретных операторов высокого порядка представляет интерес в связи с изучением решёточных моделей квантовой механики и пороговых эффектов для дискретных операторов. В данной работе основное внимание уделяется собственным значениям, возникающим из нижнего края существенного спектра возмущённого дискретного билапласиана.

В импульсном представлении оператор $\widehat{H}_\mu$ унитарно эквивалентен оператору

$$H_\mu = H_0 - \mu V$$

в пространстве $L^2(\mathbb{T})$, где $\mathbb{T} = (-\pi, \pi]$,

$$(H_0 f)(p) = (1 - \cos p)^2 f(p),$$

а оператор возмущения V определяется формулой

$$(V f)(p) = \frac{1}{2\pi}\int_{\mathbb{T}} \big(a + 2b\cos(p-q)\big) f(q)\, dq, \qquad a > 0, \quad b > 0.$$

Оператор V имеет ранг три, а его образ порождается функциями 1, $\cos p$ и $\sin p$. Поэтому спектральная задача для H_μ сводится к исследованию конечномерной системы уравнений.

Существенный спектр оператора H_μ имеет вид

$$\sigma_{\text{ess}}(H_\mu) = [0, 4].$$

Следовательно, отрицательные собственные значения H_μ образуют дискретный спектр и могут накапливаться только к нижнему краю существенного спектра $z = 0$.

Для $z \in \mathbb{C} \setminus [0, 4]$ положим

$$I_j(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{\cos^j q}{(1-\cos q)^2 - z}\, dq, \qquad j = 0, 1, 2,$$

и

$$I_s(z) = \frac{1}{2\pi}\int_{\mathbb{T}} \frac{\sin^2 q}{(1-\cos q)^2 - z}\, dq.$$

Число $z \notin [0, 4]$ является собственным значением оператора H_μ тогда и только тогда, когда

$$\Delta(\mu; z) = 0,$$

где

$$\Delta(\mu;z)=\Delta_e(\mu;z)\Delta_o(\mu;z),$$
$$\Delta_e(\mu;z)=(1-\mu aI_0(z))(1-2\mu bI_2(z))-2\mu^2 abI_1^2(z),$$
$$\Delta_o(\mu;z)=1-2\mu bI_s(z).$$

Функции $\Delta_e(\mu;z)$ и $\Delta_o(\mu;z)$ соответствуют соответственно чётной и нечётной частям пространства $L^2(\mathbb{T})$. Тем самым задача о дискретном спектре оператора H_μ распадается на два независимых спектральных канала.

Основные результаты получены при исследовании поведения нулей функций $\Delta_e(\mu;z)$ и $\Delta_o(\mu;z)$ в окрестности нижнего края существенного спектра.

Теорема. Пусть $a>0$ и $b>0$. Тогда при всех достаточно малых $\mu>0$ оператор H_μ имеет два отрицательных собственных значения $e_1(\mu)$ и $e_2(\mu)$. При $\mu\to 0+$ имеют место асимптотические формулы

$$e_1(\mu)=-\left(\frac{a+2b}{2}\right)^{4/3}\mu^{4/3}+O(\mu^2),$$

и

$$e_2(\mu)=-16b^4\mu^4+128b^5\mu^5+O(\mu^6).$$

Кроме того, число

$$\mu_0=\frac{1}{2b}$$

является критическим значением константы связи для второй ветви в чётном спектральном канале. При $\mu>\mu_0$, достаточно близких к μ_0, существует отрицательное собственное значение $e_3(\mu)$, причём

$$e_3(\mu)\sim -16b^4\left(\mu-\frac{1}{2b}\right)^4,\qquad \mu\downarrow\mu_0+.$$

Таким образом, два собственных значения, существующих при малых значениях константы связи, имеют существенно различные асимптотические порядки:

$$|e_1(\mu)|\asymp\mu^{4/3},\qquad |e_2(\mu)|\asymp\mu^4.$$

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ГЕОМЕТРИЯ КУБИЧЕСКИХ ВЕКТОРНЫХ ПОЛЕЙ НА S^3 И $\mathbb{RP}^3$

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Рассмотрим единичную сферу $S^3 \subset \mathbb{R}^4$ и положим $r_1^2 = x_1^2 + x_2^2$, $r_2^2 = x_3^2 + x_4^2$, $c = r_1^2$. Обозначим через

$$A_1 = -x_2\frac{\partial}{\partial x_1} + x_1\frac{\partial}{\partial x_2}, \qquad A_2 = -x_4\frac{\partial}{\partial x_3} + x_3\frac{\partial}{\partial x_4},$$

которые являются образующми стандартного действия тора $T^2 = S^1 \times S^1$. Рассмотрим следующее двухпараметрическое семейство

$$X_{\alpha,\beta} = \big(1 + \alpha(1-c)\big)A_1 + \big(1+\beta c\big)A_2 = \omega_1(c)A_1 + \omega_2(c)A_2, \qquad \alpha,\beta \in \mathbb{R}. \tag{1}$$

векторных полей на $\mathbb{R}^4$, которые являются кубическими. Эти поля нечётны относительно антиподального отображения и потому корректно индуцируют векторные поля на $\mathbb{RP}^3$. Динамическая структура семейства была описана в [1].

Функция c является первым интегралом. При $0 < c < 1$ её уровни

$$T_c = \{x \in S^3 : r_1^2 = c\}$$

суть торы Клиффорда, а в угловых координатах поток поля (1) имеет вид

$$\Phi_t(c, \theta_1, \theta_2) = \big(c, \theta_1 + \omega_1(c)t, \theta_2 + \omega_2(c)t\big). \tag{2}$$

Теорема. Для семейства (1) справедливы следующие утверждения.

1. Относительно стандартной индуцированной римановой метрики на S^3 ковариантное ускорение равно

$$\nabla_{X_{\alpha,\beta}} X_{\alpha,\beta} = \big(\omega_1(c)^2 - \omega_2(c)^2\big)\big(cu_2 - (1-c)u_1\big), \tag{3}$$

где $u_1 = (x_1, x_2, 0, 0)$ и $u_2 = (0, 0, x_3, x_4)$. Следовательно, орбита на T_c геодезична тогда и только тогда, когда $\omega_1(c)^2 = \omega_2(c)^2$. При $\alpha + \beta \neq 0$ первая ветвь $\omega_1 = \omega_2$ даёт

$$c = c_+ = \frac{\alpha}{\alpha+\beta} \in (0,1) \quad \Longleftrightarrow \quad \alpha\beta > 0,$$

а при $\alpha \neq \beta$ вторая ветвь $\omega_1 = -\omega_2$ даёт

$$c = c_- = \frac{\alpha+2}{\alpha-\beta} \in (0,1) \quad \Longleftrightarrow \quad (\alpha+2)(\beta+2) < 0.$$

При $(\alpha,\beta) = (0,0)$ выполнено $\omega_1 = \omega_2 = 1$ при всех c, и геодезическими являются все регулярные орбиты. На указанных торах ненулевые орбиты являются большими окружностями двух противоположных расслоений Хопфа. Равенство $\nabla_{X_{\alpha,\beta}} X_{\alpha,\beta} \equiv 0$ на всей сфере выполнено ровно при

$$(\alpha,\beta) = (0,0) \qquad \text{или} \qquad (\alpha,\beta) = (-2,-2). \tag{4}$$

2. Для любых $\alpha, \beta \in \mathbb{R}$ существует глобальная гладкая T^2-инвариантная метрика

$$\widehat{g}_{\alpha,\beta} = \left[\left(1 + \beta r_1^2 + \frac{\beta^2 r_1^4}{3}\right)(dx_1^2 + dx_2^2) + \left(1 + \alpha r_2^2 + \frac{\alpha^2 r_2^4}{3}\right)(dx_3^2 + dx_4^2)\right]\Big|_{TS^3}, \tag{5}$$

для которой $\nabla^{\widehat{g}_{\alpha,\beta}}_{X_{\alpha,\beta}} X_{\alpha,\beta} = 0$. Метрика (5) спускается на $\mathbb{RP}^3$. Поле $X_{\alpha,\beta}$ обращается в нуль тогда и только тогда, когда $\alpha\beta + \alpha + \beta = 0$, $\alpha < -1$ и $\beta < -1$; во всех остальных случаях

$$Y = \frac{X_{\alpha,\beta}}{\|X_{\alpha,\beta}\|_{\widehat{g}_{\alpha,\beta}}}$$

является единичным геодезическим полем, и потому ориентированное одномерное слоение, порождённое орбитами $X_{\alpha,\beta}$, является вполне геодезическим. При $\alpha > -1$, $\beta > -1$ явная геодезирующая форма задаётся равенством

$$\eta = \left(1 + \frac{\beta r_1^2}{2}\right)(x_1 dx_2 - x_2 dx_1) + \left(1 + \frac{\alpha r_2^2}{2}\right)(x_3 dx_4 - x_4 dx_3); \tag{6}$$

для неё $\eta(X_{\alpha,\beta}) > 0$ и $\iota_{X_{\alpha,\beta}} d\eta = 0$, где $\iota_X d\eta$ есть 1-форма $Y \mapsto d\eta(X, Y)$. По критерию Уодсли и Салливана существование такой формы равносильно геодезируемости порождённого полем слоения, см. [2, 3].

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Число целых точек в области ограниченной петлей лист Декарта

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Основным результатом этой работы является вывод асимптотической формулы для число целых точек в области, ограниченной петлёй декартова листа с остаточным членом. Декартов лист это плоская алгебраическая кривая третьего порядка, задаваемая в декартовых прямоугольных координатах следующим уравнением:

$$x^3 + y^3 - 3Nxy = 0,$$

где параметр $3N$ определяется как диагональ квадрата, сторона которого равна наибольшей хорде петли.

В теорема 1, для числа целых точек в области, ограниченной петлёй декартова листа выводится точная формула.

Теорема 1. *Пусть N — натуральное число, тогда для $T(N)$ — количества целых точек в области, ограниченной петлёй декартова листа $x^3 + y^3 - 3Nxy = 0$, $x > 0$, $y > 0$, справедлива формула*

$$T(N) = \frac{3}{2}N^2 + \Delta(N), \qquad \Delta(N) = (2\mathbb{R}_0(N) - \mathbb{R}_1(N) + \mathbb{R}_2(N)) + c,$$

где c — абсолютная постоянная, причём $-4.25 \le c \le 2.8$, а суммы $\mathbb{R}_1(N)$ и $\mathbb{R}_2(N)$ определяются соответственно соотношениями

$$\mathbb{R}_1(N) = \sum_{0<x\le\frac{3N}{2}} \rho\big(y(x)\big), \qquad \mathbb{R}_2(N) = \sum_{\frac{3(\sqrt{5}-1)N}{4}<x\le\frac{3N}{2}} \rho\big(y(x)\big),$$

где функция $y = y(x)$ в некоторой окрестности каждой точки (x, y) петли декартова листа локально представляет собой график функции, при этом в сумме $\mathbb{R}_1(N)$ точка (x, y) движется против часовой стрелки, а в сумме $\mathbb{R}_2(N)$ — по часовой стрелке, а $\mathbb{R}_0(N)$ — число целых точек лежащих в куске петли лист Декарта, соединяющие по направлению против часовой стрелки начало координат и её точку пересечения с прямым $x = \frac{3N}{2}$.

Пусть $F(x, y) \in \mathbb{Z}[x, y]$ — неприводимый многочлен степени $d \geq 2$ и $C = \big\{(x, y) \in \mathbb{R}^2 : \ F(x, y) = 0\big\}$ — соответствующая вещественная алгебраическая кривая. Для $B \geq 1$ обозначим через

$$N_C(B) = \#\big\{(x, y) \in \mathbb{Z}^2 : \ F(x, y) = 0, \ |x| \leq B, \ |y| \leq B\big\}$$

число её целых точек в квадрате $[-B, B]^2$.

В 1989 году Э. Бомбьери и Дж. Пилы [1] доказали: *для любого сколь угодно малого положительного ε существует постоянная $c(F, \varepsilon) > 0$ такая, что*

$$N_C(B) \leq c(F, \varepsilon) B^{\frac{1}{d}+\varepsilon}.$$

Иными словами, для любой неприводимой вещественной алгебраической кривой степени $d \geq 2$ число её целых точек в квадрате $|x|, |y| \leq B$ растёт не быстрее, чем $B^{\frac{1}{d}+\varepsilon}$ при любом $\varepsilon > 0$. В частности, для кубической кривой ($d = 3$) справедлива оценка

$$N_C(B) \ll_\varepsilon B^{\frac{1}{3}+\varepsilon}.$$

Теорема Э. Бомбьери и Дж. Пилы играет фундаментальную роль в теории целых точек на алгебраических кривых.

При натуральном числе N рассмотрим петлю декартова листа, то есть кривую

$$C_N : \ x^3 + y^3 - 3Nxy = 0, \qquad x > 0, \qquad y > 0.$$

являющиеся неприводимым алгебраическим кривым степени $d = 3$. По теореме Бомбьери–Пила имеем

$$\#\big\{(x, y) \in \mathbb{Z}^2 : \ (x, y) \in C_N, \ |x| \leq B, \ |y| \leq B\big\} \leq cB^{\frac{1}{3}+\varepsilon},$$

где $c = c(\varepsilon)$ абсолютная постоянная зависящая только от ε.

На петле декартова листа координаты x и y имеют порядок N, поэтому $B \asymp N$, и, следовательно,

$$D(N) \ll_\varepsilon cN^{\frac{1}{3}+\varepsilon}.$$

Следующая теорем посвящена уточнению этой оценки для петли декартова листа, являющиеся неприводимым алгебраическим кривым степени $d = 3$.

Теорема 2. *Пусть N — достаточно большое натуральное число, $\varepsilon > 0$ — сколь угодно малое положительное число. Тогда для $D(N)$ — числа целых решений уравнения*

$$x^3 + y^3 - 3Nxy = 0, \qquad x > 0, \quad y > 0,$$

справедлива оценка

$$D(N) \leq 2\tau_3(3N),$$

где $\tau_3(n)$ — число представлений n в виде произведения трёх натуральных чисел.

Из этой леммы и известной оценки $\tau_3(n) \ll_\varepsilon n^\varepsilon$ следует, что

$$D(N) \ll_\varepsilon N^\varepsilon.$$

Пользуясь модифицированным методом «стаканчиков И. М. Виноградова», которого разработали Г. И. Архипов и В. Н. Чубариков [2], а также пользуясь теоремой 1 оценки сумм $\mathbb{R}_1(N)$ и $\mathbb{R}_2(N)$ в остаточном члене $\Delta(N)$ в теореме 1 сведем к оценкам специальных тригонометрических сумм $W_1(h,N)$ b $W_2(h,N)$.

Теорема 3. *Пусть N — достаточно большое натуральное число, $M > 100$ — натуральное число и $1 \le H < M \ln M$. Тогда для $T(N)$ — количество целых точек в области, ограниченной петли лист Декарта $x^3+y^3-3Nxy=0$, $x>0$, $y>0$, справедлива формула:*

$$T(N) = \frac{3}{2}N^2 + \Delta(N),$$
$$\Delta(N) \ll \left(\frac{N}{M} + \max_{1\le h\le H}|W_1(h,N)| + \max_{1\le h\le H}|W_2(h,N)|\right)\ln^2 M + \tau_3(3N),$$
$$W_1(h,N) = \sum_{0<x\le\frac{3N}{2}} e(hy(x)), \qquad W_2(h,N) = \sum_{\frac{3(\sqrt{5}-1)N}{4}<x\le\frac{3N}{2}} e(hy(x)),$$

причём функция $y=y(x)$ в некоторой окрестности каждой точки (x,y) петли декартова листа локально представляет собой график функции, при этом в сумме $W_1(h,N)$ точка (x,y) движется против часовой стрелки, а в сумме $W_1(h,N)$ — по часовой стрелке.

Оценивая тригонометрические суммы $W_1(h,N)$ и $W_2(h,N)$ по величине производного второго порядка докажем следующую теорему.

Теорема 4. *Пусть N — натуральное число, тогда для $T(N)$ — количества целых точек в области, ограниченной петлёй декартова листа*

$$x^3+y^3-3Nxy=0, \qquad x>0,\ y>0,$$

справедлива асимптотическая формула

$$T(N) = \frac{3}{2}N^2 + O\left(N^{\frac{2}{3}}(\ln N)^{\frac{7}{3}}\right).$$

где постоянная под знаком $\ll$ является абсолютной величиной.

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Обобщение тернарной проблемы Эстермана с почти пропорциональными слагаемыми

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Эстерман [1] при $n=2$ доказал асимптотическую формулу для числа решений уравнения

$$p_1+p_2+m^n=N, \tag{1}$$

где p_1, p_2 — простые числа, m — натуральное число. В работах [2, 3, 4] при $n = 2, 3, 4$ эта задача исследована с более жесткими условиями, а именно, когда слагаемые почти равны, то есть выведена асимптотическая формула для числа решений диофантова уравнения (1) с условиями

$$\left|p_i - \frac{N}{3}\right| \le H, \quad i = 1, 2, \qquad \left|m^n - \frac{N}{3}\right| \le H, \qquad H \ge N^{1-\theta(n)}\mathscr{L}^{c_n}, \qquad \mathscr{L} = \ln N,$$

соответственно при

$$\theta(2) = \frac{1}{4}, \quad c_2 = 2; \qquad \theta(3) = \frac{1}{6}, \quad c_3 = 3; \qquad \theta(4) = \frac{1}{12}, \quad c_4 = \frac{40}{3}. \tag{2}$$

Доклад посвящен выводу асимптотической формулы в обобщении проблемы Эстермана с почти пропорциональными слагаемыми при произвольном фиксированном $n \ge 3$.

Теорема 1. Пусть N — достаточно большое натуральное число, $n \ge 3$ — фиксированное натуральное число, $\rho(N, p)$ — число решений сравнения $x^n \equiv N \pmod p$, μ_1, μ_2, μ_3 — положительные фиксированные числа, удовлетворяющие условию $\mu_1 + \mu_2 + \mu_3 = 1$, $J_n(N, H)$ — число решений диофантова уравнения (1) с условиями

$$|p_k - \mu_k N| \le H, \quad k = 1, 2, \qquad |m^n - \mu_3 N| \le H.$$

Тогда, при $H \ge N^{1-\frac{1}{n(n-1)}}\mathscr{L}^{\frac{2^{n+1}}{n-1}+n-1}$, справедлива асимптотическая формула:

$$J_n(N, H) = \frac{3\mathfrak{S}(N)H^2}{n\mu_3^{1-\frac{1}{n}}N^{1-\frac{1}{n}}\mathscr{L}^2} + O\left(\frac{H^2}{N^{1-\frac{1}{n}}\mathscr{L}^3}\right), \qquad \mathfrak{S}(N) = \prod_p \left(1 + \frac{\rho(N, p)}{(p-1)^2}\right)$$

где постоянная под знаком O зависит от чисел μ_1, μ_2, μ_3 и n.

Из теоремы 0.1 при $\mu_1 = \mu_2 = \mu_3 = \frac{1}{3}$ следует асимптотическая формула в обобщении проблемы Эстермана с почти равными слагаемыми.

Следствие 1. Пусть N — достаточно большое натуральное число, $n \ge 3$ — фиксированное натуральное число, $\rho(N, p)$ — число решений сравнения $x^n \equiv N \pmod p$, $J_n(N, H)$ — число решений диофантова уравнения (1) с условиями

$$\left|p_k - \frac{N}{3}\right| \le H, \quad k = 1, 2, \qquad \left|m^n - \frac{N}{3}\right| \le H.$$

Тогда, при $H \ge N^{1-\frac{1}{n(n-1)}}\mathscr{L}^{\frac{2^{n+1}}{n-1}+n-1}$, справедлива асимптотическая формула:

$$J_n(N, H) = \frac{3^{2-\frac{1}{n}}\mathfrak{S}(N)H^2}{nN^{1-\frac{1}{n}}\mathscr{L}^2} + O\left(\frac{H^2}{N^{1-\frac{1}{n}}\mathscr{L}^3}\right), \qquad \mathfrak{S}(N) = \prod_p \left(1 + \frac{\rho(N, p)}{(p-1)^2}\right),$$

где постоянная под знаком O зависит только от n.

Заметим, что полученные ранее в работах [3, 4] асимптотические формулы в обобщении проблемы Эстермана с почти равными слагаемыми при $n = 3$ и $n = 4$, приведенные нами в формуле (2), являются частными случаями следствия 1.

Доказательство теоремы 1 проводится круговым методом Харди-Литтлвуда-Рамануджана в форме тригонометрических сумм И. М. Виноградова в сочетании с методом, описанным в работах [5, 6, 7, 8, 9], где исследовано поведение короткой суммы Г. Вейля вида

$$T(\alpha; x, y) = \sum_{x-y<m\le x} e(\alpha m^n),$$

в больших дугах и доказана асимптотическая формула в обобщении проблемы Варинга с почти пропорциональными слагаемыми. Основными утверждениями, позволившими доказать теорему 1, являются:

- асимптотическая формула для коротких тригонометрических сумм Г. Вейля вида $T(\alpha; x, y)$ в малой окрестности центра больших дуг (следствие 2 леммы 1);
- нетривиальная оценка сумм $T(\alpha; x, y)$ в больших дугах за исключением малой окрестности их центров (следствие 3 леммы 1);
- нетривиальная оценка сумм $T(\alpha; x, y)$ в малых дугах (теорема 2).

Лемма 1. [5]. Пусть $\alpha = \frac{a}{q} + \lambda, \quad (a, q) = 1$, а $\tau \ge 2n(n-1)x^{n-2}y$, тогда при $\{n|\lambda|x^{n-1}\} \le \frac{1}{2q}$ имеет место формула

$$T(\alpha; x, y) = \frac{S(a, q)}{q} T(\lambda; x, y) + O\left(q^{\frac{1}{2}+\varepsilon}\right),$$

а при $\{n|\lambda|x^{n-1}\} > \frac{1}{2q}$ имеет место оценка

$$|T(\alpha; x, y)| \ll q^{1-\frac{1}{n}} \ln q + \min(yq^{-\frac{1}{n}}, \lambda^{-\frac{1}{2}} x^{1-\frac{n}{2}} q^{-\frac{1}{n}}).$$

Следствие 2. Пусть $\tau \ge 2n(n-1)x^{n-2}y$, $|\lambda| \le \frac{1}{2nqx^{n-1}}$, тогда имеет место соотношение

$$T(\alpha; x, y) = \frac{y}{q} S(a, q)\gamma(\lambda; x, y) + O(q^{\frac{1}{2}+\varepsilon}).$$

Следствие 3. Пусть $\tau \ge 2n(n-1)x^{n-2}y$, $\frac{1}{2nqx^{n-1}} < |\lambda| \le \frac{1}{q\tau}$, тогда имеет место оценка

$$T(\alpha; x, y) \ll q^{1-\frac{1}{n}} \ln q + \min\left(yq^{-\frac{1}{n}}, x^{\frac{1}{2}} q^{\frac{1}{2}-\frac{1}{n}}\right).$$

При решении ряда аддитивных задач с почти пропорциональными слагаемыми, к которым относятся проблема Варинга и обобщение проблемы Эстермана, возникают задачи о поведении коротких тригонометрических сумм Г. Вейля вида $T(\alpha; x, y)$ как в больших так и в малых дугах. Для произвольного $n \ge 3$ в больших дугах они изучены в работах [5, 8]. Для тригонометрических сумм $T(\alpha; x, y)$ в малых дугах при $q \gg y^{\varepsilon}$ были получены нетривиальные оценки в работе [10].

Воспользовавшись методами работ [11, 12, 13, 14], нетривиальная оценка получена при

$$(\ln y)^{(n-1)^2} \ll q \ll y^n (\ln y)^{-(n-1)^2}.$$

Теорема 2. Пусть $x \ge x_0 > 0$, $\ln x < y \le x(\ln x)^{-1}$, α — вещественное число,

$$\left|\alpha - \frac{a}{q}\right| \le \frac{1}{q^2}, \qquad (a, q) = 1,$$

тогда при $n \ge 3$ справедлива оценка

$$|T(\alpha; x, y)| \ll y \left(\frac{1}{q} + \frac{1}{y} + \frac{q}{y^n}\right)^{2^{-n}} (\ln qy)^{(n-1)^2 2^{-n}},$$

где постоянная под знаком $\ll$ зависит только от n.

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Асимптотическая формула в обобщённой проблеме Эстермана для нецелых степеней с почти пропорциональными слагаемыми

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Эстерман [1] при $n = 2$ доказал асимптотическую формулу для числа решений уравнения

$$p_1 + p_2 + m^n = N, \tag{1}$$

где p_1, p_2 — простые числа, m — натуральное число. В работах [2,3,4] при $n = 2, 3, 4$ эта задача исследована при более жёстких условиях, а именно, когда слагаемые почти равны, то есть получена асимптотическая формула для числа решений диофантова уравнения (1) с условиями

$$\left|p_i - \frac{N}{3}\right| \le H, \quad i = 1, 2, \qquad \left|m^n - \frac{N}{3}\right| \le H, \qquad H \ge N^{1-\theta(n)}\mathfrak{L}^{c_n}, \quad \mathfrak{L} = \ln N,$$

соответственно при

$$\theta(2)=\frac{1}{4},\quad c_2=2;\qquad \theta(3)=\frac{1}{6},\quad c_3=3;\qquad \theta(4)=\frac{1}{12},\quad c_4=\frac{40}{3}. \tag{2}$$

В работе [5] при произвольном фиксированном $n\geq 3$, опираясь на новые оценки коротких тригонометрических сумм Г. Вейля, исследована проблема Эстермана с почти пропорциональными слагаемыми и получена асимптотическая формула для числа решений уравнения (1) с условиями

$$|p_k-\mu_k N|\leq H,\qquad k=1,2,\qquad |m^n-\mu_3 N|\leq H,\qquad H\geq N^{1-\frac{1}{n(n-1)}\mathfrak{L}^{\frac{2^{n+1}}{n-1}+n-1}}. \tag{3}$$

Заметим, что полученная асимптотическая формула при $\mu_1=\mu_2=\mu_3=\frac{1}{3}$ превращается в асимптотическую формулу для обобщённой проблемы Эстермана с почти равными слагаемыми, а результаты [3,4], приведённые в (2), являются частными случаями (3).

В. Н. Чубариков поставил следующую задачу: *для фиксированных нецелых значений c исследовать короткие тригонометрические суммы вида*

$$S_c(\alpha;x,y)=\sum_{x-y<n\leq x} e\big(\alpha[n^c]\big),$$

а также уравнение (1), в котором слагаемое m^n заменяется на $[m^c]$, причём рассматривается случай почти равных слагаемых.

Первая часть этой задачи была решена в работах [7,8]. Для сумм $S_c(\alpha;x,y)$ получена оценка, равномерная по c, для всех $\alpha\in[-0.5,0.5]$, за исключением малой окрестности нуля, а также получена асимптотическая формула с остаточным членом, равномерная по c при α из малой окрестности нуля. Эти результаты позволили доказать асимптотическую формулу в обобщённой тернарной проблеме Эстермана для нецелых степеней с почти равными слагаемыми [9,10], то есть при $H\geq N^{1-\frac{1}{2c}}\mathfrak{L}^2$ найдена асимптотическая формула для числа решений уравнения

$$p_1+p_2+[n^c]=N,\qquad \left|p_i-\frac{N}{3}\right|\leq H,\ i=1,2,\qquad \left|[n^c]-\frac{N}{3}\right|\leq H,$$

в простых числах p_1, p_2 и натуральном n, где c — фиксированное нецелое число, удовлетворяющее условиям

$$\|c\|\geq 3c\big(2^{[c]+1}-1\big)\frac{\ln\mathfrak{L}}{\mathfrak{L}},\qquad c>\frac{4}{3}+\mathfrak{L}^{-0.3}.$$

В настоящем докладе для произвольного фиксированного нецелого c, удовлетворяющего условию (4), доказана теорема об асимптотической формуле в обобщённой проблеме Эстермана с почти пропорциональными слагаемыми. Полученный результат представляет собой обобщение и уточнение основной теоремы из работы [9].

Теорема 1. Пусть N — достаточно большое натуральное число, $\mathfrak{L}=\ln N$, μ_1, μ_2, μ_3 — положительные фиксированные числа, удовлетворяющие $\mu_1+\mu_2+\mu_3=1$, а c — фиксированное нецелое число, удовлетворяющее условиям

$$\|c\|\geq 3c\big(2^{[c]+1}-1\big)\frac{\ln\mathfrak{L}}{\mathfrak{L}},\qquad c>\frac{4}{3}\left(1+\frac{52\ln\mathfrak{L}}{\mathfrak{L}}\right). \tag{4}$$

Пусть $J_c(N,H)$ обозначает число решений диофантова уравнения

$$p_1+p_2+[n^c]=N$$

в простых числах p_1, p_2 и натуральном n при условиях

$$|p_k - \mu_k N| \le H, \quad k = 1, 2, \qquad |[n^c] - \mu_3 N| \le H.$$

Тогда при $H \ge N^{1-\frac{1}{2c}}\mathfrak{L}^2$ справедлива асимптотическая формула

$$J_c(N,H) = \frac{3H^2}{c(\mu_3 N)^{1-\frac{1}{c}}\mathfrak{L}^2} + O\left(\frac{H^2}{N^{1-\frac{1}{c}}\mathfrak{L}^3}\right),$$

где постоянная под знаком O зависит от μ_1, μ_2, μ_3 и c.

С помощью кругового метода Харди-Литтлвуда-Рамануджана в форме тригонометрических сумм И. М. Виноградова доказательство теоремы сведено к трём следующим задачам:

- нахождение нетривиальной оценки суммы $S_c(\alpha; x, y)$, равномерной по параметру c, для всех $\alpha \in [-0.5, 0.5]$ за исключением малой окрестности нуля;
- получение асимптотической формулы суммы $S_c(\alpha; x, y)$ с остаточным членом, равномерной по c при α из малой окрестности нуля;
- вывод асимптотической формулы для коротких линейных тригонометрических сумм с простыми числами вида

$$S_1(\alpha; x, y) = \sum_{x-y<n\le x} \Lambda(n) e(\alpha n),$$

в малой окрестности нуля.

Эти задачи решены соответственно в леммах 1, 2 и 3.

Лемма 1. [6]. Пусть $x \ge x_0 > 0$, $\mathfrak{L}_x = \ln x$, A – фиксированное число больше единицы, c – нецелое число с условиями

$$1 < c \le \log_2 \mathfrak{L}_x - \log_2 \ln \mathfrak{L}_x^{6A}, \qquad \|c\| \ge \left(2^{[c]+1} - 1\right)(A+1)\,\mathfrak{L}_x^{-1} \ln \mathfrak{L}_x. \tag{5}$$

Тогда при $y \ge \sqrt{2cx}\,\mathfrak{L}_x^{A+\theta}$ и $x^{1-c}y^{-1}\mathfrak{L}_x^A \le |\alpha| \le 0,5$ справедлива оценка

$$S_c(\alpha; x, y) \ll y\mathfrak{L}_x^{-A},$$

где $\theta = 0$ при $c \ge 1,1$ и $\theta = 0,5$ при $c < 1,1$.

Лемма 2. [6]. Пусть $x \ge x_0 > 0$, A – фиксированное положительное число больше единицы, c – нецелое число, удовлетворяющее условиям (5). Тогда при $y \ge \sqrt{2c}x^{\frac{1}{2}}\mathfrak{L}_x^A$ и $|\alpha| \le x^{1-c}y^{-1}\mathfrak{L}_x^A$ справедлива асимптотическая формула

$$S_c(\alpha; x, y) = \frac{\sin \pi\alpha}{\pi\alpha} \int_{x-y}^{x} e(\alpha(t^c - 0{,}5))dt + O\left(\frac{y|\sin \pi\alpha|}{\mathfrak{L}_x^A}\right).$$

Лемма 3. [6]. Пусть $x \ge x_0$, A — произвольное фиксированное положительное число, $y \ge x^{\frac{5}{8}}\mathfrak{L}_x^{1.5A+15}$ и $|\alpha| \le x(2\pi y^2)^{-1}$. Тогда справедлива асимптотическая формула:

$$S_1(\alpha; x, y) = \sum_{x-y<n\le x} \Lambda(n) e(\alpha n) = \frac{\sin \pi\alpha y}{\pi\alpha} e\left(\alpha\left(x - \frac{y}{2}\right)\right) + O\left(\frac{y}{\mathfrak{L}_x^A}\right).$$

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Нижние оценки трёхканальных гамильтонианов, возникающих в нерелятивистской квантовой механике

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Исследование спектральных свойств многоканальных гамильтонианов является одним из важных направлений современной математической физики и связано с изучением связанных состояний, пороговых явлений и рассеяния в квантовых системах. В частности, трёхканальные модели возникают естественным образом при рассмотрении взаимодействия двух ограниченных каналов, описывающих, например, систему кварк-антикварк, с каналом рассеяния, соответствующим двухадронной системе [1]. Такие модели позволяют исследовать взаимодействие различных физических каналов в рамках нерелятивистской квантовой механики и приводят к операторным матрицам, действующим в прямой сумме трёх гильбертовых пространств.

Заметим, что в общем случае любой линейный ограниченный оператор, действующий в прямой сумме трёх гильбертовых пространств, допускает представление в виде блочно-

операторной матрицы третьего порядка. В случае неограниченного оператора ситуация существенно отличается: его область определения, вообще говоря, не обязана представляться в виде прямой суммы трёх подпространств. Поэтому представление неограниченного оператора в виде блочно-операторной матрицы третьего порядка является дополнительным предположением, требующим соответствующего обоснования.

Ради удобства мы приведем понятие относительной ограниченности операторов [2].

Пусть $(E, \|\cdot\|_E)$, $(F, \|\cdot\|_F)$, $(G, \|\cdot\|_G)$ банахово пространство и

$$T : E \supset D(T) \to F, \quad S : E \supset D(S) \to G$$

линейные операторы. Если $D(T) \subset D(S)$ и для некоторых констант a_S, $b_S \geq 0$ имеет место неравенство

$$\|Sx\|_G \leq a_S\|x\|_E + b_S\|Tx\|_F, \quad x \in D(T), \tag{1}$$

то оператор S назовем *ограниченным относительно* T или просто T*-ограниченным*. Нижняя грань δ_S всевозможных констант b_S в (1) называется относительной границей (относительной гранью) оператора S по отношению к T или просто T-границей (T-гранью) оператора S.

В настоящей работе рассматривается трёхканальный гамильтониан

$$\mathcal{A} := \begin{pmatrix} -\nabla^2/2 + U_1 & V_{12} & 0 \\ \overline{V}_{12} & -\nabla^2/2 + U_2 & V_{23} \\ 0 & \overline{V}_{23} & -\nabla^2/2, \end{pmatrix}$$

действующий в пространстве

$$L_2(\mathbb{R}^3) \oplus L_2(\mathbb{R}^3) \oplus L_2(\mathbb{R}^3),$$

который представляется в виде

$$\mathcal{A} = \mathcal{A}_0 + V. \tag{2}$$

Здесь $\hbar$ масса кварка и масса адронной основной состояния нормалиризован к единицу и

$$\mathcal{A}_0 := \begin{pmatrix} H_{\rm c}^{(1)} & 0 & 0 \\ 0 & H_{\rm c}^{(2)} & 0 \\ 0 & 0 & H_{\rm s} \end{pmatrix}, \quad V := \begin{pmatrix} 0 & V_{12} & 0 \\ \overline{V}_{12} & 0 & V_{23} \\ 0 & \overline{V}_{23} & 0 \end{pmatrix}.$$

Для гамильтонианов $H_{\rm c}^{(i)} = -\nabla^2/2 + U_i$, мы предположим, что потенциалы U_i такие, что $H_{\rm c}^{(i)}$ является самосопряженным в $L_2(\mathbb{R}^3)$ и ограничены снизу с дискретном спектром, $H_{\rm c}^{(i)} \geq -\omega_i$, где $\omega_i > 0$,

$$\sigma(H_{\rm c}^{(i)}) = \sigma_{\rm p}(H_{\rm c}^{(i)}) = \{\mu_n^{(i)} : n \in \mathbb{N}\} \subset [-\omega_i, \infty)$$

стремящимся к ∞.

Невозмущенный гамильтониан $H_{\rm s} = -\nabla^2/2$ есть кинетический оператор энергии для относительной движений между двумя адроны. Оно является самосопряженным и неотрицательным в $L_2(\mathbb{R}^3)$ с непрерывным спектром $\sigma(H_{\rm s}) = \sigma_{\rm c}(H_{\rm s}) = [0, \infty)$.

Вне диагональные элементы V_{ij} и $V_{ji} = \overline{V}_{ij}$, $|i-j| = 1$, $i, j = 1, 2, 3$ представляет взаимодействие между каналами. Предположим, что оператор $V_{21} = \overline{V}_{12}$ ограничен относительно $H_{\rm c}^{(1)}$ с относительной гранью δ_{21}, операторы V_{12} и $V_{32} = \overline{V}_{23}$ ограничены относительно $H_{\rm c}^{(2)}$

с относительной гранью δ_{12} и δ_{32}, соответственно, и оператор V_{23} ограничен относительно $H_{\rm s}$ с относительной гранью δ_{23}. Если

$$\sqrt{2}\max\{\delta_{21}, \sqrt{\delta_{12}^2+\delta_{32}^2}, \delta_{32}\} < 1,$$

то $\mathcal{A}$ диагонально доминирующая с порядком < 1, причем оно является самосопряженным в области

$$D(\mathcal{A}) = D(H_{\rm c}^{(1)}) \oplus D(H_{\rm c}^{(2)}) \oplus D(H_{\rm s})$$

и ограничен снизу.

Основной интерес представляет получение количественных оценок снизу для спектра трёхканального гамильтониана. Такие оценки имеют существенное значение для установления ограниченности гамильтониана снизу, определения области возможных энергетических уровней и анализа устойчивости соответствующей квантовой системы. В связи с этим целью настоящей работы является получение явных нижних оценок для трёхканального гамильтониана с учётом как диагональных взаимодействий внутри каналов, так и межканальных связей V_{12} и V_{23}.

Перейдём к формулировке основного результата настоящей работы.

Теорема 1. Если потенциалы V_{ij} существенны ограничены, $V_{ij} \in L_\infty(\mathbb{R}^3)$, то нижняя грань оператора $\mathcal{A}$ удовлетворяет следующим оценкам

$$\min\sigma(\mathcal{A}) \geq -\frac{1}{3}(\omega_1+\omega_2) + 2\sqrt{-\frac{p_0}{3}}\cos\frac{\varphi_0+2\pi}{3} > \min\{-\omega_1,-\omega_2\} - (\|V_{12}\|^2+\|V_{23}\|^2)^{1/2},$$

где

$$\begin{aligned}
p_0 &:= -\frac{1}{6}\{(\omega_1-\omega_2)^2+\omega_1^2+\omega_2^2\} - \|V_{12}\|^2 - \|V_{23}\|^2;\\
q_0 &:= \|V_{23}\|^2\omega_1 - \frac{2}{27}(\omega_1+\omega_2)^3 + \frac{1}{3}(\omega_1\omega_2 - \|V_{12}\|^2 - \|V_{23}\|^2)(\omega_1+\omega_2);\\
\varphi_0 &:= \arccos\Big(-\frac{3q_0}{2p_0}\sqrt{-\frac{3}{p_0}}\Big).
\end{aligned}$$

Доказательство теоремы 1 основано на использовании альтернативной формулы для кубической числовой области значений гамильтониана $\mathcal{A}$, полученной в работе [3].

Следует отметить, что в работе [3] доказан аналог классической теоремы Гершгорина для блочно-операторных матриц произвольного порядка, сформулированный в терминах элементов строк и столбцов. Данный результат позволяет локализовать спектр рассматриваемого блочно-операторного гамильтониана и, в частности, получить явную нижнюю оценку для его спектра. Применяя указанный результат к оператору $\mathcal{A}$, получаем

$$\min\sigma(\mathcal{A}) \geq \min\{-\omega_1 - \|V_{12}\|, -\omega_2 - \|V_{12}\| - \|V_{23}\|\}.$$

Полученная оценка показывает, что нижняя граница спектра гамильтониана определяется, с одной стороны, нижними границами спектров соответствующих диагональных операторов, а с другой - нормами операторов межканального взаимодействия V_{12} и V_{23}. Таким образом, усиление межканального взаимодействия приводит к соответствующему смещению нижней границы спектра в сторону меньших значений. В частности, наличие связи между соседними каналами оказывает непосредственное влияние на положение спектрального дна рассматриваемой системы.

Наряду с указанной оценкой можно получить более общую нижнюю границу, непосредственно следуя из основной теоремы классической теории возмущений [2]. Из представления (2) гамильтониана $\mathcal{A}$ непосредственно получаем

$$\min\sigma(\mathcal{A}) \geq \min\{-\omega_1,\, -\omega_2\} - \|V\|,$$

где

$$\|V\| = \max_{\|f\|=1} \|Vf\|.$$

Полученные результаты могут быть использованы для дальнейшего спектрального анализа многоканальных моделей и исследования расположения их дискретного спектра.

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Уравнение Фаддеева для операторной матрицы с некомпактным возмущением

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Матрицы, элементами которых являются операторы, действующие в банаховых или гильбертовых пространствах, называются операторными матрицами. Со спектральной теорией таких операторных матриц можно ознакомиться в монографии [1]. Например, спин-бозонная модель на решётке, с не более двумя фотонами, представляется в виде операторной матрицы третьего порядка [2].

В данной работе рассматривается операторная матрица третьего порядка с некомпактным возмущением, действующей обрезанном трехчастичном подпространстве пространства Фока и построено аналог уравнения Фаддеева для собственных вектор-функций операторной матрицы.

Через $L_2(\mathbb{T}^d)$ обозначается гильбертово пространство комплекснозначных функций, квадратично интегрируемых на d-мерном торе $\mathbb{T}^d$ относительно поля комплексных чисел $\mathbb{C}$, а $L_2^{\rm s}((\mathbb{T}^d)^2)$ обозначается пространство симметричных квадратично интегрируемых комплекснозначных функций, определённых на $(\mathbb{T}^d)^2$. Пусть $\mathcal{H} := \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2-$ прямая сумма пространств $\mathcal{H}_0 := \mathbb{C}, \quad \mathcal{H}_1 := L_2(\mathbb{T}^d), \quad \mathcal{H}_2 := L_2^{\rm s}((\mathbb{T}^d)^2)$.

Рассмотрим матричный оператор $H_{\mu,\lambda}$, действующий в гилбертовом пространстве $\mathcal{H}$ и задающийся как:

$$H_{\mu,\lambda} := \begin{pmatrix} H_{00} & \mu H_{01} & 0 \\ \mu H_{01}^* & H_{11} & \mu H_{12} \\ 0 & \mu H_{12}^* & H_{22}^0 - \lambda V \end{pmatrix},$$

где операторы $H_{ij} : \mathcal{H}_j \longrightarrow \mathcal{H}_i, i,j = 0,1,2$, определяются по формулам:

$$H_{00}f_0 = \omega_0 f_0, \quad H_{01}f_1 = \int_{\mathbb{T}^d} v(t) f_1(t)dt, \quad (H_{11}f_1)(x) = \omega_1(x) f_1(x);$$

$$(H_{12}f_2)(x)=\int_{\mathbb{T}^d}v_1(t)f_2(x,t)dt,\quad (H_{22}^0f_2)(x,y)=\omega_2(x,y)f_2(x,y);$$

$$V:=V_1+V_2,\quad (V_1f_2)(x,y)=v_2(x)\int_{\mathbb{T}^d}v_2(t)f_2(t,y)dt,\quad (V_2f_2)(x,y)=v_2(y)\int_{\mathbb{T}^d}v_2(t)f_2(x,t)dt.$$

Здесь $f_i\in\mathcal{H}_i$, ω_0- фиксированное вещественное число, параметры μ и λ считаются положительными вещественными числами, $v(\cdot),\omega_1(\cdot),v_1(\cdot)$ и $v_2(\cdot)-$ вещественнозначные непрерывные (ненулевые) функции на $\mathbb{T}^d, i=0,1,2$, а $\omega_2(\cdot,\cdot)-$ вещественнозначная непрерывная симметричная функция на$(\mathbb{T}^d)^2$.

Согласно определению, элементы H_{01}^* и H_{12}^*, оператора $H_{\mu\lambda}$ имеют следующий вид:

$$(H_{01}^*f_0)=v(x)f_0,\quad (H_{12}^*f_1)(x,y)=\frac{v_1(y)f_1(x)+v_1(x)f_1(y)}{2}.$$

Оператор V представлен в виде суммы двух интегральных операторов и принадлежит классу некомпактных операторов. В связи с этим операторная матрица $H_{\mu,\lambda}$ рассматривается как оператор с некомпактным возмущением.

В данной работе определяется аналог системы интегральных уравнений типа Фаддеева для собственных векторов, соответствующих собственным значениям, лежащим вне существенного спектра оператора $H_{\mu,\lambda}$. Сначала введем следующее определение:

$$\widehat{\mathcal{H}}_0:=\mathcal{H}_0,\quad \widehat{\mathcal{H}}_1:=\mathcal{H}_1,\quad \widehat{\mathcal{H}}_2:=\mathcal{H}_1,\quad \widehat{\mathcal{H}}:=\widehat{\mathcal{H}}_0\oplus\widehat{\mathcal{H}}_1\oplus\widehat{\mathcal{H}}_2.$$

Существенный спектр операторной матрицы $H_{\mu,\lambda}$ обозначим через $\sigma_{\rm ess}(H_{\mu,\lambda})$. Его структура и свойства исследованы в работах [3, 4].

Пусть операторные матрицы $A(\mu,\lambda,z)$ и $K(\mu,\lambda,z)$ при каждом $z\in\mathbb{C}\setminus\sigma_{\rm ess}(H_{\mu,\lambda})$ действуют в пространстве $\widehat{\mathcal{H}}$ по формулам

$$A(\mu,\lambda,z):=\begin{pmatrix}A_{00}(\mu,\lambda,z) & A_{01}(\mu,\lambda,z) & A_{02}(\mu,\lambda,z)\\ A_{10}(\mu,\lambda,z) & A_{11}(\mu,\lambda,z) & A_{02}(\mu,\lambda,z)\\ A_{20}(\mu,\lambda,z) & A_{21}(\mu,\lambda,z) & A_{22}(\mu,\lambda,z)\end{pmatrix}$$

и

$$K(\mu,\lambda,z):=\begin{pmatrix}K_{00}(\mu,\lambda,z) & K_{01}(\mu,\lambda,z) & K_{02}(\mu,\lambda,z)\\ K_{10}(\mu,\lambda,z) & K_{11}(\mu,\lambda,z) & K_{12}(\mu,\lambda,z)\\ K_{20}(\mu,\lambda,z) & K_{21}(\mu,\lambda,z) & K_{22}(\mu,\lambda,z)\end{pmatrix},$$

где $A_{ij}(\mu,\lambda,z):\widehat{\mathcal{H}}_j\to\widehat{\mathcal{H}}_i, i,j=0,1,2,-$ оператор умножения на функцию $a_{ij}(x,\mu,\lambda,z)$,

$$a_{00}(x,\mu,\lambda,z)\equiv 1,\quad a_{0\alpha}(x,\mu,\lambda,z)\equiv 0,\quad \alpha=1,2;$$

$$a_{11}(x,\mu,\lambda,z)=\omega_1(x)-z-\frac{\mu^2}{2}\int_{\mathbb{T}^d}\frac{v_1^2(t)dt}{\omega_2(x,t)-z},\quad a_{12}(x,\mu,\lambda,z)=\mu\lambda\int_{\mathbb{T}^d}\frac{v_1(t)v_2(t)dt}{\omega_2(x,t)-z};$$

$$a_{\alpha0}(x,\mu,\lambda,z)\equiv 0,\quad \alpha=1,2,\quad a_{21}(x,\mu,\lambda,z)=\frac{1}{2\lambda}a_{12}(x,\mu,\lambda,z);$$

$$a_{22}(x,\mu,\lambda,z)=1-\lambda\int_{\mathbb{T}^d}\frac{v_2^2(t)dt}{\omega_2(x,t)-z},$$

а операторы $K_{ij}(\mu,\lambda,z):\widehat{\mathcal{H}}_j\to\widehat{\mathcal{H}}_i, i,j=0,1,2$, определяются формулами

$$K_{00}(\mu,\lambda,z)g_0=(\omega_0-z+1)g_0,\quad K_{01}(\mu,\lambda,z)\equiv\mu H_{01},\quad K_{02}(\mu,\lambda,z)\equiv 0;$$

$$(K_{10}(\mu,\lambda,z))(x) \equiv -\mu H_{01}^*, \quad (K_{11}(\mu,\lambda,z)g_2)(x) = \frac{\mu^2}{2} v_1(x) \int_{\mathbb{T}^d} \frac{v_2(t)g_1(t)dt}{\omega_2(x,t)-z};$$

$$(K_{12}(\mu,\lambda,z)g_2)(x) = -\mu\lambda v_2(x) \int_{\mathbb{T}^d} \frac{v_1(t)g_2(t)dt}{\omega_2(x,t)-z};$$

$$(K_{21}(\mu,\lambda,z)g_1)(x) = -\frac{\mu}{2} v_1(x) \int_{\mathbb{T}^d} \frac{v_2(t)g_1(t)dt}{\omega_2(x,t)-z};$$

$$(K_{22}(\mu,\lambda,z)g_2)(x) = \lambda v_2(x) \int_{\mathbb{T}^d} \frac{v_2(t)g_2(t)dt}{\omega_2(x,t)-z}.$$

Отметим, что операторы $K_{ij}(\mu,\lambda,z), i,j=0,1,2$ при каждом $z \in \mathbb{C} \setminus \sigma_{\text{ess}}(H_{\mu,\lambda})$ принадлежат классу Гильберта–Шмидта, следовательно, $K(\mu,\lambda,z)$ является компактным оператором.

Из формул действий элементов оператора $A(\mu,\lambda,z)$ следует, что он имеет блочно-диагональную структуру. Поэтому операторную матрицу $A(\mu,\lambda,z)$ можно представить в следующем виде:

$$A(\mu,\lambda,z) = I \oplus \begin{pmatrix} A_{11}(\mu,\lambda,z) & A_{12}(\mu,\lambda,z) \\ A_{21}(\mu,\lambda,z) & A_{22}(\mu,\lambda,z) \end{pmatrix}.$$

Здесь I обозначает единичный оператор в гильбертовом пространстве $\mathcal{H}_0$. Для удобства введём следующее обозначение:

$$\Delta(x,\mu,\lambda,z) := a_{11}(x,\mu,\lambda,z)a_{22}(x,\mu,\lambda,z) - a_{12}^2(x,\mu,\lambda,z).$$

Тогда обратный оператор $A(\mu,\lambda,z)$ определяется следующей леммой.

Lemma. Оператор $A(\mu,\lambda,z), z \in \mathbb{C} \setminus \sigma_{\text{ess}}(H_{\mu,\lambda})$, является ограниченным, обратимым,

$$A^{-1}(\mu,\lambda,z) = I \oplus \begin{pmatrix} B_{11}(\mu,\lambda,z) & B_{12}(\mu,\lambda,z) \\ B_{21}(\mu,\lambda,z) & B_{22}(\mu,\lambda,z) \end{pmatrix},$$

где $B_{ij}(\mu,\lambda,z) : \widehat{\mathcal{H}}_j \to \widehat{\mathcal{H}}_i, i,j=1,2,-$ оператор умножения на функцию $b_{ij}(x,\mu,\lambda,z)$,

$$b_{11} = \frac{a_{22}(x,\mu,\lambda,z)}{\Delta(x,\mu,\lambda,z)}, \quad b_{12}(x,\mu,\lambda,z) = -\frac{a_{12}(x,\mu,\lambda,z)}{\Delta(x,\mu,\lambda,z)};$$

$$b_{21} = -\frac{a_{21}(x,\mu,\lambda,z)}{\Delta(x,\mu,\lambda,z)}, \quad b_{22} = \frac{a_{11}(x,\mu,\lambda,z)}{\Delta(x,\mu,\lambda,z)}.$$

Следующее операторно-матричное уравнение

$$T(\mu,\lambda,z) = A^{-1}(\mu,\lambda,z)K(\mu,\lambda,z)$$

называется уравнением Фаддеева для операторной матрицы $H_{\mu,\lambda}$.

Следующая теорема описывает связь между собственными значениями операторных матриц $H_{\mu,\lambda}$ и $T(\mu,\lambda,z)$.

Теорема. Число $z \in \mathbb{C} \setminus \sigma_{\text{ess}}(H_{\mu,\lambda})$ является собственным значением оператора $H_{\mu,\lambda}$ тогда и только тогда, когда оператор $T(\mu,\lambda,z)$ имеет собственное значение, равное единице.

Уравнение Фаддеева в основном используется для доказательства результата о расположении существенного спектра модельного оператора $H_{\mu,\lambda}$.

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Аннотация. Исследуется система функций, составленная из разностей присоединенных функций Лежандра. Показано, что эта система не полна в $L_2(0,\pi/2)$, однако полна в замкнутом подпространстве функций, антисимметричных относительно середины интервала. Для доказательства строится унитарный оператор из указанного подпространства на $L_2(0,\pi/2)$, сводящий рассматриваемую систему к полной системе Моисеева–Могими.

Ключевые слова: функция Лежандра, система функций, полнота, антисимметричное подпространство, унитарный оператор.

Системы разностей присоединенных функций Лежандра возникают в спектральных задачах для уравнений смешанного типа. В работе [1] полнота одной из таких систем с аргументом $\cos\theta$ доказана методом интегрального уравнения. Ниже этот результат применяется для установления полноты системы с аргументом $\cos 2\varphi$ в ее естественном замкнутом подпространстве.

Пусть $0<\beta<1/2$. В пространстве $L_2(0,\pi/2)$ рассмотрим систему

$$\Phi_n(\varphi)=(\sin 2\varphi)^{1/2-\beta}\left[P_{n-1}^{\beta-1/2}(\cos 2\varphi)-P_{n-1}^{\beta-1/2}(-\cos 2\varphi)\right],\qquad n\in\mathbb{N}. \tag{1}$$

Здесь P_ν^μ означает функцию Феррерса первого рода, то есть функцию Лежандра на отрезке. Сначала убедимся, что функции (1) принадлежат $L_2(0,\pi/2)$. Положим $\alpha=1/2-\beta\in(0,1/2)$, $m=n-1$ и $x=\cos 2\varphi$. Согласно гипергеометрическому представлению [2],

$$P_m^{-\alpha}(x)=\frac{1}{\Gamma(1+\alpha)}\left(\frac{1-x}{1+x}\right)^{\alpha/2}{}_2F_1\left(-m,m+1;1+\alpha;\frac{1-x}{2}\right). \tag{2}$$

Так как первый параметр гипергеометрического ряда равен $-m$, этот ряд обрывается. Учитывая $(\sin 2\varphi)^\alpha=(1-x^2)^{\alpha/2}$, из (2) получаем

$$\Phi_n(\varphi)=\frac{1}{\Gamma(1+\alpha)}\left[(1-x)^\alpha F_m\left(\frac{1-x}{2}\right)-(1+x)^\alpha F_m\left(\frac{1+x}{2}\right)\right], \tag{3}$$

где $F_m(z) = {}_2F_1(-m, m+1; 1+\alpha; z)$ является многочленом. Следовательно, Φ_n ограничена на отрезке и $\Phi_n \in L_2(0, \pi/2)$.

Лемма (Моисеев–Могими [1]). Если $0 < \delta < 1/4$, то система

$$v_k(\theta) = (\sin\theta)^{2\delta} \left[P_{2k+1}^{-2\delta}(\cos\theta) - P_{2k+1}^{-2\delta}(-\cos\theta) \right], \qquad k = 0, 1, \ldots, \tag{4}$$

полна в $L_2(0, \pi/2)$.

Основная теорема. Пусть $0 < \beta < 1/2$. Тогда система (1) полна в замкнутом подпространстве

$$H_- := \left\{ f \in L_2(0, \pi/2) : f\left(\frac{\pi}{2} - \varphi\right) = -f(\varphi) \text{ почти всюду} \right\}. \tag{5}$$

Более того, в H_- полна уже подсистема $\{\Phi_{2k+2}\}_{k=0}^{\infty}$.

Доказательство. В пространстве $H = L_2(0, \pi/2)$ введем оператор отражения

$$(Rf)(\varphi) = f\left(\frac{\pi}{2} - \varphi\right). \tag{6}$$

Замена переменной дает $\|Rf\|_H = \|f\|_H$. Поэтому R является унитарным оператором и $R^2 = I$. В частности, $H_- = \ker(R + I)$ есть замкнутое подпространство. Кроме того,

$$H = H_+ \overset{\perp}{\oplus} H_-, \qquad H_\pm = \{f \in H : Rf = \pm f\}. \tag{7}$$

Действительно, $f_+ = (f + Rf)/2$, $f_- = (f - Rf)/2$ принадлежат соответственно H_+ и H_-, причем $f = f_+ + f_-$. Если $u \in H_+$ и $w \in H_-$, то унитарность R дает $\langle u, w\rangle = \langle Ru, Rw\rangle = -\langle u, w\rangle$; следовательно, $\langle u, w\rangle = 0$.

Далее,

$$\sin\left(2\left(\frac{\pi}{2} - \varphi\right)\right) = \sin 2\varphi, \qquad \cos\left(2\left(\frac{\pi}{2} - \varphi\right)\right) = -\cos 2\varphi. \tag{8}$$

Поэтому две функции Лежандра в (1) меняются местами и

$$\Phi_n\left(\frac{\pi}{2} - \varphi\right) = -\Phi_n(\varphi), \qquad n \in \mathbb{N}. \tag{9}$$

Таким образом, все $\Phi_n \in H_-$ и

$$\overline{\operatorname{span}\{\Phi_n : n \in \mathbb{N}\}} \subset H_-. \tag{10}$$

Определим оператор $U : H_- \to L_2(0, \pi/2)$ равенством

$$(Uf)(\theta) = f\left(\frac{\theta}{2}\right), \qquad 0 < \theta < \frac{\pi}{2}. \tag{11}$$

Так как $f \in H_-$, функция $|f|^2$ симметрична относительно точки $\pi/4$. Следовательно,

$$\|f\|_H^2 = 2\int_0^{\pi/4} |f(\varphi)|^2\, d\varphi = \int_0^{\pi/2} \left| f\left(\frac{\theta}{2}\right)\right|^2 d\theta = \|Uf\|_{L_2(0,\pi/2)}^2. \tag{12}$$

Значит, U является изометрией. Докажем его сюръективность. Для любого $g \in L_2(0, \pi/2)$ положим

$$(Vg)(\varphi) = \begin{cases} g(2\varphi), & 0 < \varphi < \pi/4, \\ -g(\pi - 2\varphi), & \pi/4 < \varphi < \pi/2. \end{cases} \tag{13}$$

Тогда $Vg \in H_-$ и $U(Vg) = g$. Следовательно, U унитарен.

Положим

$$\delta = \frac{\alpha}{2} = \frac{1}{4} - \frac{\beta}{2}. \tag{14}$$

Из $0 < \beta < 1/2$ следует $0 < \delta < 1/4$. Подставляя в (1) $n = 2k+2$ и полагая $\theta = 2\varphi$, получаем

$$\begin{aligned}(U\Phi_{2k+2})(\theta) &= \Phi_{2k+2}\left(\frac{\theta}{2}\right) \\ &= (\sin\theta)^{2\delta}\left[P_{2k+1}^{-2\delta}(\cos\theta) - P_{2k+1}^{-2\delta}(-\cos\theta)\right] = v_k(\theta).\end{aligned} \tag{15}$$

Условия леммы выполнены, поэтому система $\{v_k\}_{k=0}^{\infty}$ полна в $L_2(0, \pi/2)$.

Пусть теперь $f \in H_-$ и $\langle f, \Phi_{2k+2}\rangle_H = 0$ при всех $k = 0, 1, \ldots$. В силу унитарности U и равенства (15)

$$0 = \langle f, \Phi_{2k+2}\rangle_H = \langle Uf, U\Phi_{2k+2}\rangle_{L_2} = \langle Uf, v_k\rangle_{L_2}, \qquad k = 0, 1, \ldots. \tag{16}$$

Из полноты системы в лемме следует $Uf = 0$, а из унитарности U получаем $f = 0$. По критерию полноты в гильбертовом пространстве подсистема $\{\Phi_{2k+2}\}_{k=0}^{\infty}$ полна в H_-. Поскольку она содержится в (1), а включение (10) уже доказано, имеем

$$\overline{\operatorname{span}\{\Phi_n : n \in \mathbb{N}\}} = H_-. \tag{17}$$

Теорема полностью доказана.

Следствие. Согласно (7), ортогональное дополнение системы (1) в $H = L_2(0, \pi/2)$ в точности равно H_+. В частности, система не полна во всем H, поскольку $1 \in H_+$ и $\int_0^{\pi/2} \Phi_n(\varphi)\, d\varphi = 0$.

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О внутренних неподвижных точках некоторых семейств 2-вольтерра кубических стохастических операторов

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В симплексе $S^{m-1} = \{x = (x_1, \ldots, x_m) : x_i \geq 0, \sum_{i=1}^m x_i = 1\}$ рассмотрим кубический оператор $W : S^{m-1} \to S^{m-1}$, определяемый равенствами:

$$W : x'_q = \sum_{i,j,k=1}^{m} P_{ijk,q} x_i x_j x_k, \quad q = \overline{1, m} \tag{1}$$

где $P_{ijk,q} \geq 0$, $\sum_{q=1}^{m} P_{ijk,q} = 1$, кроме того, значения $P_{ijk,q}$ не меняются при любой перестановке i, j, k.

Определение 1. Кубический оператор (1) называется l-вольтерровским, если $P_{ijk,q} = 0$ при $q \notin \{i, j, k\}$, $q = \overline{1, l}$ $(l < m)$, $i, j, k = \overline{1, m}$, но $P_{ijk,q} > 0$ хотя бы для одного набора (i, j, k, q), $i \neq q$, $j \neq q$, $k \neq q$ при $q = \overline{l+1, m}$.

Канонический вид, например, $(m-1)$-вольтерровских кубических операторов будет в виде:

$$x'_q = x_q\Big[P_{qqq,q}x_q^2 + 3\sum_{k=1,k\neq q}^{m} P_{qqk,q}x_q x_k + 3\sum_{k=1,k\neq q}^{m} P_{qkk,q}x_k^2 + 6\sum_{j,k=1}^{m} P_{qjk,q}x_j x_k\Big] \text{ при } q = \overline{1, m-1},$$
$$x'_m = 1 - \sum_{q=1}^{m-1} x'_q. \tag{2}$$

Пусть $x^{(0)} \in S^{m-1}$ — начальная точка, $\{x^{(n)}, n = 0, 1, 2, \dots\}$ траектория точки $x^{(0)}$ при действии оператора (1), т.е. $x^{(n)} = W(x^{(n-1)})$, $n = 1, 2, \dots$.

Определение 2. Точка $x^* \in S^{m-1}$ называется неподвижной относительно W, если $W(x^*) = x^*$.

Рассмотрим 2-вольтерровский кубический стохастический оператор $W : S^2 \to S^2$ вида

$$\begin{aligned} x' &= x[ax^2 + 3by + 3cz], \\ y' &= y[ay^2 + 3bx + 3cz], \\ z' &= z^3 + (1-a)(x^3 + y^3) + 3(1-z)[(1-c)z + 3xy], \end{aligned} \tag{3}$$

где $a, b, c \in [0, 1)$, $x, y, z \geq 0$, $x + y + z = 1$.

Внутренние неподвижные точки оператора (3) удовлетворяют системе $x' = x$, $y' = y$. Вычитая соответствующие уравнения, получаем $(x-y)[a(x+y)-3b] = 0$, откуда следует, что внутренние неподвижные точки могут существовать лишь при условии $x = y$ или $x+y = \frac{3b}{a}$.

Случай $x = y$. Полагая $z = 1-2x$ и используя $x' = x$ (при $x \neq 0$), получаем квадратное уравнение

$$ax^2 + (3b - 6c)x + (3c - 1) = 0. \tag{4}$$

Теорема 1. *Пусть $a, b, c \in (0, 1)$. Если*

$$(3c-1)\left(\frac{a}{4} + \frac{3b}{2} - 1\right) < 0, \tag{5}$$

то уравнение (4) имеет ровно один корень $x^* \in (0, \frac{1}{2})$, и, следовательно, оператор (3) имеет ровно одну внутреннюю неподвижную точку вида $(x^*, x^*, 1-2x^*)$ на прямой $x = y$.

при $a = \frac{1}{2}$, $b = \frac{1}{12}$, $c = \frac{3}{5}$ имеем

$$(3c-1)\left(\frac{a}{4} + \frac{3b}{2} - 1\right) = \frac{4}{5} \cdot \left(-\frac{3}{4}\right) < 0,$$

т.е. условие (5) выполнено, и оператор (3) обладает внутренней неподвижной точкой

$$\left(\frac{67 - \sqrt{3849}}{20}, \frac{67 - \sqrt{3849}}{20}, \frac{\sqrt{3849} - 57}{10}\right)$$

на прямой $x = y$.

Случай $x+y=\frac{3b}{a}$. При этом условии уравнения $x'=x$ и $y'=y$ становятся равносильными, и, полагая $y=\frac{3b}{a}-x$, $z=1-\frac{3b}{a}$, из $x'=x$ (при $x\neq 0$) получаем

$$a^2x^2-3abx+9b^2-9bc+3ac-a=0. \tag{6}$$

Теорема 2. *Пусть* $a,b,c\in(0,1)$ *и* $\Delta:=4a-12ac+36bc-27b^2$. *Если*

$$0<\Delta<9b^2, \tag{7}$$

то уравнение (6) имеет два различных вещественных корня $x_{1,2}=\frac{3b\pm\sqrt{\Delta}}{2a}$, *оба лежащие в интервале* $(0,\frac{3b}{a})$, *и, следовательно, оператор (3) имеет ровно две внутренние неподвижные точки*

$$\left(x_1,\frac{3b}{a}-x_1,1-\frac{3b}{a}\right),\quad \left(x_2,\frac{3b}{a}-x_2,1-\frac{3b}{a}\right)$$

на прямой $x+y=\frac{3b}{a}$.

при тех же значениях $a=\frac{1}{2}$, $b=\frac{1}{12}$, $c=\frac{3}{5}$ имеем

$$\Delta=\frac{1}{80}\in\left(0,\frac{1}{16}\right)=(0,9b^2),$$

т.е. условие (7) выполнено, и оператор (3) обладает двумя внутренними неподвижными точками

$$\left(\frac{5-\sqrt{5}}{20},\frac{5+\sqrt{5}}{20},\frac{1}{2}\right),\quad \left(\frac{5+\sqrt{5}}{20},\frac{5-\sqrt{5}}{20},\frac{1}{2}\right)$$

на прямой $x+y=\frac{3b}{a}$. Таким образом, при этих значениях параметров условия (5) и (7) обеих теорем выполняются одновременно, и оператор (3) обладает, помимо неподвижной точки $M_3(0,0,1)$, тремя внутренними неподвижными точками.

Таким образом, теоремы 1 и 2 дают точные, проверяемые в терминах параметров a,b,c условия существования внутренних неподвижных точек оператора (3) на каждой из двух линий $x=y$ и $x+y=3b/a$ соответственно.

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Об оценке одной тригонометрической суммы с использованием метода экспоненциальных пар

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Оценки тригонометрических сумм играют важную роль в аналитической теории чисел. Они находят широкое применение в задачах о распределении простых чисел, в аддитивных проблемах, а также при исследовании различных арифметических функций. Классические методы, разработанные И.М. Виноградовым, Г.Харди и Дж.Литтлвудом, послужили основой для дальнейшего развития данного направления.

Одним из эффективных инструментов получения нетривиальных оценок является метод экспоненциальных пар [1]. Этот метод позволяет систематически улучшать показатели в оценках экспоненциальных сумм и применяется в широком круге задач, включая проблему Вейля и различные ее обобщения.

Определение. *Если* $B \geq 1$, $0 < h \leq B$, $F(u) \in C^{\infty}(B, 2B)$, $A \geq 1$,

$$AB^{1-r} \ll |F^{(r)}(u)| \ll AB^{1-r},\ r = 1, 2, 3, \ldots,$$

здесь постоянные в знаке $\ll$ *зависят только от* r, *и имеет место оценка*

$$\sum_{B \leq n \leq B+h} e(F(n)) \ll A^{\kappa} B^{\lambda}, \quad 0 \leq \kappa \leq 0.5 \leq \lambda \leq 1,$$

тогда пара (κ, λ) *называется экспоненциальной парой.*

Из тривиальная оценка видно, что (0; 1) является экспоненциальная пара. E.Phillips показал, что если (κ, λ) является экспоненциальная пара, тогда

$$A(\kappa, \lambda) = \left(\frac{\kappa}{2\kappa + 2},\ \frac{1}{2} + \frac{\lambda}{2\kappa + 2} \right), \qquad (A - \text{процесс})$$

$$B(\kappa, \lambda) = \left(\lambda - \frac{1}{2},\ \kappa + \frac{1}{2} \right), \qquad (B - \text{ процесс})$$

также являются экспоненциальными парами. A – процесс выводится из неравенства Вейля, а B – процесс доказывается с помощью неравенства Коши, метода разностей и метода Вейля.

При оценке числа нулей дзета-функция Римана в окрестности критической прямой возникает необходимость в получении более точной оценки специальных кратных тригонометрических сумм рассматриваемого вида

$$W = \sum_{0 \leq h \leq \frac{N}{H} \ln T} \left| \sum_{N_1 < n \leq N_2} e\left(\frac{T}{2\pi} \ln \frac{n + h + \xi}{n + \eta} \right) \right| \tag{1}$$

с условиями $\xi \geq 0$, $\eta < 1$, h – целое неотрицательное число, $x^{-2} \leq |\xi - \eta + h|$, $x = T^{0,01\varepsilon}$, $N \leq x \cdot \sqrt{T}$, $c_1 N \leq N_1 < N_2 < c_2 N$, $\varepsilon \leq 0,0001$.

Тригонометрические суммы данного типа ранее были исследованы А.А. Карацуба [2]. В своих работах при оценке указанных сумм он применял метод Йоханнес Ван дер Корпут. В результате была получена следующая оценка:

$$W \ll NT^{-\frac{4\varepsilon}{3}}.$$

В настоящей работе с использованием современного метода экспоненциальных пар и методами работы [3], получена нетривиальная оценка для рассматриваемых тригонометрических сумм.

Теорема 1. *Пусть* $(\kappa,\ \lambda)$ *– любая экспоненциальная пара,* $\xi \geq 0$, $\eta < 1$, h *– целое неотрицательное число и* $x^{-2} \leq |\xi - \eta + h|$, $x = T^{0,01\varepsilon}$, $N \leq x \cdot \sqrt{T}$, $c_1 N \leq N_1 < N_2 < c_2 N$, $\varepsilon \leq 0,0001$. *Тогда для тригонометрической суммы (1) справедлива оценка*

$$W \ll NT^{-(1+1.01\kappa-0,01\lambda)\varepsilon}(\ln T)^{\kappa+1}.$$

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ВИРТУАЛЬНЫЕ УРОВНИ ДВУХЧАСТИЧНОГО ГАМИЛЬТОНИАНА ДВУХ БОЗОНОВ НА ТРЕХМЕРНОЙ РЕШЕТКЕ

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Рассматривается двухчастичный гамильтониан системы двух одинаковых бозонов на трехмерной решетке $\mathbb{Z}^3$ с короткодействующим взаимодействием, сосредоточенным на совпадающих и ближайших соседних узлах. Благодаря трансляционной инвариантности исходный гамильтониан допускает разложение в прямой интеграл семейства операторов Шредингера $h(k)$, зависящих от полного квазиимпульса $k \in \mathbb{T}^3$. В настоящей работе основное внимание уделяется пороговым спектральным свойствам оператора $h(\mathbf{0})$.

В импульсном представлении оператор $h(k)$ действует в пространстве четных функций $L_2^e(\mathbb{T}^3)$ и имеет вид

$$h(k) = h_0(k) - \mathbf{v},$$

где свободная часть $h_0(k)$ является оператором умножения на функцию

$$\mathcal{E}_k(p) = \frac{1}{m}\varepsilon\left(p + \frac{k}{2}\right) + \frac{1}{m}\varepsilon\left(p - \frac{k}{2}\right), \qquad \varepsilon(p) = \sum_{i=1}^{3}(1 - |\cos p_i|), \tag{1}$$

а оператор взаимодействия задается формулой

$$(\mathbf{v}f)(p) = \lambda \int_{\mathbb{T}^3} f(s)\,ds + \mu \sum_{\alpha=1}^{3} \cos p_\alpha \int_{\mathbb{T}^3} \cos s_\alpha f(s)\,ds, \qquad \lambda, \mu \geq 0. \tag{2}$$

Оператор $\mathbf{v}$ имеет конечный ранг, поэтому существенный спектр оператора $h(k)$ совпадает со спектром свободного оператора:

$$\sigma_{\text{ess}}(h(k)) = [m(k), M(k)],$$

где

$$m(k) = \min_{p\in\mathbb{T}^3} \mathcal{E}_k(p), \qquad M(k) = \max_{p\in\mathbb{T}^3} \mathcal{E}_k(p).$$

Для исследования дискретного спектра введем функции

$$d(k;z) = \int_{\mathbb{T}^3} \frac{ds}{\mathcal{E}_k(s) - z}, \qquad c_i(k;z) = \int_{\mathbb{T}^3} \frac{\cos^2 s_i}{\mathcal{E}_k(s) - z}\, ds, \quad i = 1,2,3, \tag{3}$$

при $z < m(k)$ и функцию

$$\Delta(k;z) = \big(1 - \lambda d(k;z)\big) \prod_{i=1}^{3} \big(1 - \mu c_i(k;z)\big). \tag{4}$$

Число $z < m(k)$ является собственным значением оператора $h(k)$ тогда и только тогда, когда $\Delta(k;z) = 0$, причем кратность нуля функции $\Delta(k;\cdot)$ совпадает с кратностью соответствующего собственного значения.

Рассмотрим теперь пороговый случай $k = \mathbf{0}$. В силу симметрии

$$c_1(\mathbf{0};0) = c_2(\mathbf{0};0) = c_3(\mathbf{0};0).$$

Определим критические значения параметров взаимодействия

$$\lambda_0(\mathbf{0}) = \frac{1}{d(\mathbf{0};0)}, \qquad \mu_0(\mathbf{0}) = \frac{1}{c_1(\mathbf{0};0)}. \tag{5}$$

Для исследования виртуального уровня воспользуемся оператором Бирмана–Швингера G, действующим в $C(\mathbb{T}^3)$ и имеющим ядро

$$G(p,q) = \frac{v(p-q)}{\mathcal{E}_{\mathbf{0}}(q)}.$$

Будем говорить, что оператор $h(\mathbf{0})$ имеет виртуальный уровень в точке $z = 0$, если число 1 является собственным значением оператора G и соответствующая собственная функция ψ удовлетворяет условию

$$\frac{\psi(p)}{\mathcal{E}_{\mathbf{0}}(p)} \in L_1(\mathbb{T}^3) \setminus L_2(\mathbb{T}^3). \tag{6}$$

Теорема 1. Пусть $\lambda, \mu \geq 0$. Тогда виртуальный уровень оператора $h(\mathbf{0})$ в точке $z = 0$ может иметь только кратность $0, 1, 3$ или 4. Более точно: если $\lambda = \lambda_0(\mathbf{0})$ и $\mu \neq \mu_0(\mathbf{0})$, то виртуальный уровень является простым; если $\lambda \neq \lambda_0(\mathbf{0})$ и $\mu = \mu_0(\mathbf{0})$, то его кратность равна 3; если одновременно

$$\lambda = \lambda_0(\mathbf{0}), \qquad \mu = \mu_0(\mathbf{0}),$$

то кратность виртуального уровня равна 4. Во всех остальных случаях точка $z = 0$ не является виртуальным уровнем.

Доказательство. При $\lambda = \lambda_0(\mathbf{0})$ число 1 является собственным значением оператора G на одномерном подпространстве, порожденном функцией $\psi_0(p) = 1$. При $\mu = \mu_0(\mathbf{0})$

число 1 является собственным значением G на трехмерном подпространстве, порожденном функциями

$$\psi_i(p) = \cos p_i, \qquad i = 1, 2, 3.$$

В окрестности каждой точки минимума функции $\mathcal{E}_{\mathbf{0}}$ справедливо $\mathcal{E}_{\mathbf{0}}(p) \asymp |p - p_0|^2$. Поэтому функции $\psi_i(p)/\mathcal{E}_{\mathbf{0}}(p)$ принадлежат $L_1(\mathbb{T}^3) \setminus L_2(\mathbb{T}^3)$. Отсюда непосредственно следуют утверждения теоремы о кратности виртуального уровня.

Кроме того, из монотонности функций $d(\mathbf{0}; z)$ и $c_1(\mathbf{0}; z)$ на интервале $(-\infty, 0)$ следует, что при $\lambda > \lambda_0(\mathbf{0})$ возникает одно простое собственное значение ниже существенного спектра, а при $\mu > \mu_0(\mathbf{0})$ возникает собственное значение кратности 3. Таким образом, критические значения $\lambda_0(\mathbf{0})$ и $\mu_0(\mathbf{0})$ разделяют области параметров, соответствующие отсутствию связанных состояний, появлению виртуального уровня и возникновению дискретного спектра.

Полученные результаты дают полное описание возможной кратности виртуального уровня на нижнем краю существенного спектра для рассматриваемой двухбозонной модели и устанавливают ее связь с параметрами короткодействующего взаимодействия.

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Об интегрируемых функциях с обобщенным граничным свойством Морера

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Ключевые слова: голоморфное продолжение, росток вещественно-аналитического порождающего подмногообразия, свойство Морера, интеграл Бохнера–Мартинелли, интегрируемые функции.

Граничная теорема Морера в многомерном комплексном анализе утверждает, что непрерывная функция на границе ограниченной области $D \subset \mathbb{C}^n$ голоморфно продолжается в D, если интегралы от неё по пересечениям границы с комплексными прямыми равны нулю. Классические результаты в этом направлении получены Е. Гринбергом для шара [1], И. Глобевником и Е.Л. Стаутом для произвольных областей с гладкой границей [2], а также в работах [3, 4, 5, 6, 7, 8]. В монографии [9] приведен обзор ряда результатов.

В большинстве этих работ предполагается, что функция на границе является непрерывной. В отличие от случая $f \in C(\partial D)$, когда внутреннее и внешнее граничные значения интеграла Бохнера-Мартинелли определены в каждой точке границы, для функций $f \in L^p(\partial D)$,

$1 \le p < \infty$, интеграл Бохнера–Мартинелли уже не обязан быть непрерывным вплоть до границы, а его граничные значения существуют лишь почти всюду в смысле пределов по некасательным путям.

Цель настоящего исследования – обобщить известные результаты о голоморфном продолжении функций с обобщенным граничным свойством Морера на класс $L^p(\partial D)$, $p \ge 1$. В качестве достаточного множества комплексных прямых рассматривается семейство прямых, пересекающих росток порождающего многообразия $\Gamma \subset D$.

Пусть $D \subset \mathbb{C}^n$ $(n > 1)$ – ограниченная область со связной C^2 гладкой границей вида $D = \{z \in \mathbb{C}^n : \rho(z) < 0\}$, где $\rho(z)$ – C^2 гладкая, вещественнозначная функция в окрестности $\overline{D}$ с $d\rho|_{\partial D} \neq 0$. Мы отождествляем $\mathbb{C}^n$ с $\mathbb{R}^{2n}$: $z_j = x_j + iy_j$.

Рассмотрим комплексные прямые

$$l_{z,b} = \{\zeta \in \mathbb{C}^n : \zeta_j = z_j + b_j t, \quad j = 1, \dots, n, \quad t \in \mathbb{C}\}, \tag{1}$$

проходящие через точку z в направлении $b \in \mathbb{C}P^{n-1}$.

Определение 1. Функция $f \in L^p(\partial D)$, $p \ge 1$, удовлетворяет *обобщенному свойству Морера* вдоль $l_{z,b}$, если для $k = 0, 1, 2, 3$

$$\int_{\partial D \cap l_{z,b}} f(\zeta)\, \frac{dt}{\bar{t}^k} = 0. \tag{2}$$

Плоскость l пересекает ∂D трансверсально для почти всех b, а интеграл понимается в смысле Лебега.

Пусть $\Gamma \subset D$ – росток вещественно-аналитического порождающего многообразия, т.е. для каждой точки $z \in \Gamma$ комплексная линейная оболочка касательного пространства $T_z(\Gamma)$ совпадает с $\mathbb{C}^n$. Обозначим через Σ_Γ семейство всех комплексных прямых, пересекающих Γ.

Интеграл Бохнера-Мартинелли:

$$U(\zeta, z) = \frac{(n-1)!}{(2\pi i)^n} \sum_{k=1}^{n} (-1)^{k-1} \frac{\bar{\zeta}_k - \bar{z}_k}{|\zeta - z|^{2n}}\, d\bar{\zeta}[k] \wedge d\zeta \tag{3}$$

$$Mf(z) = \int_{\partial D} f(\zeta) U(\zeta, z), \quad z \notin \partial D. \tag{4}$$

Обозначим $M^+ f(z)$ при $z \in D$, $M^- f(z)$ при $z \notin \overline{D}$. Функция Mf гармоническая вне ∂D и $Mf(z) = O(|z|^{1-2n})$ при $|z| \to \infty$.

Теорема 1 (*о скачке интеграла Бохнера-Мартинелли для интегрируемых функций*). Пусть D – ограниченная область с C^2-гладкой границей, $f \in L^p(\partial D)$, $p \ge 1$. Тогда для почти всех $z \in \partial D$

$$M^+ f(z) - M^- f(z) = f(z), \tag{5}$$

где $M^+ f(z)$ и $M^- f(z)$ – предельные значения интеграла Бохнера–Мартинелли (4) изнутри и снаружи D соответственно, понимаемые в смысле *пределов по некасательным путям* (т.е. пределов при приближении к границе внутри любого конуса с вершиной в точке границы, ось которого совпадает с внутренней нормалью к ∂D).

Для гладких функций формула (5) является классическим следствием формулы Бохнера-Грина (см. [9]). Для $f \in L^p(\partial D)$ она получается предельным переходом и доказана в [9].

Рассмотрим

$$M_j f(z) = \int_{\partial D} (\zeta_j - z_j) f(\zeta) U(\zeta, z). \tag{6}$$

Согласно [9, гл. 2], функции $M_j f$ являются *вещественно-аналитическими* в D и бигармоническими.

Теорема 2. Пусть D – ограниченная область со связной границей класса C^2, Γ – росток вещественно-аналитического порождающего многообразия в D, функция $f \in L^p(\partial D)$, $p \geq 1$, обладает обобщенным свойством Морера (2) при $k \leq 3$ вдоль почти всех комплексных прямых из Σ_Γ, и связные компоненты пересечения $l \cap D$ обладают усиленным свойством Неванлинны. Тогда существует голоморфная функция $F \in \mathcal{O}(D)$ такая, что для почти всех $z \in \partial D$

$$F(z) = f(z)$$

в смысле пределов по некасательным путям (т.е. пределов при приближении к границе внутри любого конуса с вершиной в точке границы, ось которого совпадает с внутренней нормалью к ∂D), и F является голоморфным продолжением f в D.

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О РАЗРЕШИМОСТИ ДИОФАНТОВЫХ НЕРАВЕНСТВ С ПРОСТЫМИ ЧИСЛАМИ

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В современной аддитивной теории чисел значительный интерес представляет исследование уравнений и неравенств с нецелыми степенями простых чисел, восходящее к известной проблеме Пятецкого-Шапиро [1]. Пусть $c > 1$ — нецелое вещественное число, близкое к единице. Рассматривается разрешимость диофантова неравенства

$$|p_1^c + p_2^c + \cdots + p_r^c - N| < \varepsilon \tag{1}$$

в простых числах $p_1, \ldots, p_r$ для достаточно больших значений вещественного параметра N.

В работе [1] доказано, что при $1 < c < 3/2$ неравенство (1) всегда разрешимо при $r = 5$. Тернарный случай ($r = 3$), представляющий наибольшую аналитическую сложность и являющийся прямым вещественным аналогом тернарной проблемы Гольдбаха, впервые детально исследован Д. И. Толевым [2]. Заменяя постоянное $\varepsilon > 0$ на монотонно стремящуюся к нулю функцию $\varepsilon(N)$, Д. И. Толев установил разрешимость неравенства (1) для всех показателей c из интервала $1 < c < 15/14 \approx 1{,}0714$.

Целью настоящего исследования является улучшение результата Д. И. Толева посредством уточнения оценок тригонометрических сумм по простым числам и расширение области допустимых показателей c до интервала $1 < c < 11/10 = 1{,}1000$.

Теорема. *Пусть c — фиксированное вещественное число, удовлетворяющее условию $1 < c < 11/10$. Тогда существует такое эффективное число $N_0(c) > 0$, что для любого $N > N_0(c)$ диофантово неравенство*

$$|p_1^c + p_2^c + p_3^c - N| < N^{-\frac{1}{c}\left(\frac{11}{10} - c\right)} \ln^{10} N \tag{2}$$

имеет решение в простых числах p_1, p_2, p_3.

Аналитический метод и структура доказательства. Для доказательства теоремы полагаем $X = (N/2)^{1/c}$, $\varepsilon = X^{-(11/10-c)} \ln^9 X$, $\Delta = \frac{1}{10}\varepsilon$ и $r = [\ln X]$. Пусть $\varphi(y)$ — бесконечно дифференцируемая весовая функция с компактным носителем, построенная в [2], обладающая свойствами: $\varphi(y) = 1$ при $|y| \le \varepsilon - \Delta$, $\varphi(y) = 0$ при $|y| \ge \varepsilon + \Delta$, и $0 \le \varphi(y) \le 1$. Ее преобразование Фурье $\Phi(x) = \int_{-\infty}^{\infty} e(-xy)\varphi(y)\,dy$ подчиняется фундаментальной оценке:

$$|\Phi(x)| \le \min\left(2\varepsilon,\ \frac{1}{\pi|x|},\ \frac{1}{\pi|x|}\left(\frac{r}{2\pi|x|\Delta}\right)^r\right).$$

Определим взвешенное количество решений неравенства (2):

$$R(N) = \sum_{X/2 < p_1, p_2, p_3 \le X} (\ln p_1 \ln p_2 \ln p_3) \cdot \varphi(p_1^c + p_2^c + p_3^c - N).$$

Применяя формулу обращения Фурье, приводим $R(N)$ к интегралу по всей числовой прямой:

$$R(N) = \int_{-\infty}^{\infty} S^3(x) e(-Nx) \Phi(x)\,dx, \quad \text{где} \quad S(x) = \sum_{X/2 < p \le X} \ln p \cdot e(p^c x). \tag{3}$$

Полагая параметры разделения спектра $\tau = X^{12/25 - c/2}$ и $K = X^{11/10 - c} \ln^{-6} X$, разобьем интеграл (3) на три составляющие:

$$R(N) = D_1 + D_2 + D_3 = \int_{|x| \le \tau} + \int_{\tau < |x| \le K} + \int_{|x| > K} S^3(x) e(-Nx) \Phi(x)\,dx.$$

1. Оценка тригонометрической суммы в промежуточной области (D_2). Главное аналитическое препятствие состоит в получении нетривиальной степенной оценки для $S(x)$

при $\tau \le |x| \le K$. Переходя к функции фон Мангольдта $\Lambda(n)$ и применяя тождество Вогана, раскладываем сумму на три составляющие $S(x) = S_1 - S_2 - S_3 + O(X^{1/2})$:

$$S_1 = \sum_{d \le X^{1/4}} \mu(d) \sum_{l \le X/d} \ln l \cdot e((ld)^c x), \quad S_2 = \sum_{k \le X^{1/2}} \sum_{m \le X/k} c_k e((km)^c x),$$

$$S_3 = \sum_{X^{1/4} < m \le X^{3/4}} \sum_{X^{1/4} < n \le X/m} a_m \Lambda(n) e((mn)^c x).$$

Суммы S_1 и $S_2^{(1)}$ (при $k \le X^{1/4}$) оцениваются стандартно с помощью одномерной леммы ван дер Корпута величиной $O(X^{11/14})$. Суммы типа II (S_3) приводятся к билинейным формам вида:

$$W = \sum_{L < n \le L_1} \sum_{M < m \le M_1} a_m b_n e((mn)^c x).$$

Применяя неравенство Коши–Буняковского, сглаживание по одной переменной и двумерный аппарат экспоненциальных пар (p, q), получаем усиленную оценку:

$$\max_{\tau \le |x| \le K} |S(x)| \ll X^{9/10} \ln^4 X. \tag{4}$$

Используя интегральное неравенство $\int_\tau^K |S(x)|^2 |\Phi(x)|\, dx \ll X \ln^4 X$, выводим пренебрежимость интеграла по промежуточной области:

$$|D_2| \le \max_{\tau \le |x| \le K} |S(x)| \int_\tau^K |S(x)|^2 |\Phi(x)|\, dx \ll \varepsilon \frac{X^{3-c}}{\ln X}.$$

2. Главный член в окрестности нуля (D_1). При $|x| \le \tau$ аппроксимируем сумму $S(x)$ интегралом $I(x) = \int_{X/2}^X e(t^c x)\, dt$. По явной формуле Римана–Мангольдта для функции Чебышева:

$$S(x) - I(x) = -\sum_{|\gamma| \le T} \int_{X/2}^X t^{\rho - 1} e(t^c x)\, dt + O\left(\frac{\tau X^{1+c}}{T} \ln^2 X \right).$$

Выбирая параметр высоты $T = X^{41/56 + c/4}$ и применяя современные теоремы плотности нулей дзета-функции Римана Ингхэма–Хаксли $\sum_{0 < \gamma \le T_1} X^{\beta}$, получаем экспоненциально малую погрешность $|S(x) - I(x)| \ll X \exp(-(\ln X)^{1/5})$. Отсюда следует асимптотическое равенство:

$$D_1 = \int_{-\infty}^{\infty} I^3(x) e(-Nx) \Phi(x)\, dx + O\left(\varepsilon X^{3-c} e^{-(\ln X)^{1/6}} \right) \gg \varepsilon X^{3-c}.$$

3. Оценка остаточной области (D_3). В силу выбора параметров весовой функции при $|x| > K$ выполнено $|\Phi(x)| \ll |x|^{-1} (r/(2\pi\Delta|x|))^r$, откуда тривиально следует $|D_3| \ll 1$.

Заключение. Объединяя полученные оценки для областей D_1, D_2, D_3, находим:

$$R(N) = D_1 + O(|D_2|) + O(|D_3|) \gg \varepsilon X^{3-c} \to \infty \quad (N \to \infty),$$

что завершает полное доказательство разрешимости диофантова неравенства (2).

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УДК 517.3

ИНТЕГРАЛНЫЕ ПРЕДСТАВЛЕНИЯ ДЛЯ УСЕЧЕННЫХ ДРОБНЫХ ПРОИЗВОДНЫХ ТИПА ψ–МАРШО

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В работах [1, 2] представлена обобщенная форма для дробных операторов Римана-Лиувилля, названных дробными операторами типа ψ –Римана-Лиувилля, получены и доказаны важные свойства новых обобщенных операторов ψ –Римана-Лиувилля в пространстве $L^p(a,b)$.

Дробные производные типа ψ –Маршо еще не изучены. В данной работе мы вводим новые обобщения дробных производных типа ψ –Маршо на отрезке. В настоящей работе рассматривается интегральное представление так называемой усеченной дробной производной ψ –Маршо.

Определение 1 ([1]). Пусть $\alpha > 0$, $-\infty \le a < b \le \infty$ и пусть функция φ интегрируема на отрезке $[a,b]$ и $\psi \in C^1([a,b])$ является положительной возрастающей функцией такой, что $\psi'(x) \neq 0$ для всех $x \in (a,b)$. Левосторонние и правосторонние дробные интегралы типа ψ –Римана-Лиувилля функции φ относительно другой функции ψ на отрезке $[a,b]$, с порядком α, определяются следующим образом:

$\left(I_{a+}^{\alpha,\psi}\varphi\right)(x) = \frac{1}{\Gamma(\alpha)}\int_a^x \psi'(t)\left(\psi(x)-\psi(t)\right)^{\alpha-1}\varphi(t)\,dt$ при $x > a$

и

$\left(I_{b-}^{\alpha,\psi}\varphi\right)(x) = \frac{1}{\Gamma(\alpha)}\int_x^b \psi'(t)\left(\psi(t)-\psi(x)\right)^{\alpha-1}\varphi(t)\,dt$ при $x < b$

соответственно.

Определение 2 ([1]). Пусть$\alpha > 0$, $n-1 < \alpha \le n$, $n \in \mathbb{N}$, $-\infty \le a < b \le \infty$ и пусть функция f интегрируема на отрезке [a,b] и $\psi \in C^1([a,b])$ является положительной возрастающей функцией такой, что $\psi'(x) \neq 0$ для всех $x \in (a,b)$. Левосторонние и правосторонние производные ψ –Римана-Лиувилля функции f относительно другой функции ψ на $[a,b]$, с порядком α, определяются следующим образом:

$$\left(D_{a+}^{\alpha,\psi}f\right)(x) = \left(\frac{1}{\psi'(x)}\frac{d}{dx}\right)^n I_{a+}^{n-\alpha,\psi}f(x), x > a \tag{1}$$

и

$$\left(D_{b-}^{\alpha,\psi}f\right)(x) = \left(-\frac{1}{\psi'(x)}\frac{d}{dx}\right)^n I_{b-}^{n-\alpha,\psi}f(x), x < b \tag{2}$$

соответственно, где$n = [\alpha]+1$. При $0 < \alpha < 1$ формулы (1) и (1) принимают вид

$$\left(D_{a+}^{\alpha,\psi}f\right)(x) = \frac{1}{\psi'(x)}\frac{d}{dx}\frac{1}{\Gamma(1-\alpha)}\int_a^x \psi'(t)\left(\psi(x)-\psi(t)\right)^{-\alpha}f(t)\,dt, x > a$$

и

$$\left(D_{b-}^{\alpha,\psi}f\right)(x)=-\frac{1}{\psi'(x)}\frac{d}{dx}\frac{1}{\Gamma(1-\alpha)}\int_x^b\psi'(t)\left(\psi(t)-\psi(x)\right)^{-\alpha}f(t)\,dt, x<b$$

соответственно.

При $0<\alpha<1,\ 0<\varepsilon<1$ мы полагаем:

$$\left(D_{a+,\varepsilon}^{\alpha,\psi}f\right)(x)=$$

$$=\begin{cases}\frac{f(x)}{\Gamma(1-\alpha)(\psi(x)-\psi(a))^{\alpha}}+\frac{\alpha}{\Gamma(1-\alpha)}\int_{\varepsilon}^{\psi(x)-\psi(a)}\frac{f(x)-f\left(\psi^{-1}(\psi(x)-\tau)\right)}{\tau^{\alpha+1}}d\tau, \delta(\varepsilon)\le x\le b,\\ \frac{f(x)}{\Gamma(1-\alpha)\varepsilon^{\alpha}}, \qquad a\le x\le\delta(\varepsilon),\end{cases}$$

которую будем называть «усеченной» дробной производной типа ψ − Маршо, где $\delta(\varepsilon)=\psi^{-1}(\psi(x)+\varepsilon)$.

Теорема 1. Пусть $f(x)=\left(I_{a+}^{\alpha,\psi}\varphi\right)(x)$, $\varphi\in L^p(a,b)$, где $0<\alpha<1$, $1\le p<\infty$, и $0<\varepsilon<1$ и пусть $\psi\in C^1([a,b])$ является положительной возрастающей функцией такой, что $\psi'(x)\neq0$ для всех $x\in(a,b)$. Тогда усеченная дробная производная $\left(D_{a+}^{\alpha,\psi}f\right)(x)$ имеет следующее интегральное представление

$$\left(D_{a+,\varepsilon}^{\alpha,\psi}f\right)(x)=\int_0^{r(x,\varepsilon)}K_{\alpha}^{+}(t)\,\varphi\left(\psi^{-1}(\psi(x)-\varepsilon t)\right)dt,$$

где $r(x,\varepsilon)=\frac{1}{\varepsilon}(\psi(x)-\psi(a))$, ядро $K_{\alpha}^{+}(t)=\frac{\sin\alpha\pi}{\pi}\left[t_{+}^{\alpha}-(t-1)_{+}^{\alpha}\right]\in L^1(\mathbb{R})$ и является усредняющим

$$\int_0^{\infty}K_{\alpha}^{+}(t)\,dt=1$$

и $K_{\alpha}^{+}(t)>0$(.**[3]**,.105).

Следствие. Пусть $f(x)=\left(I_{a+}^{\alpha,x}\varphi\right)(x)$, $\varphi\in L^p(a,b)$, где $0<\alpha<1$, $1\le p<\infty$, и $0<\varepsilon<1$. Тогда усеченная дробная производная $\left(D_{a+}^{\alpha,x}f\right)(x)$ имеет следующее интегральное представление

$$\left(D_{a+,\varepsilon}^{\alpha,\psi}f\right)(x)=\int_0^{\frac{x-a}{\varepsilon}}K_{\alpha}^{+}(t)\,\varphi(x-\varepsilon t)\,dt,$$

где ядро $K_{\alpha}^{+}(t)=\frac{\sin\alpha\pi}{\pi}\left[t_{+}^{\alpha}-(t-1)_{+}^{\alpha}\right]\in L^1(\mathbb{R})$.

Теорема 2. Пусть $f(x)=\left(I_{a+}^{\alpha,\psi}\phi\right)(x)$,$\varphi\in L^p([a,b])$, где $0<\alpha<1$, $1<p<\infty$, и пусть $\psi\in C^1([a,b])$ является положительной возрастающей функцией такой, что $\psi'(x)\neq0$ для всех $x\in[a,b]$. Тогда

$$\left\|(\psi(x)-\psi(a))^{-\alpha}\left(I_{a+}^{\alpha,\psi}\varphi\right)(x);\ L^p\right\|\le\frac{\Gamma\left(1-\frac{1}{p}\right)}{\Gamma\left(1+\alpha-\frac{1}{p}\right)}\left[\frac{\psi'(b)}{\psi'(a)}\right]^{1/p}\|\varphi;\ L^p\|.$$

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5 – SHO'BA. SONLI USULLAR VA MATEMATIK MODELLASHTIRISH

СЕКЦИЯ 5 – ЧИСЛЕННЫЕ МЕТОДЫ И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

УДК 539.3, 519.6

Silindrik jismni bir o'lchovli termoelastik jarayoni chekli elementlar usuli yordamida modellashtirish

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Ushbu ishda temperatura va tashqi mexanik yuklama ta'siridagi bir o'lchovli sterjenning termoelastik holatini chekli elementlar usuli asosida sonli modellashtirish masalasi qaralgan. Masalani yechish uchun issiqlik o'tkazuvchanlik va elastiklik jarayonlarini o'zaro bog'lovchi hisoblash algoritmi ishlab chiqilgan hamda Python dasturlash muhitida dasturiy ta'minot yaratilgan. Dastur yordamida sterjen bo'ylab temperatura taqsimoti, tugunlarning chiziqli siljishlari, elementlarning nisbiy deformatsiyasi va ichki termomexanik kuchlanishlar hisoblangan.

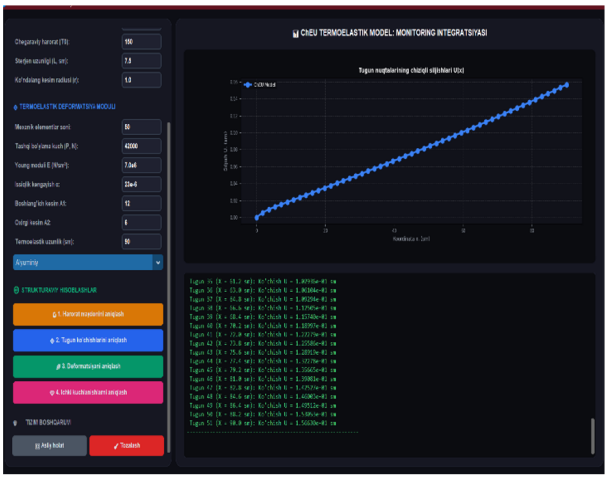
Konstruktiv materiallarda temperatura o'zgarishi ularning mexanik holatiga bevosita ta'sir ko'rsatadi. Mazkur tadqiqotda bir o'lchovli termoelastik masalani chekli elementlar usuli (ChEU) asosida yechish va natijalarni kompyuter yordamida vizuallashtirish masalasi qaraladi. Tadqiqotning asosiy maqsadi temperatura ta'sirida sterjenning kuchlanganlik-deformatsiyalanganlik holatini aniqlash hamda hisoblash jarayonini avtomatlashtirishdan iborat. Ishda issiqlik o'tkazuvchanlik va elastiklik tenglamalarining o'zaro bog'langan matematik modeli asosida hisoblash algoritmi ishlab chiqilgan. Hisoblash jarayonida sterjen chekli elementlarga ajratiladi va har bir element uchun tegishli issiqlik o'tkazuvchanlik hamda bikrlik matritsalari shakllantiriladi. Hosil qilingan mahalliy matritsalar asosida global algebraik tenglamalar sistemasi tuziladi. Chegaraviy shartlar hisobga olingandan so'ng sistema sonli usulda yechiladi.

Ishlab chiqilgan dasturda hisoblashlar quyidagi ketma-ketlikda amalga oshiriladi: dastlabki parametrlarni kiritish, material turini tanlash, temperatura maydonini hisoblash, tugun ko'chishlarini aniqlash, deformatsiyalarni hisoblash va ichki kuchlanishlarni aniqlash. Natijalar jadval va grafik ko'rinishda chiqariladi.

Blok-sxemadan ko'rinadiki, dasturda hisoblash jarayoni temperatura maydonini aniqlashdan boshlanib, undan keyin mexanik holat — siljish, deformatsiya va kuchlanishlar hisoblanadi. Bunday ketma-ketlik termoelastik jarayonlarning o'zaro bog'liqligini hisobga olish imkonini beradi. Hisoblash natijalari Ishlab chiqilgan modelning ishlash qobiliyatini tekshirish uchun test masalasi yechildi. Hisoblashlarda sterjenning boshlang'ich temperaturasi 150 °C, atrof-muhit temperaturasi 40 °C va konveksiya koeffitsiyenti h=10 qiymatlarida qaraldi. Temperaturaning sterjen bo'ylab taqsimlanishi chekli elementlar usuli yordamida aniqlandi.

1-rasmdagi grafikdan sterjenning boshlang'ich qismida temperatura yuqori bo'lib, koordinata ortishi bilan temperatura kamayib borishi ko'rinadi. Hisoblash natijalarini elementlar soniga bog'liq holda tekshirishda n=80 va n=160 holatlarida natijalar bir xil qiymatlarga kelgani aniqlandi. Jumladan, n=80 holatda tugunlardagi temperatura qiymatlari 150; 89,93; 62,74; 50,51; 45,21 va 43,34 °C ni tashkil qilgan. n=160 holatda ham xuddi shu qiymatlar olingan. Bu holat sonli yechimning barqarorlashganini ko'rsatadi. Temperatura maydoni aniqlangandan so'ng uning ta'sirida sterjenning tugunlaridagi chiziqli siljishlar hisoblandi. Hisoblashlarda tashqi mexanik yuklama P=42000 N deb qabul qilingan. 2-rasmda siljishning sterjen uzunligi bo'ylab





1-rasm.Sterjen bo‘ylab temperatura taqsimoti 2-rasm. Tugun nuqtalarining chiziqli siljishlari

o‘zgarishi keltirilgan. Birinchi tugun mahkamlanganligi sababli undagi siljish nolga teng. Keyingi tugunlarda esa siljish qiymati ortib boradi. Hisoblash natijalarida tugunlarning ko‘chishi mos ravishda 0; 0,03736; 0,06729 va 0,10319 sm qiymatlarni tashkil qilgan. Tugun siljishlari asosida elementlarning nisbiy deformatsiyalari hisoblandi. Deformatsiya

$$\varepsilon = (u(i+1) - u(i))/L$$

formula yordamida aniqlanadi.





3-rasm. Elementlar bo‘yicha nisbiy deformatsiya 4-rasm Tugunlardagi termomexanik kuchlanishlar

3-rasmdagi grafikdan deformatsiyaning elementlar bo‘yicha o‘zgarishi kuzatiladi. Natijalar temperatura maydoni va tashqi mexanik yuklamaning birgalikdagi ta’siri natijasida sterjenning turli qismlarida deformatsiya holati shakllanishini ko‘rsatadi. Hisoblashning yakuniy bosqichida deformatsiya va temperatura natijalari asosida sterjendagi termomexanik ichki kuchlanishlar aniqlandi. 4-rasmdagi natijalar ChEU yordamida aniqlangan kuchlanishlarning sterjen bo‘ylab taqsimlanishini ko‘rsatadi. Olingan natijalarni analitik yechim bilan taqqoslash uchun tugunlardagi kuchlanish qiymatlari solishtirildi. ChEU bo‘yicha hisoblangan kuchlanishlar 3598,4 dan 6582,9 N/sm? gacha o‘zgargan. Eng katta nisbiy farq sterjenning chekka qismida kuzatilib, 5,95 Olingan natijalar ishlab chiqilgan kompyuter modelining termoelastik masalani

yechishda yetarli aniqlik bilan ishlashini ko‘rsatadi. Shuningdek, dasturiy ta’minotda kirish parametrlarini o‘zgartirib, turli fizik-mexanik xususiyatlarga ega materiallar uchun ketma-ket hisoblash tajribalarini bajarish imkoniyati mavjud.

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HETEROGEN FAZOVIY-VAQTIY MA’LUMOTLARNI QAYTA ISHLASH ZANJIRIDA XATOLIKLARNING TARQALISHI VA OPTIMAL TO‘R QADAMI

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Turli tabiatga ega kuzatuv manbalaridan olingan fazoviy-vaqtiy ma’lumotlarni birgalikda qayta ishlash masalasi hisoblash matematikasi, geodeziya va deformatsiya monitoringi uchun dolzarb bo‘lib, u nazariy va amaliy jihatdan bir vaqtning o‘zida qiziqish uyg‘otadi. Masalaning dolzarbligi kuzatuv vositalarining rivojlanishi, yer usti va sun’iy yo‘ldosh ma’lumotlarining bir tizimda birlashtirilishi hamda hosila kattaliklarga — qiyalik, siljish tezligi va tezlanishga — qo‘yiladigan aniqlik talablarining ortishi bilan bog‘liqdir. O‘lchov noaniqligini baholash bilan bog‘liq turli masalalar ilmiy adabiyotlarda [1, 2, 4] va boshqa joylarda tasvirlangan.

Ushbu ilmiy tadqiqot ishida ketma-ket bosqichlardan tashkil topgan qayta ishlash zanjirida xatolikning bosqichdan bosqichga o‘tishi o‘rganiladi. Ishning maqsadi xatolik byudjetining yopiq ifodalarini keltirib chiqarish, ularni sonli modellashtirish orqali tekshirish hamda hosila kattaliklarni hisoblashda to‘r qadamini tanlashning miqdoriy mezonini olishdan iborat.

Aytaylik, $\Omega \subset \mathbb{R}^2$ chegaralangan sohada N ta nuqtada kuzatuv olingan bo‘lsin. Qayta ishlash to‘rt bosqichni o‘z ichiga oladi: dastlabki ishlov berish, sochma nuqtalarni muntazam to‘rga o‘tkazish, ma’lumotlarni birlashtirish va hosila kattaliklarni hisoblash. Har bir o‘lchovni uch komponentli model ko‘rinishida yozamiz

$$z_i = z(P_i) + b_k + \varepsilon_i + g_i, \qquad \varepsilon_i \sim N(0, \sigma^2), \tag{1}$$

bu yerda b_k – manbaga xos tizimli siljish, ε_i – markazlashgan tasodifiy xatolik, g_i – qo‘pol xatolik. K bosqichli zanjir uchun yakuniy dispersiya xatoliklarning tarqalishi qonuniga muvofiq

$$\sigma_{nat}^2 = \sum_{k=1}^{K} \left(\frac{\partial F}{\partial p_k} \right)^2 \sigma_k^2 \tag{2}$$

ko‘rinishida ifodalanadi, bunda $\partial F/\partial p_k$ – yakuniy natijaning k-bosqich parametriga sezgirligi. (2) dan muhim xulosa kelib chiqadi: butun tizim aniqligini eng katta hissali bitta bo‘g‘in belgilaydi va optimallashtirishni aynan shundan boshlash lozim.

Qayta ishlash amallari xatolikka ta’siri bo‘yicha ikki guruhga ajraladi. Birinchi guruh – o‘rtachalash amallari. Ular k ta qo‘shni kuzatuvni birlashtirgani uchun tasodifiy xatolikni kamaytiradi

$$\sigma_{tiklash} = \frac{\sigma_{o‘lchov}}{\sqrt{k}}. \tag{3}$$

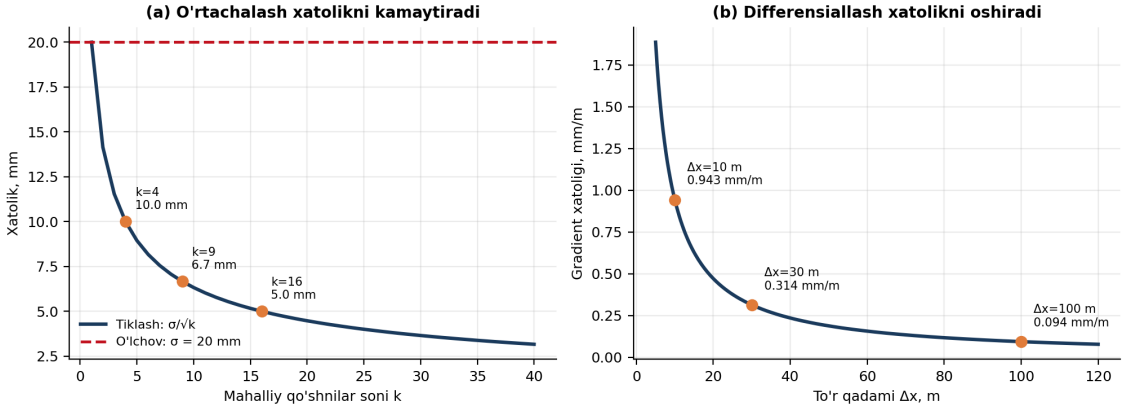
Ikkinchi guruh – differensiallash amallari. Ular ikki qiymatning ayirmasiga tayangani uchun xatolikni, aksincha, oshiradi

$$\sigma_{grad} = \frac{\sqrt{2}\,\sigma_{tiklash}}{\Delta x}. \tag{4}$$

(4) ifoda qo‘shni to‘r tugunlarining xatoliklari o‘zaro bog‘liq emas deb faraz qiladi. Amalda ular qisman bir xil kuzatuvlarga tayanadi va korrelyatsiyalangan bo‘ladi; korrelyatsiya koeffitsienti ρ hisobga olinganda

$$\sigma_{grad} = \frac{\sigma_{tiklash}\sqrt{2(1-\rho)}}{\Delta x} \tag{5}$$

hosil bo‘ladi, ya’ni (4) yuqoridan baho beradi. Ikki qonuniyatning qarama-qarshi xarakteri 1-rasmda ko‘rsatilgan.



1-rasm. Xatolikning o‘zgarishi: (a) o‘rtachalash bosqichida; (b) gradient hisoblash bosqichida

Ifodalar sonli tajribada tekshirildi. Ochiq kon bortlari monitoringiga xos shartlarda uch bosqichli zanjir qaraldi: dastlabki o‘lchov xatoligi 20 mm, tiklashda to‘qqizta qo‘shni hisobga olinadi, gradient esa 30 m qadamli to‘rda hisoblanadi. Bosqichma-bosqich natijalar 1-jadvalda keltirilgan.

1-jadval. Xatolikning zanjir bo‘ylab o‘zgarishi

Bosqich	**Amal**	**Xatolik**	**O‘zgarish**
1. O‘lchov	—	20,00 mm	—
2. Tiklash ($k = 9$)	o‘rtachalash	6,67 mm	3,0 barobar kamaydi
3. Gradient ($\Delta x = 30$ m)	differensiallash	0,314 mm/m	birlik o‘zgardi

Nazariy bahoning to‘g‘riligi Monte-Karlo modellashtirish bilan tekshirildi: o‘n ming sinovda tasodifiy xatoliklar generatsiya qilinib, butun zanjir bo‘ylab o‘tkazildi va yakuniy tarqoqlik o‘lchandi

$$\sigma_{nazariy} = 0{,}31427 \text{ mm/m}, \qquad \sigma_{empirik} = 0{,}31260 \text{ mm/m}, \qquad \delta = 0{,}53\%. \tag{6}$$

Farqning kichikligi (2)–(4) ifodalarga asoslangan modelning ichki izchilligini tasdiqlaydi. Tizimli siljish esa tasodifiy xatolikdan tubdan farq qiladi: u (3) bo‘yicha kamaymaydi va kuzatuvlar soni ortgani sari tobora ishonchliroq ko‘rina boshlaydi. Gradient bosqichida manzara teskari bo‘lib, fazoviy jihatdan doimiy siljish ayirmada qisqaradi va natijaga umuman o‘tmaydi; faqat fazoviy o‘zgaruvchi qismi saqlanib qoladi.

Turli qadam va qo‘shnilar soni uchun hisoblangan gradient xatoligi 2-jadvalda keltirilgan.

2-jadval. Gradient xatoligining parametrlarga bog‘liqligi, mm/m

Qo‘shnilar soni k	$\Delta x = 10$ **m**	$\Delta x = 30$ **m**	$\Delta x = 100$ **m**
4	1,414	0,471	0,141
9	0,943	0,314	0,094
16	0,707	0,236	0,071

Jadvaldan aniq nomutanosiblik ko‘rinadi: qadamni uch barobar oshirish xatolikni uch barobar kamaytiradi, qo‘shnilar sonini to‘rt barobar oshirish esa faqat ikki barobar. Demak fazoviy aniqlikni qurbon qilish qo‘shimcha kuzatuv jalb qilishdan samaraliroq, biroq bu mayda detallarni yo‘qotish hisobiga amalga oshadi.

Aynan shu qarama-qarshilik optimal to‘r qadamini belgilaydi. Markaziy ayirmali sxema uchun umumiy xatolik ikki haddan iborat bo‘lib, shovqin hissasi qadamga teskari, kesish xatoligi esa qadam kvadratiga proportsional o‘sadi

$$E(\Delta x) = \frac{A}{\Delta x} + \frac{M_3\,\Delta x^2}{6}, \qquad A = \sigma_{tiklash}\sqrt{2(1-\rho)}, \qquad M_3 = \max_{\Omega} \left| z''' \right|. \tag{7}$$

$E(\Delta x)$ ni minimallashtirish optimal qadam uchun quyidagi ifodani beradi

$$\Delta x_{opt} = \left(\frac{3A}{M_3} \right)^{1/3}. \tag{8}$$

Yuqoridagi sonli misolda $A = 9{,}43$ mm va $M_3 = 10^{-3}$ mm/m^3 bo‘lganda $\Delta x_{opt} \approx 30{,}5$ m chiqadi, umumiy xatolik esa 0,46 mm/m ni tashkil etadi. Demak tanlangan 30 m qadam optimalga juda yaqin. (8) ifodaning amaliy qiymati shundaki, u to‘r qadamini urinib ko‘rish yo‘li bilan emas, kuzatilayotgan maydonning silliqligi orqali oldindan hisoblash imkonini beradi.

Sonli eksperimentlar o‘tkazilgan. Ularning natijalarining tahlili shuni ko‘rsatadiki, hosila kattaliklar har doim asosiy kattaliklardan kam aniq bo‘ladi va ularni talqin qilishda buni hisobga olish lozim, juda mayda to‘r qadami esa aniqlikni oshirish o‘rniga uni keskin yomonlashtiradi. Aniq formulalar asosida olingan natijalarni ochiq kon bortlari deformatsiyasini kuzatish tizimlarini loyihalashda, heterogen fazoviy-vaqtiy ma’lumotlarni qayta ishlash algoritmlarini ishlab chiqishda hamda monitoring dasturlarining aniqlik ko‘rsatkichlarini asoslashda foydalanish mumkin.

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UDC 519.6

A TWO-LEVEL MULTIGRID METHOD WITH A PHYSICS-INFORMED NEURAL NETWORK ON THE COARSE LEVEL

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Multigrid methods efficiently solve discretized elliptic partial differential equations by combining fine-grid smoothing with coarse-grid correction [1–3]. While classical relaxation rapidly reduces high-frequency errors, the remaining smooth error is transferred to a coarser grid and corrected there. In this work, we propose a hybrid two-level multigrid method in which a physics-informed neural network (PINN) [4,5] replaces the conventional coarse-grid solver. The fine-grid problem is treated by weighted Jacobi smoothing, while the PINN approximately solves the coarse-grid residual equation using a physics-informed loss function. Since the error remaining after smoothing is predominantly smooth, the neural network only needs to approximate the coarse error correction rather than the complete fine-grid solution.

The proposed approach is tested on a one-dimensional Poisson problem with a known analytical solution. Using 127 fine-grid and 63 coarse-grid interior points, the PINN-based method reduces the relative residual from 1 to $3.70 \cdot 10^{-2}$ after one multigrid cycle and to $1.54 \cdot 10^{-4}$ after two cycles. The relative solution error reaches approximately $5.0 \cdot 10^{-5}$, which is close to the discretization-error level. The first-cycle convergence is nearly identical to that obtained with an exact coarse-grid solver. These results demonstrate that a PINN can effectively approximate smooth coarse-grid corrections and can be successfully incorporated into a classical multigrid framework.

Consider the one-dimensional Poisson problem

$$-u''(x) = f(x), \qquad x \in (0,1), \tag{1}$$

with homogeneous Dirichlet boundary conditions

$$u(0) = u(1) = 0. \tag{2}$$

Let

$$x_i = ih, \qquad h = \frac{1}{N+1}, \qquad i = 0, \ldots, N+1.$$

The standard second-order finite-difference discretization of (1) gives

$$A_h u_h = f_h, \tag{3}$$

where

$$A_h = \frac{1}{h^2} \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -1 & 2 & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{pmatrix}.$$

Let $u_h^{(k)}$ be the current approximation. First, ν_1 weighted Jacobi iterations are applied,

$$u_h^{(k,j+1)} = u_h^{(k,j)} + \omega D_h^{-1} \left(f_h - A_h u_h^{(k,j)} \right), \tag{4}$$

where $D_h = \mathrm{diag}(A_h)$ and ω denotes the damping parameter. These iterations efficiently reduce oscillatory error components.

After pre-smoothing, the residual is

$$r_h = f_h - A_h u_h. \tag{5}$$

Let $H = 2h$ and let I_h^H denote the full-weighting restriction operator. The coarse-grid residual is given by

$$r_H = I_h^H r_h, \qquad (r_H)_j = \frac{1}{4}(r_h)_{2j-1} + \frac{1}{2}(r_h)_{2j} + \frac{1}{4}(r_h)_{2j+1}. \tag{6}$$

In the classical two-grid method, one solves the coarse error equation

$$A_H e_H = r_H. \tag{7}$$

The principal modification considered here is to replace the conventional solution of (7) by a physics-informed neural-network approximation.

Let $N_\theta(x)$ be a fully connected neural network with trainable parameters θ. Since the coarse-level error satisfies homogeneous boundary conditions, we introduce

$$e_\theta(x) = x(1-x)N_\theta(x). \tag{8}$$

Consequently,

$$e_\theta(0) = e_\theta(1) = 0$$

is satisfied exactly without introducing an additional boundary penalty in the loss function.

Let x_j^H, $j = 1, \ldots, N_H$, denote the interior coarse-grid points and define

$$\mathbf{e}_\theta = \left(e_\theta(x_1^H), \ldots, e_\theta(x_{N_H}^H)\right)^T.$$

The network is trained by minimizing the discrete physics-informed loss

$$\mathcal{L}(\theta) = \frac{1}{N_H} \left\| A_H \mathbf{e}_\theta - r_H \right\|_2^2. \tag{9}$$

Hence, no exact coarse-grid solution is used during training. The restricted residual and the coarse differential operator provide the training information.

After training, the neural correction is transferred to the fine grid by linear interpolation:

$$u_h \leftarrow u_h + I_H^h \mathbf{e}_\theta. \tag{10}$$

Finally, ν_2 post-smoothing iterations of (4) are performed. Thus, one cycle of the proposed method can be summarized as

$$\begin{gathered} \text{pre-smoothing} \longrightarrow r_h = f_h - A_h u_h \longrightarrow r_H = I_h^H r_h \\ \longrightarrow \text{PINN coarse solve} \longrightarrow u_h + I_H^h \mathbf{e}_\theta \longrightarrow \text{post-smoothing}. \end{gathered} \tag{11}$$

The important feature of this construction is the separation of error scales. The classical smoother treats the oscillatory components, whereas the neural network is applied to the smoother coarse-level correction. This task is substantially simpler than asking a neural network to approximate the complete fine-grid solution.

To demonstrate the approach, consider

$$-u''(x) = \pi^2 \sin(\pi x), \qquad x \in (0,1), \tag{12}$$

with homogeneous Dirichlet boundary conditions. The analytical solution is

$$u_{\text{ex}}(x) = \sin(\pi x). \tag{13}$$

For the numerical experiment, $N = 127$ interior fine-grid points and $N_H = 63$ coarse-grid points are used. Three weighted Jacobi iterations are applied before and after coarse correction, $\nu_1 = \nu_2 = 3$, with $\omega = 2/3$. The PINN consists of two hidden layers with 32 neurons per layer and hyperbolic tangent activation functions. The Adam algorithm is used to minimize (9).

The relative solution error and relative residual are defined by

$$E_k = \frac{\|u_h^{(k)} - u_{\text{ex}}\|_2}{\|u_{\text{ex}}\|_2}, \qquad R_k = \frac{\|f_h - A_h u_h^{(k)}\|_2}{\|f_h\|_2}. \tag{14}$$

The numerical results for the PINN-based two-level method are presented in Table 1.

Table 1. Convergence of the PINN-based two-level method

k	Relative error E_k	Relative residual R_k
0	1.0000	1.0000
1	$1.34 \cdot 10^{-4}$	$3.70 \cdot 10^{-2}$
2	$5.01 \cdot 10^{-5}$	$1.54 \cdot 10^{-4}$
3	$5.02 \cdot 10^{-5}$	$4.39 \cdot 10^{-5}$
4	$5.02 \cdot 10^{-5}$	$3.53 \cdot 10^{-5}$

For comparison, when (7) is solved exactly, the relative residuals after the first three two-grid cycles are approximately

$$3.69 \cdot 10^{-2}, \qquad 5.60 \cdot 10^{-5}, \qquad 8.50 \cdot 10^{-8}.$$

Figure 1 compares these values with the convergence of the proposed PINN coarse solver.

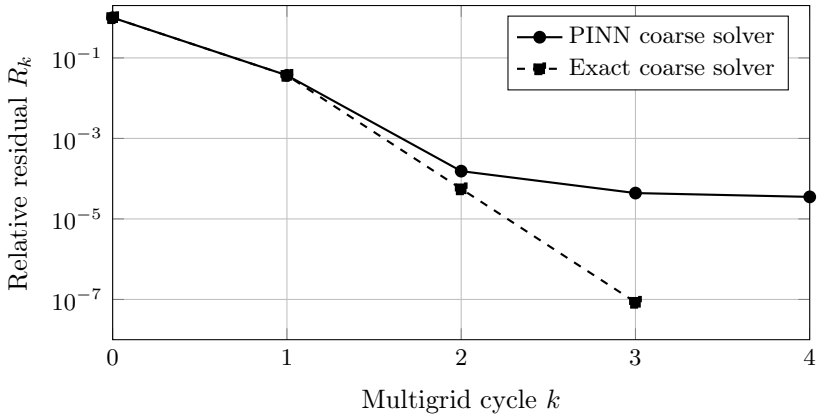


Рис. 6: Relative residual versus two-level multigrid cycle for the PINN and exact coarse-grid solvers.

As can be seen from Fig. 6, both methods show nearly identical behavior during the first cycle. The PINN method gives $R_1 = 3.70 \cdot 10^{-2}$, while the exact coarse solver gives $R_1 = 3.69 \cdot 10^{-2}$. This indicates that the neural network successfully captures the dominant smooth error correction generated after the first smoothing stage.

After two cycles, the PINN method reduces the residual to $1.54 \cdot 10^{-4}$, whereas the exact coarse solver reaches $5.60 \cdot 10^{-5}$. Thus, both methods reduce the initial residual by approximately four orders of magnitude within two cycles. The difference becomes more pronounced during subsequent iterations: the exact coarse solve reaches $8.50 \cdot 10^{-8}$ after the third cycle, while the

PINN residual approaches a level of order 10^{-5}. This stagnation is associated with the finite accuracy of the neural optimization in the coarse-grid equation.

At the same time, the relative solution error approaches approximately $5.0 \cdot 10^{-5}$. This value is essentially the discretization error associated with the second-order finite difference approximation on the selected grid. Therefore, reducing the algebraic residual considerably below this level does not result in a comparable improvement of the error with respect to the continuous solution (13).

For the simple one-dimensional Poisson equation, solving the small coarse system directly is clearly cheaper than training a neural network. The purpose of this example is therefore not to claim a computational advantage over classical multigrid, but to demonstrate the feasibility of a PINN as an approximate coarse solver. The approach becomes potentially more attractive when the coarse problem is nonlinear or parameter-dependent, or when the trained network can be reused for many related problems.

In particular, instead of initializing the PINN independently at every multigrid cycle, the network parameters obtained from a previous cycle can be used as an initial approximation for the next one. For a family of parameter-dependent PDEs, the physical parameters can also be included as neural-network inputs. In such situations, the training cost may be amortized over many subsequent coarse solves.

Thus, a two-level multigrid method with a physics-informed neural-network coarse solver has been proposed. The numerical experiment demonstrates that the neural correction can effectively represent the smooth error remaining after fine-grid relaxation. The PINN-based method produces a substantial residual reduction during the first multigrid cycles and reaches the discretization-error level of the considered finite-difference problem. Further investigations will consider two-dimensional problems, nonlinear equations, parameter-dependent operators and pretrained neural coarse solvers.

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UDK 539.3

Optimal Quadrature Formulas for Cauchy-Type Integrals with Trigonometric Weights in the space $L_2^{(1)}(-1,1)$.

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This study develops optimal quadrature formulas for the numerical evaluation of singular integrals with highly oscillations in the space $L_2^{(1)}(-1,1)$. The quadrature coefficients are obtained by minimizing the norm of the corresponding error functional using the Sobolev method. Explicit analytical expressions for the coefficients are derived, accounting for both the singularity and rapid oscillations of the integrand. Numerical results demonstrate the accuracy and effectiveness of the proposed formulas. The developed approach provides a rigorous and efficient framework for numerical treatment of singular oscillatory integrals and related integral operators.

This work is devoted to the construction of the optimal quadrature formula for the approximate calculation of highly oscillatory Cauchy-type singular integrals. In this regard, we will consider the following quadrature formula

$$\int_{-1}^{1} \frac{\sin(\omega x)\varphi(x)}{x-t}\,dx \cong \sum_{\beta=0}^{N} A_\beta\,\varphi(x_\beta). \tag{1}$$

where $-1 < t < 1$, $x_\beta = h\beta - 1$ are the nodes of the quadrature formula, $h = \frac{2}{N}$, N is a natural number, $\beta = \overline{0,N}$, $\varphi(x) \in L_2^{(1)}(-1,1)$, and $\omega \neq 0$. A_β are the coefficients of the quadrature formula (1). The difference between the integral and the the corresponding quadrature sum

$$(R,\varphi) = \int_{-1}^{1} \frac{\sin(\omega x)\varphi(x)}{x-t}\,dx - \sum_{\beta=0}^{N} A_\beta\,\varphi(x_\beta). \tag{2}$$

is called the error of the quadrature formula (**??**).

The space $L_2^{(1)}(-1,1)$ is a Hilbert space, and the inner product of two functions is defined as follows:

$$\langle \varphi, \psi \rangle = \int_{-1}^{1} \varphi'(x)\ \psi'(x)dx,$$

The norm of the function is defined by the following formula:

$$\|\varphi\|^2_{L_2^{(1)}} = \langle \varphi, \varphi \rangle = \int_{-1}^{1} (\varphi'(x))^2\,dx.$$

The error functional corresponding to the error is given by

$$R(x) = \frac{\varepsilon_{[-1,1]}(x)\sin(\omega x)}{x-t} - \sum_{\beta=0}^{N} A_\beta\,\delta\left(x - x_\beta\right). \tag{3}$$

where $\varepsilon_{[-1,1]}(x)$ is the indicator of the interval $[-1,1]$, and $\delta(x)$ is Dirac's delta-function. By the Riesz theorem, there exists a unique function function $\psi_R \in L_2^{(1)}(-1,1)$ such that

$$(R,\varphi) = \langle \psi_R, \varphi \rangle, \qquad \|R\|_{L_2^{(1)*}(-1,1)} = \|\psi_R\|_{L_2^{(1)}(-1,1)}.$$

According to the Sobolev method, the extremal function ψ_R corresponding to the error functional R has the form

$$\psi_R(x) = -\big(R * G_1\big)(x) - P_0, \qquad G_1(x) = \frac{|x|}{2}.$$

After substituting the extremal function and expanding the convolution terms, we obtain the following theorem.

Theorem 1. *The squared norm of the error functional R corresponding to the quadrature formula is given by*

$$\|R\|^2_{L_2^{(1)*}(-1,1)} = -\Bigg[\int_{-1}^{1}\int_{-1}^{1} \frac{\sin(\omega x)\sin(\omega y)|x-y|}{2(x-t)(y-t)}\,dx\,dy$$

$$-\sum_{\beta=0}^{N} A_\beta \int_{-1}^{1} \frac{\sin(\omega x)|x-x_\beta|}{x-t}\,dx + \sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} A_\beta A_\gamma \frac{|x_\beta - x_\gamma|}{2}\Bigg].$$

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UDK 519.6

HETEROGEN MANBALARDAN OLINGAN BAHOLARNI BIRLASHTIRISHNING SONLI TAHLILI

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Bir necha manbadan olingan o‘lchov natijalarini birlashtirish masalasi zamonaviy monitoring tizimlarini loyihalashda dolzarb bo‘lib, u matematik statistika, sonli usullar va amaliy geodeziyada bir vaqtning o‘zida qiziqish uyg‘otadi. Masalaning dolzarbligi kuzatuv vositalarining tez rivojlanishi, turli tabiatga ega o‘lchov kanallarining bir tizimda birlashtirilishi hamda

yakuniy natijaning ishonchliligiga qo‘yiladigan talablarning ortishi bilan bog‘liqdir. Heterogen ma’lumotlarni birlashtirish bilan bog‘liq turli masalalar ilmiy adabiyotlarda [1, 2, 4] va boshqa joylarda tasvirlangan.

Ushbu ilmiy tadqiqot ishida o‘lchov aniqligi bo‘yicha sezilarli farq qiluvchi manbalardan olingan baholarni yagona bahoga keltirish masalasi o‘rganiladi. Ishning maqsadi optimal og‘irliklarning yopiq ifodasini keltirib chiqarish, qo‘shimcha manba jalb qilishning samarasini miqdoriy baholash hamda olingan ifodalarni sonli modellashtirish orqali tekshirishdan iborat.

Aytaylik, bitta noma’lum z kattalik haqida m ta o‘zaro bog‘liq bo‘lmagan manbadan z_k baholar olingan bo‘lsin va ularning har biri σ_k noaniqlik bilan tavsiflansin ($k = 1, 2, \ldots, m$). Har bir bahoni uch komponentli model ko‘rinishida yozamiz

$$z_k = z + b_k + \varepsilon_k, \qquad \varepsilon_k \sim N(0, \sigma_k^2), \tag{1}$$

bu yerda b_k – manbaga xos tizimli siljish, ε_k – markazlashgan tasodifiy xatolik. Birlashtirilgan baho baholarning chiziqli kombinatsiyasi sifatida quriladi

$$z^* = \sum_{k=1}^{m} w_k z_k, \qquad \sum_{k=1}^{m} w_k = 1. \tag{2}$$

(2) dagi normallash sharti bahoning siljimaganligini ta’minlaydi. Og‘irliklar z^* bahoning dispersiyasini minimallashtirish talabidan aniqlanadi. Shartli ekstremum masalasi Lagranj ko‘paytuvchilari usuli bilan yechiladi va natijada og‘irliklar noaniqlik kvadratiga teskari proportsional bo‘lib chiqadi

$$w_k = \frac{1/\sigma_k^2}{\sum_{j=1}^{m} 1/\sigma_j^2}. \tag{3}$$

Birlashtirilgan bahoning dispersiyasi esa teskari dispersiyalar yig‘indisi orqali to‘planadi

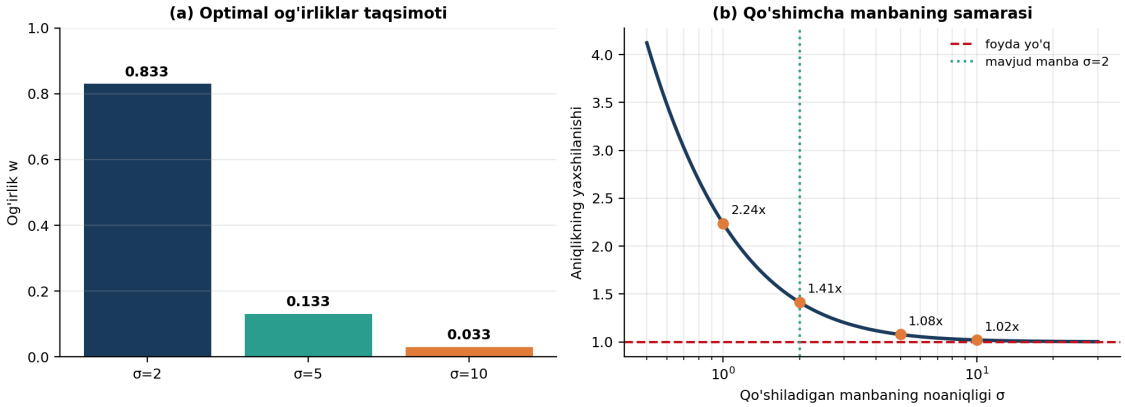
$$\frac{1}{\sigma^2} = \sum_{k=1}^{m} \frac{1}{\sigma_k^2}. \tag{4}$$

(3) va (4) munosabatlar baholarning o‘zaro bog‘liq emasligiga tayanadi. Manbalar umumiy tashqi omilga bog‘liq bo‘lganda ularning xatoliklari korrelyatsiyalanadi va (4) ifoda noaniqlikni past baholaydi. Bunday holatda kovariatsiya matritsasi $\mathbf{\Sigma}$ orqali umumiy ifoda o‘rinli bo‘ladi

$$\mathbf{w} = \frac{\mathbf{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{\Sigma}^{-1}\mathbf{1}}, \tag{5}$$

bunda $\mathbf{1}$ – barcha komponentalari birga teng bo‘lgan vektor; (3) uning diagonal $\mathbf{\Sigma}$ uchun xususiy holi hisoblanadi.

Olingan ifodalar sonli tajribada tekshirildi. Noaniqliklari mos ravishda 2, 5 va 10 birlikni tashkil etuvchi uchta manba qaraldi. (3) formula bo‘yicha hisoblangan og‘irliklar hamda qo‘shimcha manbaning aniqlikka ta’siri 1-rasmda keltirilgan.



1-rasm. Birlashtirish samaradorligi: (a) optimal og‘irliklar taqsimoti; (b) qo‘shimcha manbaning aniqlikka ta’siri

Chap paneldan ko‘rinadiki, og‘irliklar taqsimoti keskin nomutanosib: eng aniq manba og‘irlikning 83,3% ini, eng noaniq manba esa atigi 3,3% ini oladi. (4) ga ko‘ra birlashtirilgan bahoning noaniqligi 1,826 birlikka teng bo‘lib, u eng aniq manbadan 1,095 barobar yaxshiroqdir. Uch usulning qiyosiy natijalari 1-jadvalda keltirilgan.

1-jadval. Birlashtirish usullarining taqqoslanishi

Usul	**Noaniqlik**	**Eng aniq manbaga nisbatan**	**Baho**
Og‘irliklangan (optimal)	1,826	1,095 barobar yaxshi	maqbul
Eng aniq manbani olish	2,000	—	qabul qilinadi
Arifmetik o‘rtacha	3,786	1,893 barobar yomon	yaroqsiz

Jadvaldan ko‘rinadiki, arifmetik o‘rtacha optimal usuldan 2,07 barobar va hatto eng aniq manbadan ham 1,89 barobar yomon natijaga olib keladi, ya’ni sifatsiz manbani o‘ylamay qo‘shish natijani yaxshilash o‘rniga buzadi. Nazariy baholarning to‘g‘riligi Monte-Karlo modellashtirish bilan tasdiqlandi: ikki yuz ming sinovda empirik tarqoqlik 1,822 ni tashkil etib, nazariy qiymatdan 0,2% ga farq qildi.

Qo‘shimcha manba jalb qilishning samarasi alohida ko‘rib chiqildi. Mavjud noaniqlik σ_0 bo‘lganda yangi manbaning bergan yutug‘i faqat noaniqliklar nisbatiga bog‘liq

$$G = \frac{\sigma_0}{\sigma} = \sqrt{1 + \frac{\sigma_0^2}{\sigma_{yangi}^2}}. \tag{6}$$

(6) dan ko‘rinadiki, $\sigma_{yangi} \gg \sigma_0$ bo‘lganda $G \to 1$ va yaxshilanish yo‘qoladi. Mavjud manba $\sigma_0 = 2{,}0$ bo‘lgan hol uchun hisob 2-jadvalda keltirilgan.

2-jadval. Qo‘shimcha manbaning samarasi (mavjud manba $\sigma_0 = 2{,}0$)

Yangi manba σ	**Birlashgan σ**	**Yaxshilanish**	**Xulosa**
1,0 (aniqroq)	0,894	2,24 barobar	juda foydali
2,0 (teng)	1,414	1,41 barobar	foydali
5,0 (2,5 barobar noaniq)	1,857	1,08 barobar	kam foyda
10,0 (5 barobar noaniq)	1,961	1,02 barobar	deyarli foydasiz
20,0 (10 barobar noaniq)	1,990	1,005 barobar	foydasiz

Jadvaldan aniq qonuniyat ko‘rinadi: teng aniqlikdagi manba natijani 1,41 barobar yaxshilaydi, mavjud manbadan ikki yarim barobar noaniq manba atigi 8% ga, besh barobar noaniq manba esa

2% ga yaxshilaydi. Bu monitoring dasturini rejalashtirishda qo‘shimcha o‘lchovga sarflanadigan resursning qaytimini oldindan baholash imkonini beradi.

Keltirilgan apparat baholarning siljimaganligiga ham tayanadi. Manbalardan birida $b_k \neq 0$ bo‘lsa, og‘irliklangan o‘rtacha siljishni bartaraf etmaydi, balki uni yakuniy natijaga o‘tkazadi; aynan eng aniq manba katta og‘irlik olgani uchun uning siljishi eng katta zarar keltiradi. Shu sababli birlashtirishdan oldin manbalar orasidagi nomuvofiqlikni tekshirish zarur. Juftlik farqi uchun tuzilgan statistika

$$\chi^2 = \frac{(z_1 - z_2)^2}{\sigma_1^2 + \sigma_2^2} \tag{7}$$

bir erkinlik darajali χ^2 taqsimotiga bo‘ysunadi; undan keskin og‘ish yo tizimli siljish, yo noaniqlikning noto‘g‘ri baholanganidan darak beradi.

Sonli eksperimentlar o‘tkazilgan. Ularning natijalarining tahlili shuni ko‘rsatadiki, heterogen ma’lumotlarda arifmetik o‘rtachaga tayanish yakuniy aniqlikni ikki barobardan ortiq yomonlashtiradi, sifatsiz manbani o‘ylamay qo‘shish esa deyarli hech qanday foyda bermaydi. Aniq formulalar asosida olingan natijalarni o‘lchov tizimlarini loyihalashda, geodezik va masofadan zondlash ma’lumotlarini birgalikda qayta ishlashda hamda monitoring dasturlarining texnik-iqtisodiy asoslashda foydalanish mumkin.

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MAGNITLARDAGI AYLANMA TO’LQINLARNING MATEMATIK MODELINI O’RGANISH

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Kuchli dispersiv muhitda to’lqinlarning tarqalishi bilan bog’liq bo’lgan geofizika, okeanologiya, atmosfera fizikasi va magnitli tartiblangan sistemalar fizikasining ko’plab muammolari vaqt bo’yicha en yuqori hosilaga nisbatan yechib bo’lmaydigan chiziqli xususiy hosilali differensial tenglamalarga keltiriladi. Bunday tenglamalar S.L. Sobolev [1] ilmiy faoliyatidan boshlab tizimli o’rganila boshlangan bo’lib, Sobolev tipidagi tenglamalar deb ataladi. Ularni o’rganish boshlang’ich-chegaraviy masalalarning turli shakllarda yechilishini isbotlash, yechimlar asimptotikasini qurish, spektral masalalar va sonli hisoblashlar kabi yo’nalishlarda olib boriladi [1–3]. Sobolev tipidagi tenglamalardan biri magnitlardagi aylanma to’lqinlar tenglamasidir.

Masalaning qo'yilishi. Magnitlardagi aylanma to'lqinlarning tenglamasi quyidagi ko'rinishga ega [1]:

$$\left(\frac{\partial^2}{\partial t^2}+\omega_1^2\right)\Delta_3 u(x,t)+\omega_2^2\Delta_2 u(x,t)=f(x,t). \tag{1}$$

Bu yerda $\omega_1=\gamma(H_0+\beta M_0)$, $\omega_2=\gamma\sqrt{4\pi M_0(H_0+\beta M_0)}$, $\gamma=|ge|/(2mc)$, g – ferromagnitning gidromagnit nisbati, $\beta=K/M_0^2$, $M_0=m_0e_3$ – "yengil o'q" (e_3) tipidagi ferromagnetika, K – const., $H_0=H_0e_3$ – tashqi maydon, c – yorug'lik tezligi, e – elektron zaryadining absolyut qiymati.

(1) tenglamani quyidagi ko'rinishda yozib olinadi:

$$\frac{\partial^2}{\partial t^2}\Delta_3 u+\omega_1^2\Delta_3 u+\omega_2^2\Delta_2 u=f(x,t),\qquad (x,t)\in Q_T. \tag{2}$$

Tenglamaning boshlang'ich shartlari:

$$u(x,0)=u_0(x),\qquad \frac{\partial u}{\partial t}(x,0)=u_1(x),\qquad t=0,\ x\in\overline{\Omega}, \tag{3}$$

va chegaraviy shartlar:

$$u(x,t)=\mu(x,t),\qquad x\in\Gamma=\partial\Omega,\quad t\in(0,T]. \tag{4}$$

Bu yerda

$$\overline{\Omega}=\Omega+\Gamma,\qquad \Omega=\{0<x_k<l_k,\ k=1,2,3\},\qquad Q_T=\{(x,t):x\in\Omega,\ t\in(0,T]\}.$$

(2)–(4) masala uchun umumlashgan masalani qo'yamiz. Har bir $t\in[0,T]$ uchun $H=W_2^2(\Omega)\cap\mathring{W}_2^1(\Omega)$ ga tegishli bo'lgan va $\partial^2u/\partial t^2\in W_2^2(\Omega)$ hosilaga ega, barcha $t\in[0,T]$ va $\forall\vartheta(x)\in H$ uchun quyidagi tenglamani qanoatlantiruvchi $u(x,t)$ funksiya berilgan masalaning umumlashgan yechimi deyiladi:

$$a_3\left(\frac{d^2u(t)}{dt^2},\vartheta\right)+a_2(u(t),\vartheta)+a_1(u(t),\vartheta)=(f(t),\vartheta), \tag{5}$$

$$(u(0)-u_0,\vartheta)=0,\qquad \left(\frac{du}{dt}(0)-u_1,\vartheta\right)=0,\qquad \forall\vartheta(t)\in H. \tag{6}$$

Bu yerda

$$a_3(u,\vartheta)=\int_\Omega\sum_{k=1}^{3}u_{x_k}\vartheta_{x_k}\,dx,\qquad a_2(u,\vartheta)=\omega_1^2a_3(u,\vartheta),$$

$$a_1(u,\vartheta)=\omega_2^2\int_\Omega\sum_{k=1}^{2}u_{x_k}\vartheta_{x_k}\,dx.$$

$u=u(t)\in H=W_2^1(\Omega)$ sohadagi $t\in[0,T]$ abstrakt argumentning $\overline{\Omega}$ sohasining Γ chegarasida nolga aylanadigan funksiyasi, bunda soha skalyar ko'paytmaga va

$$\|u\|_1=\left(\int_\Omega\sum_{k=1}^{3}(u_{x_k})^2\,dx\right)^{1/2},\qquad dx=dx_1dx_2dx_3$$

normaga ega Sobolev fazosi. Bundan,

$$0\le a_1(u,u)\le C_1\|u\|_1^2,\qquad c_2\|u\|_1^2\le a_2(u,u)\le C_2\|u\|_1^2,\qquad c_3\|u\|_1^2\le a_3(u,u)\le C_3\|u\|_1^2$$

baholari o'rinli, bu yerda C_1, C_2, C_3, c_2, c_3 – musbat sonlar.

Sonli usul. Magnitlardagi spin to'lqinlar tenglamasi uchun boshlang'ich-chegaraviy masalalarni o'rganish muhim masalalardan biri hisoblanadi. Bu tenglamalarni dastlab faqat fazoviy o'zgaruvchilar bo'yicha chekli elementlar usulida approksimatsiyalash orqali vaqt o'zgaruvchisi differensial tarzda saqlanib qoladi. Natijada ikkinchi tartibli oddiy differensial tenglamalar sistemasi olinadi va olingan differensial tenglama chekli elementlar usuli asosida qurilgan ayirmali sxemalari bilan yechiladi [1–3]. Ayirmali sxemalarning approksimatsiya xatoligi baholari va turg'unlik shartlari o'rganiladi. Ushbu ayirmali sxemalardan foydalanib magnitlardagi aylanma to'lqinlar tenglamasini yechish va olingan algoritm asosida dasturlash tillaridan foydalanib dastur tuzish ko'zda tutiladi.

Xulosa. Ishda magnitlardagi aylanma to'lqinlar tenglamasi uchun boshlang'ich-chegaraviy masalaning matematik formulirovkasi, uning umumlashgan yechimi va chekli elementlar usuliga asoslangan sonli yondashuv ko'rib chiqildi. Fazoviy approksimatsiya natijasida hosil bo'ladigan ikkinchi tartibli oddiy differensial tenglamalar sistemasini ayirmali sxemalar yordamida yechish yondashuvi belgilandi. Sxemaning aniq approksimatsiya xatoligi va turg'unlik baholari hamda hisoblash natijalari manba materialida batafsil keltirilmaganligi sababli ushbu tezisda ular uchun qo'shimcha, taxminiy sonli natijalar kiritilmadi.

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MATHEMATICAL MODELING OF SPATIAL OBJECT RECONSTRUCTION FROM INTEGRAL DATA

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Abstract. A mathematical model for reconstructing a spatial object from integral data over a family of paraboloidal surfaces with a vertex singularity is considered. The one-sided geometry of the family leads, after applying the two-dimensional Fourier transform, to a Volterra integral equation in the vertical variable. Using the local structure of its kernel near the vertex, the first-kind equation is reduced to a family of Volterra equations of the second kind. This reduction provides the basis for a Fourier–Volterra numerical reconstruction algorithm. Synthetic data are generated independently by direct evaluation of the original surface integrals. For smooth asymmetric model objects, the accuracy of the forward simulation, the effect of vertical discretization, and the spatial resolution of the reconstruction are investigated. The numerical results demonstrate convergence of the computational procedure and show that a small global reconstruction error alone does not guarantee the resolution of closely spaced features of the object.

Keywords: mathematical modeling, integral geometry, paraboloidal surfaces, spatial reconstruction, inverse problem, Volterra equation, Fourier–Volterra method, spatial resolution.

Problems of reconstructing functions from their integrals over prescribed families of curves and surfaces constitute an important class of problems in integral geometry [1]. When passing from classical hyperplanes to curved manifolds, the geometry of the integration family directly affects the structure of the forward operator and the properties of the inverse problem. For paraboloidal surfaces, the Fourier transform combined with a reduction to Volterra integral equations provides a natural framework for analytical inversion [2]. Integral geometry problems in which inversion is accompanied by a finite loss of smoothness were investigated in [3,4]. The analytical basis for the present modeling study is the problem for paraboloidal surfaces with a vertex singularity considered in [5].

Let $\Omega = \mathbb{R}^2 \times (0, h)$ and let $u = u(\xi_1, \xi_2, \zeta)$ be an unknown function describing the spatial distribution of the object under investigation. For each point $x = (x_1, x_2, z) \in \Omega$, consider the surface

$$S_x = \{(\xi_1, \xi_2, \zeta) : (\xi_1 - x_1)^2 + (\xi_2 - x_2)^2 = (z - \zeta)^4, \quad 0 \le \zeta \le z\}. \tag{1}$$

The point x is the vertex of S_x. An essential feature of (1) is its one-sided geometry: the entire surface lies below its vertex. In any vertical section through the axis, one obtains two symmetric branches with a common sharp vertex, both directed downward. If r denotes the horizontal distance from the axis, then

$$r = (z - \zeta)^2, \qquad \zeta = z - \sqrt{r}. \tag{2}$$

A horizontal section $\zeta = \text{const} < z$ is a circle of radius $r = (z - \zeta)^2$. Consequently, the data corresponding to a fixed vertex height z depend only on the part of the object at levels $0 \le \zeta \le z$. This property gives rise to the Volterra structure of the transformed problem.

The forward measurement model is

$$\psi(x_1, x_2, z) = \int_{S_x} u(\xi_1, \xi_2, \zeta)\, dS. \tag{3}$$

Using $\xi_1 = x_1 + (z - \zeta)^2 \cos\varphi$, $\xi_2 = x_2 + (z - \zeta)^2 \sin\varphi$, one obtains

$$\psi(x_1, x_2, z) = \int_0^z \int_0^{2\pi} u\big(x_1 + (z - \zeta)^2 \cos\varphi, x_2 + (z - \zeta)^2 \sin\varphi, \zeta\big) \times (z - \zeta)^2 \sqrt{1 + 4(z - \zeta)^2}\, d\varphi\, d\zeta. \tag{4}$$

Formula (4) is also used directly to generate the synthetic data employed for verification. Thus, the reconstruction is not tested on data produced by a discrete implementation of the same transformed equation used in the inversion.

Applying the two-dimensional Fourier transform in x_1, x_2 and writing $\lambda = (\lambda_1, \lambda_2)$ gives

$$\widehat{\psi}(\lambda, z) = \int_0^z k_\lambda(t) \widehat{u}(\lambda, z - t)\, dt, \tag{5}$$

where

$$k_\lambda(t) = 2\pi t^2 \sqrt{1 + 4t^2}\, J_0(|\lambda| t^2). \tag{6}$$

As $t \to 0$,

$$k_\lambda(t) = 2\pi t^2 + 4\pi t^4 + O\big((1 + |\lambda|^2) t^6\big),$$

so $k_\lambda(0) = k'_\lambda(0) = 0$ and $k''_\lambda(0) = 4\pi$. Differentiating (5) three times therefore yields

$$\widehat{u}(\lambda, z) + \int_0^z K_\lambda(z - \eta) \widehat{u}(\lambda, \eta)\, d\eta = \frac{1}{4\pi} \partial_z^3 \widehat{\psi}(\lambda, z), \tag{7}$$

where $K_\lambda(t) = (4\pi)^{-1} k'''_\lambda(t)$. Thus the three-dimensional problem decomposes into a family of one-dimensional Volterra equations parameterized by λ.

On a uniform grid $x_{1,i}, x_{2,j}, z_k = k\Delta z$, a two-dimensional FFT is computed at each level. After forming

$$F^k_{mn} = \frac{1}{4\pi} D^{(3)}_{z,h} \widehat{\Psi}^k_{mn}, \tag{8}$$

a discrete Volterra equation is solved for each Fourier mode. With trapezoidal quadrature the system is lower triangular, hence the values $U^0_{mn}, U^1_{mn}, \ldots, U^{N_z}_{mn}$ are determined successively. The computational route is

$$\boxed{\Psi_h \xrightarrow{\mathrm{FFT}_{2D}} \widehat{\Psi}_h \xrightarrow{D^3_z} F_h \xrightarrow{\text{Volterra}} \widehat{U}_h \xrightarrow{\mathrm{IFFT}_{2D}} U_h.} \tag{9}$$

For numerical verification, a smooth asymmetric model object was used:

$$u_{\text{true}}(x_1, x_2, z) = \sum_{j=1}^{3} A_j \exp\left[-\frac{(x_1 - a_j)^2}{\sigma^2_{1j}} - \frac{(x_2 - b_j)^2}{\sigma^2_{2j}} - \frac{(z - c_j)^2}{\sigma^2_{3j}} \right]. \tag{10}$$

Its components have different locations, amplitudes and spatial scales, avoiding artificial simplification due to symmetry.

The accuracy of the forward simulation was checked first. At $z = 0.75$, using quadrature resolutions $(N_\zeta, N_\varphi) = (61, 96)$ and $(121, 192)$ gave

$$E_{\text{forward}} = \frac{\|\psi_{61,96} - \psi_{121,192}\|_2}{\|\psi_{121,192}\|_2} = 3.85 \times 10^{-6}. \tag{11}$$

To study vertical discretization, the successive step sizes $\Delta z = 0.05, 0.025, 0.0125$ were used. The corresponding diagnostic reconstruction errors were

$$1.286 \times 10^{-1}, \qquad 8.245 \times 10^{-3}, \qquad 1.158 \times 10^{-3}. \tag{12}$$

Thus refinement of the z-grid rapidly reduces the error and indicates that evaluation of the third derivative in (7) is a sensitive computational stage.

In the full experiment, integral data were generated independently from (4) and then inverted by the Fourier–Volterra algorithm. On the representative section $z = 0.5$,

$$E_2 = \frac{\|u_{\text{rec}}(\cdot, \cdot, 0.5) - u_{\text{true}}(\cdot, \cdot, 0.5)\|_2}{\|u_{\text{true}}(\cdot, \cdot, 0.5)\|_2} = 6.46 \times 10^{-3}. \tag{13}$$

Hence the relative reconstruction error on this section is approximately 0.65%.

Spatial resolution was also investigated using two identical smooth inclusions separated by a distance d. Along the line joining their centers, the profile criterion

$$\gamma(d) = \frac{\min(M_1, M_2) - m}{\min(M_1, M_2)} \tag{14}$$

was used, where M_1, M_2 are the reconstructed maxima and m is the minimum between them. The inclusions were regarded as resolved when $\gamma(d) \geq 0.10$. On a $28 \times 28 \times 81$ grid,

$$\gamma(0.35) = 0.332, \qquad \gamma(0.25) = 0.0136,$$

Therefore, the computations yield

$$\boxed{0.25 < d_{\min} \leq 0.35}. \tag{15}$$

At $d = 0.25$, although the inclusions no longer satisfy the resolution criterion, the relative L_2 error of the corresponding reconstructed section remains of order 10^{-3}. Thus a small global reconstruction error alone does not guarantee faithful recovery of local spatial structure.

The numerical modeling shows that the geometry of the original surface family directly determines the computational method: one-sided geometry leads to a Volterra reduction and then to successive reconstruction in z. Independent generation of surface-integral data followed by numerical inversion permits verification without matched forward and inverse discretizations. The results demonstrate constructive recovery of smooth spatial objects and show that both reconstruction accuracy and spatial resolution should be assessed. Reconstruction from perturbed data and stabilized algorithms are natural directions for further investigation.

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NUMERICAL RECONSTRUCTION OF A FUNCTION FROM INCOMPLETE INTEGRAL DATA OVER PARABOLAS

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Abstract. A numerical reconstruction problem for a weighted integral geometry transform with incomplete data over a family of parabolas in the plane is considered. The recovery region and the region containing the vertices of the integrating parabolas are spatially separated, which makes the corresponding inverse problem strongly ill-posed. A regularized computational method is proposed based on direct discretization of the original integral operator and minimization of a Tikhonov functional with a smoothness stabilizer. Numerical results are presented for exact and noisy data.
Keywords: integral geometry, incomplete data, parabolas, Tikhonov regularization, numerical reconstruction, mathematical modeling.

Stable recovery of functions from approximate integral data is a classical problem in the numerical theory of ill-posed inverse problems [1,2]. The need for regularization becomes particularly significant for incomplete-data geometries, where restricting the family of integrating curves leads to an additional loss of information. Such effects arise in integral geometry problems with incomplete data [3,4].

Let

$$\Omega_l = \{(\xi,\eta) \in \mathbb{R}^2 :\ \xi \in \mathbb{R},\ 0 < \eta < l\}, \qquad D_l = \{(x,y) \in \mathbb{R}^2 :\ x \in \mathbb{R},\ l < y < 2l\},$$

and for every $(x,y) \in D_l$ consider the parabola

$$P(x,y): \qquad \eta = y - (\xi - x)^2.$$

The weighted integral data

$$f(x,y) = \int_{P(x,y)\cap\Omega_l} \operatorname{sgn}(\xi - x) u(\xi,\eta)\, d\xi \tag{1}$$

are assumed to be known. The unknown quantity is the integrand $u(\xi,\eta)$.

Using the parametrization $\xi = x \pm \sqrt{y-\eta}$ of the two branches, (1) becomes

$$f(x,y) = \frac{1}{2}\int_0^l \frac{u(x+\sqrt{y-\eta},\eta) - u(x-\sqrt{y-\eta},\eta)}{\sqrt{y-\eta}}\, d\eta, \qquad l < y < 2l. \tag{2}$$

The essential feature is the spatial separation

$$0 < \eta < l < y < 2l.$$

For this incomplete-data configuration, the corresponding inverse problem is strongly ill-posed. Related incomplete-data integral geometry problems and their stability properties were studied in [3,4]. Hence direct numerical use of an exact inversion procedure is unstable in the presence of data errors.

In the present work no analytic continuation of noisy data is used. Instead, the integral operator in (2) is discretized directly in the physical variables. On the grids

$$\xi_i = -X + ih_\xi, \qquad \eta_k = kh_\eta,$$

and at the data nodes

$$x_p = -X + ph_x, \qquad y_j = l + \varepsilon + jh_y,$$

the integral in (2) is approximated by a quadrature formula. Values of

$$u\big(x_p \pm \sqrt{y_j - \eta_k}, \eta_k\big)$$

that do not coincide with grid nodes are evaluated by linear interpolation. This yields a sparse system

$$\mathbf{A}\mathbf{u} \approx \mathbf{f}^\delta. \tag{3}$$

Because of the strong ill-posedness, (3) is not solved directly. The regularized solution is defined as a minimizer of

$$J_\alpha(\mathbf{u}) = \|\mathbf{A}\mathbf{u} - \mathbf{f}^\delta\|_2^2 + \alpha\left(\|\mathbf{D}_\xi\mathbf{u}\|_2^2 + \|\mathbf{D}_\eta\mathbf{u}\|_2^2\right), \qquad \alpha > 0, \tag{4}$$

where $\mathbf{D}_\xi$ and $\mathbf{D}_\eta$ are first-order finite-difference matrices. The corresponding regularized system is

$$\left[\mathbf{A}^*\mathbf{A} + \alpha\left(\mathbf{D}_\xi^*\mathbf{D}_\xi + \mathbf{D}_\eta^*\mathbf{D}_\eta\right)\right]\mathbf{u}_\alpha^\delta = \mathbf{A}^*\mathbf{f}^\delta. \tag{5}$$

Thus, smoothness of the unknown function is incorporated as a priori information, suppressing rapidly oscillating components that are weakly determined by the incomplete integral data [1,2].

For the numerical experiment we set

$$l = 1, \qquad X = 3,$$

and used

$$N_\xi = N_x = 64, \qquad N_\eta = 21, \qquad N_y = 50.$$

Hence the discrete problem contains 1344 unknowns and 3200 integral data values.

As a model function we used

$$u_{\mathrm{ex}}(\xi,\eta) = C\psi(\xi)\varphi(\eta),$$

where

$$\psi(\xi) = \begin{cases} \exp\left(-\dfrac{1}{1-\xi^2}\right), & |\xi| < 1, \\ 0, & |\xi| \geq 1, \end{cases} \qquad \varphi(\eta) = \begin{cases} \exp\left(-\dfrac{1}{1-(2\eta-1)^2}\right), & 0 < \eta < 1, \\ 0, & \eta \notin (0,1), \end{cases}$$

and C was chosen so that $\max u_{\mathrm{ex}} = 1$. Random perturbations with relative noise levels 1%, 3%, and 5% were added to the exact synthetic data.

The reconstruction quality was measured by

$$E_2 = \frac{\|\mathbf{u}_\alpha^\delta - \mathbf{u}_{\mathrm{ex}}\|_2}{\|\mathbf{u}_{\mathrm{ex}}\|_2}, \qquad R = \frac{\|\mathbf{A}\mathbf{u}_\alpha^\delta - \mathbf{f}^\delta\|_2}{\|\mathbf{f}^\delta\|_2}.$$

The numerical results are summarized below:

δ	α	E_2	R
0	10^{-10}	0.430	0.000020
0.01	10^{-6}	0.541	0.00910
0.03	10^{-6}	0.585	0.02687
0.05	10^{-6}	0.643	0.04538

As the perturbation level increases, stronger regularization is required. At the same time, the residual remains comparable with the imposed noise level. In particular,

$$R \approx 0.0091, \qquad 0.0269, \qquad 0.0454$$

for noise levels of 1%, 3%, and 5%, respectively. The computations show that the regularized solutions preserve the main spatial characteristics of the model function under moderate noise, while the noticeable quantitative error reflects the strong ill-posedness of the original problem.

Thus, direct discretization of the integral operator combined with Tikhonov regularization using a smoothness stabilizer provides a stable computational approach to the considered incomplete-data problem. The method is constructed directly for the present parabolic geometry and does not rely on numerical discretization of inversion formulas for full-data parabolic transforms [5].

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Coefficients of the interpolation splines in the space $W_2^{(2,0)}(0,1)$

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We denote by $W_2^{(2,0)}(0,1)$ the class of functions φ defined on $[0,1]$ which possess an absolutely continuous first derivative and whose second derivative belongs to $L_2(0,1)$. This class is equipped with the pseudo-inner product

$$\langle \varphi, \psi \rangle = \int_0^1 \big(\varphi''(x) - \varphi(x)\big)\big(\psi''(x) - \psi(x)\big)\, dx. \tag{1}$$

If we identify functions differing by a solution of the equation $f''(x) - f(x) = 0$, then (1) is an inner product and $W_2^{(2,0)}(0,1)$ is a Hilbert space with the norm $\|\varphi\| = \langle \varphi, \varphi \rangle^{1/2}$. The kernel of the operator $L \equiv d^2/dx^2 - 1$ is two dimensional and is generated by the functions e^x and e^{-x}.

Let the values $\varphi(x_\beta)$ of a function $\varphi \in W_2^{(2,0)}(0,1)$ be given at the equally spaced nodes

$$x_\beta = h\beta, \qquad \beta = 0, 1, \ldots, N, \qquad h = \frac{1}{N}, \qquad N \geqslant 2. \tag{2}$$

Problem. *Among all functions $S \in W_2^{(2,0)}(0,1)$ satisfying the interpolation conditions*

$$S(x_\beta) = \varphi(x_\beta), \qquad \beta = 0, 1, \ldots, N, \tag{3}$$

find the function which gives the minimum to the norm $\|S\|$.

The problem is solved by the method of S. L. Sobolev [2]. The Euler equation of the corresponding variational problem contains the operator $L^*L = d^4/dx^4 - 2\,d^2/dx^2 + 1$, and its fundamental solution, i.e. the solution of the equation $G_2^{(4)}(x) - 2G_2''(x) + G_2(x) = \delta(x)$,

where δ is Dirac's delta function, is found by means of the direct and inverse Fourier transforms in the closed form

$$G_2(x) = \frac{\operatorname{sgn}(x)}{4}\big(x\cosh x - \sinh x\big). \tag{4}$$

Therefore the solution of the Problem has the form

$$S(x) = \sum_{\gamma=0}^{N} C_\gamma\, G_2(x - x_\gamma) + d_1 e^x + d_2 e^{-x}, \tag{5}$$

where the unknowns C_γ are the jumps of the third derivative of the spline at the nodes. Conditions (3), together with the requirement that outside the segment $[0,1]$ the spline belongs to $\ker L$, give the system of $N+3$ linear equations

$$\sum_{\gamma=0}^{N} C_\gamma\, G_2(x_\beta - x_\gamma) + d_1 e^{x_\beta} + d_2 e^{-x_\beta} = \varphi(x_\beta), \qquad \beta = 0, 1, \ldots, N, \tag{6}$$

$$\sum_{\gamma=0}^{N} C_\gamma e^{x_\gamma} = 0, \qquad \sum_{\gamma=0}^{N} C_\gamma e^{-x_\gamma} = 0. \tag{7}$$

The system (6)–(7) is solved by using the discrete analogue $D_2[\beta]$ of the operator $d^4/dx^4 - 2\,d^2/dx^2 + 1$, i.e. the discrete function satisfying $D_2[\beta] * G_2[\beta] = \delta[\beta]$ and decreasing as $|\beta| \to \infty$. Putting $\Delta_h = h\cosh h - \sinh h$ and $P = \big(\sinh 2h - 2h\cosh 2h\big)/\Delta_h$, we have

$$D_2[\beta] = \frac{2}{\Delta_h}\begin{cases} A_1\lambda_1^{|\beta|-1}, & |\beta| \geqslant 2, \\ 1 + A_1, & |\beta| = 1, \\ P + \dfrac{A_1}{\lambda_1}, & \beta = 0, \end{cases} \tag{8}$$

where λ_1 is the root of the equation $\lambda^2 + \dfrac{\sinh 2h - 2h}{\Delta_h}\,\lambda + 1 = 0$ lying in the interval $(-1, 0)$, that is

$$\lambda_1 = \frac{\sinh 2h - 2h - 2\sinh h\sqrt{\sinh^2 h - h^2}}{2\big(\sinh h - h\cosh h\big)}, \qquad A_1 = \frac{4\lambda_1^2 h^2 \sinh^4 h}{\big(\lambda_1^2 - 1\big)\Delta_h^2}. \tag{9}$$

The function $D_2[\beta]$ annihilates the discrete values of e^x and e^{-x}, and $\lambda_1 \to -(2-\sqrt{3})$ as $h \to 0$. We note that in the case $m = 1$ studied by the authors in [1] the discrete analogue is a purely three point function, while for $m = 2$ a decreasing geometric component appears.

Applying the operator $D_2[\beta]$ to the system (6)–(7) written in the convolution form, we obtain the following result.

Theorem 1. *The coefficients of the interpolation spline (5) with equally spaced nodes (2) in the space* $W_2^{(2,0)}(0,1)$ *have the form*

$$C_0 = \frac{2}{\Delta_h}\left[a_1^- e^{-h} + a_2^- e^{h} + P\varphi(0) + \varphi(h) + \frac{A_1}{\lambda_1}\Phi_0\right], \tag{10}$$

$$C_\beta = \frac{2}{\Delta_h}\left[\varphi(h\beta - h) + P\varphi(h\beta) + \varphi(h\beta + h) + \frac{A_1}{\lambda_1}\Phi_\beta\right], \qquad \beta = 1, 2, \ldots, N-1, \tag{11}$$

$$C_N = \frac{2}{\Delta_h}\left[\varphi(1 - h) + P\varphi(1) + a_1^+ e^{1+h} + a_2^+ e^{-(1+h)} + \frac{A_1}{\lambda_1}\Phi_N\right], \tag{12}$$

where λ_1, A_1 are defined by (9) and

$$\Phi_\beta = \sum_{\gamma=0}^{N} \lambda_1^{|\beta-\gamma|}\varphi(h\gamma) + \lambda_1^{\beta} M_1 + \lambda_1^{N-\beta} N_1, \tag{13}$$

$$M_1 = \frac{a_1^- \lambda_1 e^{-h}}{1-\lambda_1 e^{-h}} + \frac{a_2^- \lambda_1 e^{h}}{1-\lambda_1 e^{h}}, \qquad N_1 = \frac{a_1^+ \lambda_1 e^{1+h}}{1-\lambda_1 e^{h}} + \frac{a_2^+ \lambda_1 e^{-(1+h)}}{1-\lambda_1 e^{-h}}. \tag{14}$$

Here the constants $a_1^{\mp}$, $a_2^{\mp}$ of the exterior branches are known: they are determined from the relations $a_1^- + a_2^- = \varphi(0)$, $a_1^+ e + a_2^+ e^{-1} = \varphi(1)$ and from the two equations $C_{-1} = C_{N+1} = 0$.

Proof. We extend the coefficients by $C_\gamma = 0$ for $\gamma < 0$ and $\gamma > N$ and introduce the discrete function $u[\beta] = \sum_\gamma C_\gamma G_2(h\beta - h\gamma) + d_1 e^{h\beta} + d_2 e^{-h\beta}$ defined for all integer β. By (6) we have $u[\beta] = \varphi(h\beta)$ for $\beta = 0, 1, \ldots, N$, while conditions (7) imply that G_2 may be replaced by its exponential branches outside $[0,1]$, so that $u[\beta] = a_1^- e^{h\beta} + a_2^- e^{-h\beta}$ for $\beta \leqslant 0$ and $u[\beta] = a_1^+ e^{h\beta} + a_2^+ e^{-h\beta}$ for $\beta \geqslant N$; matching at $\beta = 0$ and $\beta = N$ gives the two relations for $a_1^{\mp}$, $a_2^{\mp}$ stated above, and the remaining two conditions are $C_{-1} = C_{N+1} = 0$.

Convolving u with D_2 and using $D_2[\beta] * G_2[\beta] = \delta[\beta]$ together with $D_2[\beta] * e^{\pm h\beta} = 0$, we get $C_\beta = \sum_{\gamma=-\infty}^{\infty} D_2[\beta-\gamma]\, u[\gamma]$. Since, by (8), $D_2[\beta] = \frac{2}{\Delta_h}\big[\frac{A_1}{\lambda_1}\lambda_1^{|\beta|} + \varepsilon_1[\beta] + P\delta[\beta]\big]$, where $\varepsilon_1[\beta] = 1$ for $|\beta| = 1$ and $\varepsilon_1[\beta] = 0$ otherwise, it follows that

$$C_\beta = \frac{2}{\Delta_h}\left[u[\beta-1] + P\,u[\beta] + u[\beta+1] + \frac{A_1}{\lambda_1}\sum_{\gamma=-\infty}^{\infty} \lambda_1^{|\beta-\gamma|} u[\gamma]\right]. \tag{15}$$

Because $|\lambda_1| < 1$, the two exterior parts of the last sum are geometric series and are summed in the closed form (14), which turns the sum into Φ_β defined by (13). Substituting in (15) the values of $u[\beta-1]$, $u[\beta]$, $u[\beta+1]$ for $\beta = 0$, $\beta = 1, \ldots, N-1$ and $\beta = N$ respectively, we obtain (10), (11) and (12). The theorem is proved.

Formulas (10)–(12) do not require the solution of a linear system of order N, so that the coefficients of the interpolation spline are computed in $O(N)$ arithmetical operations. The constructed spline is exact for the functions e^x and e^{-x} generating $\ker L$, on each mesh interval it is a linear combination of e^x, e^{-x}, xe^x, xe^{-x}, and outside the segment $[0,1]$ it belongs to $\ker L$. The obtained results complement, for the second order operator L, the formulas established by the authors in [1] for the case $m = 1$.

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An optimal quadrature formula with derivative for the approximate calculation of Fourier integrals.

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It is well known that numerical integration formulas, or quadrature formulas, are methods for the approximate evaluation of definite integrals. They are required for computing integrals when the antiderivatives of the integrand cannot be expressed in terms of elementary functions, or when the values of the integrand function are given at nodal points. The use of quadrature formulas with a high algebraic degree of precision for integrating functions with low smoothness does not lead to good results. For such functions, the use of generalized rectangle, generalized trapezoidal, or Hermite-type formulas yields good results. Therefore, the construction of optimal quadrature formulas with derivatives in spaces of differentiable functions and the estimation of their errors is one of the purposeful scientific research directions.

The approximate calculation of integrals of highly oscillatory functions is considered one of the important problems of numerical analysis, since these integrals are widely used in science and technology. Fourier integrals of the form

$$I(\varphi,\omega)=\int\limits_0^1 e^{2\pi i\omega x}\varphi(x)dx \tag{1}$$

serve as examples of highly oscillatory integrals for sufficiently large values of ω, because in this case the integrand is highly oscillatory. In the approximate calculation of such integrals, standard methods of numerical integration often require a large amount of computational work and cannot be applied efficiently.Originally, an effective formula for the approximate integration of highly oscillatory functions was given by Filon [1].

Thus here we solve the problem of constructing optimal quadrature formulas with derivatives in the space $W_2^{(2,1)}(0,1)$.

Suppose that the first derivative of functions $\varphi(x)$ is absolutely continuous, the square of its second-order derivative is integrable, and it is a complex-valued function with a real argument. This set of functions forms a Hilbert space $W_2^{(2,1)}(0,1)$, with the inner product

$$\langle\varphi,\psi\rangle=\int\limits_0^1\left(\varphi''(x)+\varphi'(x)\right)\left(\overline{\psi}''(x)+\overline{\psi}'(x)\right)dx, \tag{2}$$

where $\overline{\psi}$ is the complex conjugate of the function ψ, the norm of the function φ is defined by the formula

$$\left\|\varphi|W_2^{(2,1)}\right\|=\langle\varphi,\varphi\rangle^{\frac{1}{2}}=\left(\int\limits_0^1\left(\varphi''(x)+\varphi'(x)\right)\left(\overline{\varphi}''(x)+\overline{\varphi}'(x)\right)dx\right)^{\frac{1}{2}}. \tag{3}$$

In this space, we consider a quadrature formula with the first derivative of the following form

$$\int\limits_0^1 e^{2\pi i\omega x}\varphi(x)dx\cong\sum_{\beta=0}^{N}\left(d_0[\beta]\varphi(h\beta)+d_{1,W}[\beta]\varphi'(h\beta)\right), \tag{4}$$

where $e^{2\pi i\omega x}$ is the weight function, $[\beta]=(h\beta)$, $h=1/N$, N is a natural number, $i^2=-1$, $\omega h\in\mathbb{Z}$, $\omega\neq 0$, $d_0[\beta]$ and $d_{1,W}[\beta]$ are the coefficients of the quadrature formula (6). Here $d_0[\beta]$ are the optimal coefficients of the quadrature formula

$$\int\limits_0^1 e^{2\pi i\omega x}f(x)dx\cong\sum_{\beta=0}^{N}d_0[\beta]\varphi(h\beta)$$

at $\omega h \in \mathbb{Z}$, $\omega \neq 0$ in the space $L_2^{(1)}(0,1)$ from the work [2] and have the form

$$d_0[0] = -\frac{1}{2\pi i\omega},$$
$$d_0[\beta] = 0, \quad \beta = 1, N-1,$$
$$d_0[N] = \frac{1}{2\pi i\omega}, \tag{5}$$

At the same time, we consider $d_{1,W}[\beta]$ as the unknown coefficients of the quadrature formula (4). The primary objective of the present work is to find the coefficients $d_{1,W}[\beta]$ which give the minimum error for the formula (4).

To do this, we estimate the difference

$$(\ell_W, \varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N}\left(d_0[\beta]\varphi(h\beta) + d_{1,W}[\beta]\varphi'(h\beta)\right) = \int_{-\infty}^{\infty} \ell_W(x)\varphi(x)dx \tag{6}$$

which represents the error of formula (4). It is easy to see that the following error functional corresponds to this difference:

$$\ell_W(x) = e^{2\pi i\omega x}\varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N}\left(d_0[\beta]\delta(x-h\beta) - d_{1,W}[\beta]\delta'(x-h\beta)\right), \tag{7}$$

where $\varepsilon_{[0,1]}(x)$ is the characteristic function of the interval $[0,1]$, $\delta(x)$ is Dirac's delta-function.

The main goal of this work is to construct quadrature formulas of the form (4) in the space $W_2^{(2,1)}$ using the method of S.L. Sobolev. To achieve this, it is necessary to solve the following problems.

Problem 1. *Determine the norm of the error functional ℓ_W in the space $W_2^{(2,1)*}$ of the quadrature formula of the form (4).*

Problem 2. *Find the coefficients $d_{1,W}[\beta]$ that give the minimum value of the obtained norm of ℓ_W.*

Problem 3. *Calculate the square of the norm of the error functional ℓ_W for optimal quadrature formula.*

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Dastlab integrallash metodining murakkab gidrodinamik tizimlar turg‘unligi masalalariga tatbiqi

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Geterogen (ko‘p fazali), xususan gaz aralashmali oqimlar dinamikasini tadqiq etish murakkab modellashtirishdagi muhim yo‘nalishlardan biridir. Bunda qattiq zarrachalarni dispers faza, ularni o‘rab turgan eltuvchi fazani esa dispersion faza deyiladi. Geterogen muhitlarni tavsiflovchi to‘liq tenglamalar sistemasi Raxmatulin, Nigmatulin va Dru tomonidan olib borilgan tadqiqotlarda keltirilgan. Raxmatulin-Nigmatulin-Dru tomonidan tavsiya etilgan matematik modellardan eng kichik qo‘zg‘alishlar metodi bilan ikki fazali oqimlar gidrodinamik turg‘unligini tavsiflovchi yuqori tartibli hosila oldida kichik parametr bo‘lgan quyidagi chiziqli bo‘lmagan oddiy differensial tenglamalar uchun xos qiymat muammosiga kelinadi:

$$\frac{1}{i\alpha\,\mathrm{Re}}\left(\frac{d^4\psi}{dy^4}-2\alpha^2\frac{d^2\psi}{dy^2}+\alpha^4\psi\right)-$$

$$-\left(U(y)-\lambda-\frac{if}{\alpha\tau}\right)D\psi+\frac{d^2U}{dy^2}\psi+\frac{f}{i\alpha\tau}D\varphi=0,\quad -1<y<1 \tag{1}$$

$$\frac{1}{i\alpha\tau}D\psi-\left(U(y)-\lambda-\frac{i}{\alpha\tau}\right)D\varphi+\varphi\frac{d^2U}{dy^2}=0, \tag{2}$$

$$\psi(\pm1)=\frac{d\psi}{dy}(\pm1)=0, \tag{3}$$

$$\varphi(-1)=0. \tag{4}$$

Tenglamalar sistemasi (1)–(4) da $\mathrm{Re}=\dfrac{\rho UL}{\mu}$ - Reynolds soni, ρ_1-gazning zichligi, μ-qovushqoq -lik, $U(y)$-asosiy oqimning tezlik profili, L - xarakterli uzunlik, $\tau=S\,\mathrm{Re}$ - zarrachalarning relaksatsiya vaqti, bu yerda

$$S=\frac{2}{9}\left(\frac{a}{L}\right)^2 s_1,\qquad s_1=\frac{\rho_2}{\rho_1},$$

bu yerda ρ_2,α- mos ravishda zarrachalar materialining zichligi va radiusi, α- to‘lqin soni bo‘lib, u haqiqiy miqdor, hamda qo‘zg‘alishlarning to‘lqin uzunligi l bilan quyidagi munosabat orqali bog‘langan

$$\alpha=\frac{2\pi}{l},\qquad D=\frac{d^2}{dy^2}-\alpha^2$$

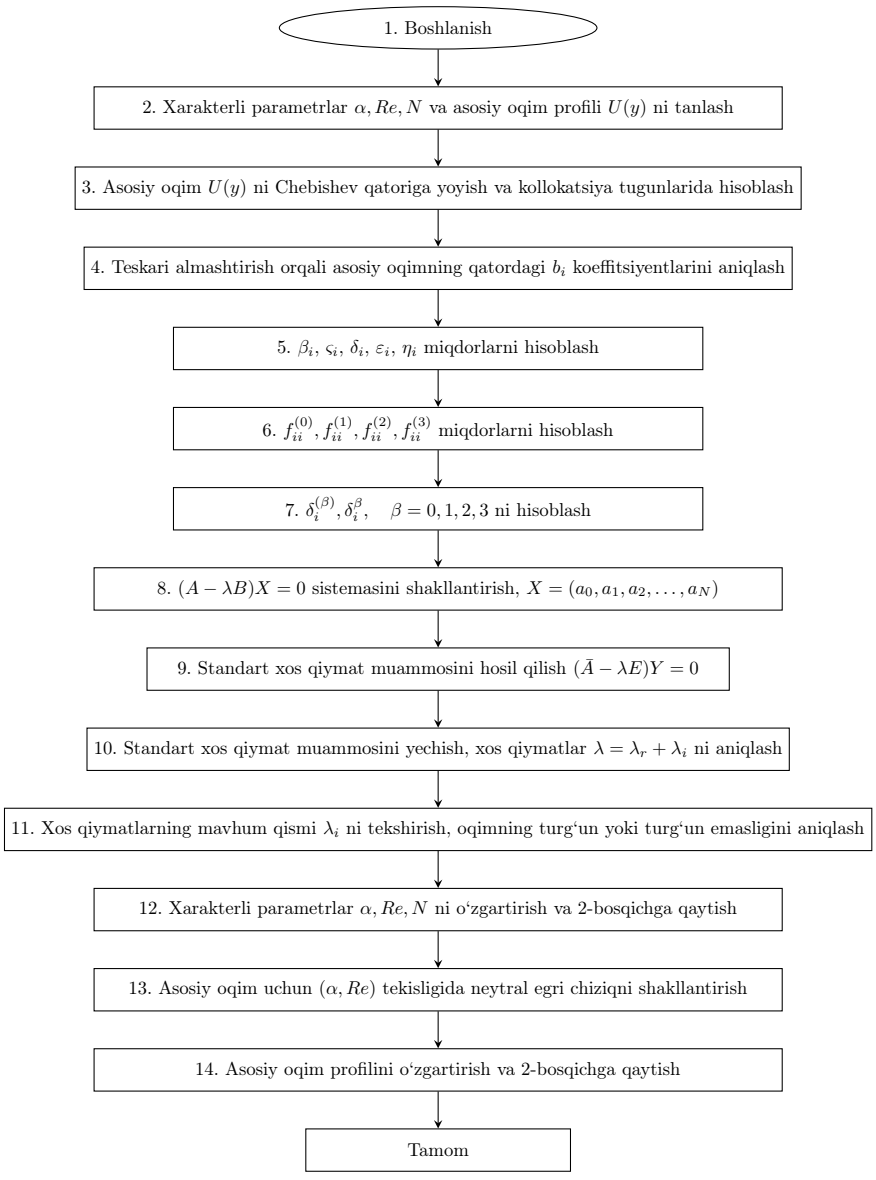
- differensial operator, $f=\alpha_0 s_1$-zarrachalarning massaviy konsentratsiyasi, α_0- zarrachalarning hajmiy konsentratsiyasi, $\lambda=\lambda_r+i\lambda_i$- aniqlanishi lozim bo‘lgan noma’lum o‘zgaruvchis, bunda λ_r- fazaviy tezlik, λ_i- o‘sish koeffitsiyenti. Agar $\lambda_i>0$ bo‘lsa, u holda qaralayotgan oqim turg‘unmas, agar $\lambda_i<0$ bo‘lsa, turg‘un bo‘ladi. Agarda $\lambda_i=0$ bo‘lsa, u holda qo‘zg‘alishlar tebranishi neytral turg‘un bo‘ladi.

Shunday qilib, ikki fazali oqimlar gidrodinamik turg‘unligi muammosi to‘rtta parametr bilan xarakterlanadi: Re - Reynolds soni, α- to‘lqin soni, τ- zarrachalarning relaksatsiya vaqti, f- zarrachalarning massaviy konsentratsiyasi. Ushbu masalani yechish o‘ta murakkab masalalardan hisoblanadi, chunki to‘rt o‘lchamli parametrlar fazosi $(\mathrm{Re},\alpha,\tau,f)$ da beshinchi parametr $\lambda=\lambda_r+i\lambda_i$ ning mavhum qismi $\lambda_i=0$ nolga aylanadigan kombinatsiyasini topish talab etiladi. $\lambda_i=0$ nolga aylanadigan egri chiziq yoki sirt neytral turg‘unlik egri chizig‘i yoki sirti deyiladi.

Ikki fazali oqimlarning turg‘unlik tenglamalarini sonli modellashtirish uchun dastlab integral -lash metodidan foydalanish taklif etiladi, bunda birinchi turdagi Chebishev ko‘phadlari bazis funksiyalari sifatida qo‘llaniladi. Metodning mohiyati shundan iboratki, masalani yechishdan oldin differensial tenglamalar sistemasidagi yuqori tartibli hosilar birinchi turdagi Chebishev ko‘phadlari qatori ko‘rinishida ifodalanadi. Differensial masalani yechishdan oldin bu qator

ketma-ket kerakli marta integrallanadi. So‘ngra differensial masalada integrallangan qatorlardan foydalaniladi [1-3].

Gidrodinamik turg‘unlik muammosini tadqiq etishga mo‘ljallangan dasturiy majmua strukturasi



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Fundamental properties of two-dimensional trigonometric basis functions

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Let us be given a domain $\Omega = [0,1] \times [0,1] \subset \mathbb{R}^2$ and a mesh

$$\Delta_{h_1,\,h_2} :=$$

$$= \left\{ (x_i,\, y_j) : x_i = ih_1,\, y_j = jh_2,\, i = 0,\, 1,\, ...,\, m,\, j = 0,\, 1,\, ...,\, n,\, h_1 m = 1,\, h_2 n = 1 \right\}$$

in this domain. Through points x_i and y_j of mesh $\Delta_{h_1,\,h_2}$, we draw lines parallel to the ordinate and abscissa axes, respectively. As a result, the domain Ω is divided into rectangular parts (i.e., finite elements) of the same shapes $[x_i,\, x_{i+1}] \times [y_j,\, y_{j+1}]\,(i = 0,\, 1,\, ...,\, m-1;\, j = 0,\, 1,\, ...,\, n-1)$.

To construct trigonometric basis functions of two variables on finite elements $[x_i,\, x_{i+1}] \times [y_j,\, y_{j+1}]\,(i = 0,\, 1,\, ...,\, m-1;\, j = 0,\, 1,\, ...,\, n-1)$ in the domain Ω, we use the concept of the tensor product of functions of one variable.

Definition 1. Let f and g be functions of one variable. We define a two-variable function $f \otimes g$ using the expression

$$f \otimes g = (f \otimes g)\,(x,\, y) = f(x)g(y). \tag{1}$$

The function $f \otimes g$ of two variables is called the tensor product of the functions f and g of one variable. Where the symbol $\otimes$ represents the tensor product.

In general, there are two methods for constructing trigonometric basis functions of two variables. These are the construction method using the tensor product and the geometric method. To simplify the calculations, we construct two-variable trigonometric basis functions using the tensor product below. For this, we assign a function $\varphi_{i,\,j}(x,\, y)$, defined in the form

$$\varphi_{i,\,j}(x,\, y) = \varphi_i \otimes \varphi_j(x,\, y) = \varphi_i(x)\varphi_j(y)\,(i = 0,\, 1,\, ...,\, m;\, j = 0,\, 1,\, ...,\, n)$$

and satisfying the relation

$$\varphi_{i,\,j}(x_p,\, y_q) = \begin{cases} 1,\, p = i \text{ and } q = j, \\ 0,\, \text{otherwise} \end{cases}$$

i.e.,

$$\varphi_{i,\,j}(x_p,\, y_q) = \delta_{i,\,p}\delta_{j,\,q}\,(p = 0,\, 1,\, ...,\, m,\, q = 0,\, 1,\, ...,\, n),$$

to each node $(x_i,\, y_j)$. Here, $(\varphi_i \otimes \varphi_j)\,(x,\, y)$ is defined by (1), $\varphi_i(x)$ and $\varphi_j(y)$ are determined using expressions (33)-(35) in [1], and $\delta_{i,\,p}, \delta_{j,\,q}$ are the Kronecker symbols.

It should be noted that the functions $\varphi_{i,\,j}(x,\, y)\,(i = 0,\, 1,\, ...,\, m,\, j = 0,\, 1,\, ...,\, n)$ form a finite-dimensional linear space in the domain Ω, and therefore these functions are uniquely defined by their values at the nodes $(x_i,\, y_j)$.

To geometrically represent the basis functions $\varphi_{i,\,j}(x,\, y)\,(i = 0,\, 1,\, ...,\, m,\, j = 0,\, 1,\, ...,\, n)$, we present their graphs for the case $m = n = 10$ and $\omega_1 = \omega_2 = 1$ in Fig. 1. Here, ω_1 and ω_2 are the parameters in the expression of the functions $\varphi_i(x)$ and $\varphi_j(y)$, respectively.

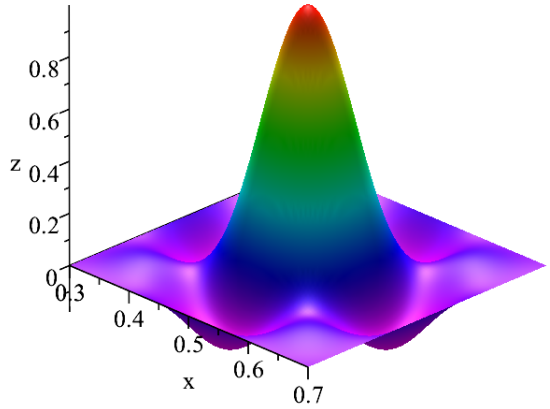


Fig.1. This figure shows the graph of the function $\varphi_{i,j}(x,y)\,(i=1,2,...,m-1, j=1,2,...,n-1)$.

Theorem 1. (Main property theorem) For functions $\varphi_{i,j}(x,\,y)$ in the domain Ω, the following relations hold:

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,\,y)=1, \tag{2}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,\,y)\sin(\omega_1 x_i)\sin(\omega_2 y_j)=\sin(\omega_1 x)\sin(\omega_2 y), \tag{3}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,\,y)\cos(\omega_1 x_i)\cos(\omega_2 y_j)=\cos(\omega_1 x)\cos(\omega_2 y), \tag{4}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,\,y)\sin(\omega_1 x_i)\cos(\omega_2 y_j)=\sin(\omega_1 x)\cos(\omega_2 y), \tag{5}$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,\,y)\cos(\omega_1 x_i)\sin(\omega_2 y_j)=\cos(\omega_1 x)\sin(\omega_2 y), \tag{6}$$

where ω_1 and ω_2 are elements of the set $\mathbb{R}\setminus\{0\}$.

From relations (2)-(6), it can be stated that the functions $\varphi_{i,j}(x,\,y)\,(i=0,\,1,\,...,\,m,\,j=0,\,1,\,...,\,n)$ accurately reconstruct any linear combination of the functions 1, $\sin(\omega_1 x)\sin(\omega_2 y)$, $\cos(\omega_1 x)\cos(\omega_2 y)$, $\sin(\omega_1 x)\cos(\omega_2 y)$ and $\cos(\omega_1 x)\sin(\omega_2 y)$.

It should be noted that two-dimensional trigonometric basis functions $\varphi_{i,j}(x,\,y)\,(i=0,\,1,\,...,\,m,\,j=0,\,1,\,...,\,n)$ can be applied to finite element methods and signal processing.

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UDC 514.7

TEMIR YO'L EGRI QISMINING BA'ZI PARAMETRLARINI MATEMATIK USUL BILAN ANIQLASH

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Temir yo'llarni loyihalashda asosan yo'lning qayrilish qismini xavfsiz va iqtisodiy barqaror qurish yoki ta'mirlash eng dolzarb masala hisoblanadi. Temir yo'lning egri qismi uning ikki to'g'ri yo'l qismini bir-biriga bog'lovchi qism shaklida paydo bo'ladi. Biz bu qismni 1-rasmda ko'rsatilgan shakldagi geometrik tasviridan foydalanamiz. To'g'ri qismlarning uchlari bo?lgan A va B nuqtalarni $AB = l$ kesma tutashtirsin, va shu nuqtalardan o'tuvchi to'g'ri chiziqlarga urinma aylana radiusi R bo'lsin [1].

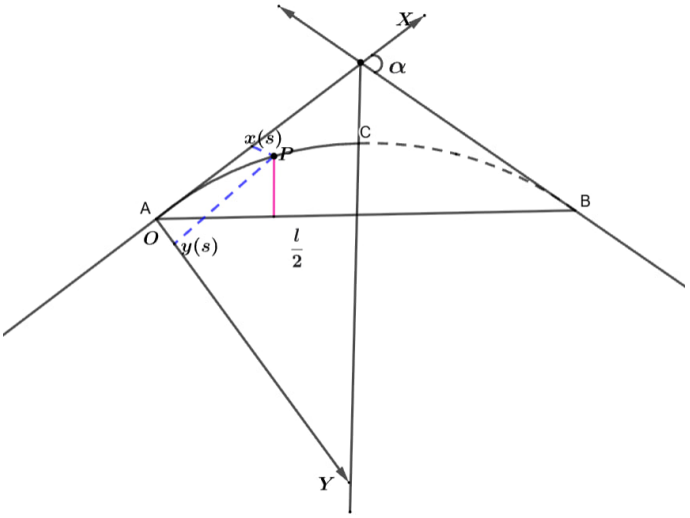
Bunda $l = 2R\Delta \cos\frac{\alpha}{2}$ ga teng bo'ladi.

Temir yo'l to'g'ri va doiraviy egri qismlarini o'zaro bog'lovchi egri chiziq, o'tish qismi deb nomlanib, u bir qancha texnik talablarni qanoatlantirishi zarur [2],[3]. Temir yo'l to'g'ri qismini ifodalovchi to'g'ri chiziqlarni davom ettirib, kesishishidan hosil bo'lgan α burchak va A nuqtani koordinata boshi, shu nuqtadan o'tgan to'g'ri qismning davomi sifatida $AX = OX$ o'qi va A nuqtaga perpendikulyar to'g'ri chiziqni $AY = OY$ o'qi deb belgilaymiz.

Ushbu sistema koordinatalar sistemasining boshi A nuqtadan o'tgan l to'g'ri chiziqning (AB kesma) tenglamasi

$$Y = \operatorname{tg}\frac{\alpha}{2} \cdot X \tag{1}$$

ko'rinishda bo'ladi.



1-rasm: Temir yo'l egri qismini loyihalash

O'tish qismini ifodalovchi chiziq sifatida kubik parabola olinganda, uning tabiiy parametrik tenglamasi:

$$\gamma : \begin{cases} X(s) = s \\ Y(s) = s^3 \end{cases} \tag{2}$$

ga teng bo'ladi.

Bundan, yuqoridagi biror $P(X(s), Y(s))$ nuqtadan l to'g'ri chiziqqacha bo'lgan masofa $d(s)$ ni topish formulasi:

$$d(s) = s \cdot \sin\frac{\alpha}{2} - s^3 \cdot \cos\frac{\alpha}{2} \tag{3}$$

ekanligi kelib chiqadi.

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UDK 517.977.5:631.672.4

Algorithms for Determining the Water Needs of Consumers of the Main Canal During the Growing Season

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The necessary expenditures and the cultivated areas related to the$m$-th plot during the n-th decade of the growing season are determined in the following way

$$Q_{mn}^R = \sum_{j \in J_m} Q_{mjn}^R \tag{1}$$

$$S_{mn}^R = \sum_{j \in J_m} S_{mjn}^R \tag{2}$$

where J_{mj}- sets of numbers of taps m- th sections of the main canal.

The process (algorithm) for calculating water flow rates for the sections of the main canal is outlined as follows:

1 To determine the necessary water flow at the endpoints of the main canal for the *n-th decade,* the following procedure is implemented*:*

$$Q^{RC}_{nm_k} = Q^{R}_{nm_k}, \ \ \forall m_k \in M_k, \ \forall n \in N_W \tag{3}$$

2 Next, beginning with the terminal hierarchical groups within the sections, the required water flow at the starting points of the main canal sections for the *n-th* decade is calculated as detailed below.*:*

$$Q^{RP}_{mn} = \frac{Q^{RC}_{mn} + QW^{R}_{Wmn} - Q^{R}_{Rmn}}{\eta_m}, \ \ \forall m \in I_{iH}, \ \ \forall n \in N_W \tag{4}$$

$$Q^{R}_{Wmn} = \sum_{j \in J^{W}_{m}} \left(Q^{R}_{IWmjn} + q_{DRmjn} \right) = Q^{R}_{IWmn} + q_{DRmn}, \quad \forall m \in I_{iH}, \ \forall n \in N \tag{5}$$

$$Q^{R}_{Rmn} = \sum_{j \in J^{R}_{m}} Q^{R}_{Rmjn}, \quad \forall m \in I_{iH}, \quad \forall n \in N_W \tag{6}$$

$$Q^{R}_{IWmn} = \sum_{j \in J^{W}_{m}} Q^{R}_{IWmjn}, \quad \forall m \in I_{iH}, \quad \forall n \in N_W \tag{7}$$

$$q_{DRmn} = \sum_{j \in J^{W}_{m}} q_{DRmjn}, \quad \forall m \in I_{iH}, \quad \forall n \in N \tag{8}$$

where $Q^{RP}{}_{mn}$, $Q^{RC}{}_{mn}$ - water consumption at the beginning and end of the site, $Q^{R}{}_{Wmn}$, $Q^{R}{}_{Rmn}$ - total water consumption of required water intakes and predicted inflows, $Q^{R}{}_{IWmn}$, $qDRmn$ - total water consumption of required water intakes for irrigation and other consumers, J^{W}_{m}- set of numbers of water intakes on the site, J^{R}_{m}- set of numbers of inflows on the site,I_{iH} - set of site numbers in the group under consideration, η_m- efficiency of the m-th section of the canal, N_H- numbers of the decade of the growing season [1,2].

3 *In groups j_{iH} in the i- hierarchy of the structure of the main channel, the balance ratios are written as:*

$$Q^{RC}_{n_{ijH}n} = \sum_{:_{ijH} \in N_{ijH}} Q^{RP}_{C_{ijH}n}, \quad \forall n_{ijH} \in I_{iH}, \ \ \forall n \in N_W \tag{9}$$

where $Q^{C}_{n_{ijH}}$ is the water flow at the end of the n_{ijH}-th section [1,3].

4 *Next, the calculation is repeated in steps 2 - 3 for the next hierarchy in the structure.*

Thus, the planned modes of operation of sections of the main canal are calculated to meet the needs of all consumers, which are characterized by the following set:

$$\Omega^{WR}_{C} = \left\{ \begin{array}{c} \left[m, \ \ Q^{RP}_{mn}, \ Q^{RC}_{mn}, \ Q^{R}_{Emn}, \ \ Q^{R}_{Rmn}, \ \ Q^{R}_{IWmn}, q^{R}_{DRmn}, \ \ S^{R}_{mn} \right], \\ \forall m \in M, \ \forall n \in N_W \end{array} \right\} \tag{10}$$

here , $Q^{RP}{}_{mn}$ and $Q^{RC}{}_{mn}$ – represent the water discharge at the beginning and end of the plot, $Q^{R}{}_{Wmn}$ and $Q^{R}{}_{Rmn}$ refer to the total water discharges from water intakes and tributaries, $Q^{R}{}_{IWmn}$, $q^{R}DRmn$ – denote the total water consumption for irrigation and other users, $S^{R}{}_{mn}$–

represents the irrigated areas of agricultural crops on plot mmm during decade n of the growing season [3].

The algorithms for calculating water demand for consumers of the main canal during both growing periods enhance the operational modes of the system, reduce water consumption losses, and enable the collection and accumulation of energy resources and essential information. This distribution within the system increases the performance of its components by speeding up processing times.

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UDK 519.63

A Finite-Difference Scheme for a Degenerate–Singular Elliptic Operator with Applications in Mathematical Oncology

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Degenerate and singular elliptic equations model transport and diffusion in heterogeneous media. We consider a second-order degenerate–singular elliptic equation, associated with a third-order elliptic-type problem studied analytically by the authors, containing a degenerating coefficient y^m and a singular term $\beta_0 u_y/y$. A second-order central finite-difference scheme is constructed on a uniform grid, with the singular coefficient regularized near $y = 0$; its accuracy is verified against a manufactured solution, achieving the expected $O(h^2)$ order, and the qualitative behaviour of the computed solution is analysed for several (m, β_0). The framework is briefly interpreted as a model for stationary transport of oxygen, nutrients, or therapeutic agents in heterogeneous tumor tissue.

Keywords: finite-difference scheme, degenerate elliptic equation, singular coefficient, manufactured solution, convergence, diffusion, mathematical oncology.

1. Mathematical model and numerical formulation. We consider the second-order elliptic equation

$$Lu \equiv y^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = F(x, y) \tag{1}$$

in the rectangular domain $\Omega = [-1,1] \times [0,1]$, $m > 0$, $-m/2 < \beta_0 < 1$. This second-order equation is the reduced equation associated with the original third-order elliptic-type equation studied analytically by the authors, since $\partial_x(y^m u_x) = y^m u_{xx}$ for y^m independent of x. Here y^m degenerates at $y = 0$ and $\beta_0 u_y/y$ is singular there, so direct numerical treatment is nontrivial. The numerical experiments use the boundary value problem

$$u(-1,y) = u(1,y) = 0, \quad u(x,0) = 0, \quad u(x,1) = \sin(\pi x), \tag{2}$$

compatible with the homogeneous conditions at $x = \pm 1$.

On a uniform grid $x_i = -1 + ih_x$, $y_j = jh_y$, central finite differences give the discrete equation

$$y_j^m \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{h_y^2} + \frac{\beta_0}{y_j} \frac{u_{i,j+1} - u_{i,j-1}}{2h_y} = F_{i,j}. \tag{3}$$

The singular coefficient is regularized at the degenerate boundary to avoid division by zero in the numerical implementation,

$$y_j \to \max(y_j, \varepsilon), \qquad \varepsilon = 10^{-6}, \tag{4}$$

which only affects the (Dirichlet) boundary row $y = 0$ itself, since $y_j \geq h_y \gg \varepsilon$ at every interior node; the resulting sparse system is solved by a direct sparse linear solver.

2. Verification by a manufactured solution. To verify the accuracy of scheme (3) we use the manufactured solution

$$u_{\text{ex}}(x,y) = \sin(\pi x) y^2 \tag{5}$$

in $\Omega = [-1,1] \times [0,1]$, $m = 2$, $\beta_0 = 0.5$; substitution of (5) into (1) gives the forcing function

$$F(x,y) = \sin(\pi x)\big[-\pi^2 y^{m+2} + 2 + 2\beta_0\big]. \tag{6}$$

The numerical solution u_h is compared with u_{ex} via the discrete error $E_h = \|u_h - u_{\text{ex}}\|_{L_2(\Omega)}$, and the observed order is computed from

$$p_h = \log_2(E_h/E_{h/2}). \tag{7}$$

The results are given in Table 1.

h	E_h	$E_h/E_{h/2}$	p_h
0.1000	9.09×10^{-4}	–	–
0.0500	2.35×10^{-4}	3.86	1.95
0.0250	5.97×10^{-5}	3.94	1.98
0.0125	1.51×10^{-5}	3.97	1.98

Table 1. Convergence of the finite-difference scheme for $m = 2$, $\beta_0 = 0.5$.

The error drops by a factor of four when h is halved and the observed order approaches 2, confirming $O(h^2)$ accuracy even with the regularized singular term.

3. Qualitative numerical experiment. In addition to the manufactured-solution test, the scheme is applied to the homogeneous equation

$$y^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0 \tag{8}$$

with boundary conditions (2), on a uniform grid with $h_x = h_y = 0.05$; several (m, β_0) combinations are considered, not to fix biological parameter values but to illustrate how degeneracy and singularity influence the computed solution.

(m, β_0)	$\max u$	$\min u$	Qualitative behaviour
(1.0, 0.2)	0.874	-0.874	fastest decay away from $y = 1$
(2.0, 0.5)	0.896	-0.896	intermediate decay
(3.0, 0.8)	0.916	-0.916	slowest decay, closest to boundary values

Table 2. Influence of m and β_0 on the interior extrema of the computed solution. (The extrema over $\overline{\Omega}$ equal ± 1 for every (m, β_0), attained on $y = 1$ where $u = \sin(\pi x)$ is prescribed; the table reports the extrema over the open domain Ω.)

Increasing m and β_0 raises the interior extremum toward the prescribed boundary value 1, i.e. the solution decays more slowly away from $y = 1$; for $m = 2$, $\beta_0 = 0.5$ the computed solution (Figure 1) is antisymmetric in x, consistent with $\sin(\pi x)$.

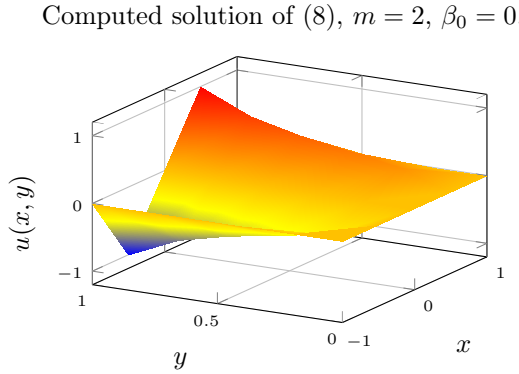


Figure 1. Finite-difference solution of the homogeneous equation (8), $m = 2$, $\beta_0 = 0.5$.

4. Interpretation in mathematical oncology. Degenerate diffusion models of this type describe transport in heterogeneous biological media, with $u(x, y)$ representing, e.g., the stationary concentration of oxygen or a therapeutic agent, y^m the diffusivity degenerating as $y \to 0$ (a poorly vascularized region), and $\beta_0 u_y / y$ an additional transport effect near that boundary. Calibrating m, β_0 against experimental data is left for future work.

5. Conclusion. A second-order finite-difference scheme has been constructed and verified for a degenerate–singular elliptic equation arising from a third-order elliptic-type problem. The manufactured-solution test confirms $O(h^2)$ convergence (Table 1), and the qualitative experiments (Table 2, Figure 1) show that the interior extremum of the solution approaches the prescribed boundary value as m and β_0 increase. The resulting scheme offers a practical tool for degenerate diffusion models in heterogeneous media, with a potential application in mathematical oncology.

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УДК 517.984

EXPLICIT DELAY-DEPENDENT STABILITY THRESHOLD FOR A NONLINEAR DISCRETE EPIDEMIC MODEL

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We study the local stability of the endemic equilibrium in a delayed discrete epidemic model with nonlinear saturated feedback. The original two-component system is reduced to a scalar delay difference equation, and a direct unit-circle analysis yields an explicit necessary and sufficient stability condition for an arbitrary integer delay. In particular, a closed-form critical threshold $\alpha_c(m)$ is obtained, showing that increasing delay shrinks the endemic stability region.

Consider

$$x_{n+1} = \frac{\alpha x_n}{1 + \beta y_{n-m}}, \qquad y_{n+1} = \gamma x_n, \tag{1}$$

where $\alpha, \beta, \gamma > 0$ and $m \in \mathbb{N}_0$. Since $y_{n+1} = \gamma x_n$, after the initial transient (1) is equivalent to

$$x_{n+1} = \frac{\alpha x_n}{1 + \delta x_{n-m-1}}, \qquad \delta = \beta\gamma. \tag{2}$$

For $\alpha > 1$, the positive equilibrium is $x^* = (\alpha - 1)/\delta$. Writing $x_n = x^* + u_n$ and linearizing (2) at x^* gives

$$u_{n+1} = u_n - c\, u_{n-m-1}, \qquad c = \frac{\alpha - 1}{\alpha}, \tag{3}$$

with characteristic equation

$$\lambda^{m+2} - \lambda^{m+1} + c = 0. \tag{4}$$

Thus the local stability problem is reduced to the Schur stability of a delay-difference trinomial.

To determine the stability boundary, put $\lambda = e^{i\theta}$ in (3). Then $\lambda^{m+1}(1 - \lambda) = c$. Since

$$1 - e^{i\theta} = 2\sin\left(\frac{\theta}{2}\right) \exp\left[i\left(\frac{\theta}{2} - \frac{\pi}{2}\right)\right],$$

we obtain $c = 2\sin\left(\frac{\theta}{2}\right) \exp\left[i\left(\left(m + \frac{3}{2}\right)\theta - \frac{\pi}{2}\right)\right]$. Because c is positive and real, the first admissible unit-circle angle is

$$\theta_m = \frac{\pi}{2m + 3},$$

and therefore the first pair of characteristic roots reaches $|\lambda| = 1$ at

$$c_m = 2\sin\left(\frac{\pi}{4m + 6}\right), \qquad \lambda_{1,2} = \exp\left(\pm\frac{i\pi}{2m + 3}\right). \tag{5}$$

For small $c > 0$ all roots lie inside the unit disk, and no root reaches its boundary before $c = c_m$. Hence (3) is Schur stable precisely for $0 < c < c_m$.

Theorem. Let $\alpha > 1$, $\beta, \gamma > 0$, and $m \in \mathbb{N}_0$. The endemic equilibrium of (1) is locally asymptotically stable if and only if

$$0 < \frac{\alpha - 1}{\alpha} < 2\sin\left(\frac{\pi}{4m + 6}\right). \tag{6}$$

For $m = 0$, it is locally asymptotically stable for every finite $\alpha > 1$. For $m \geq 1$, (6) is equivalent to

$$1 < \alpha < \alpha_c(m), \qquad \alpha_c(m) = \frac{1}{1 - 2\sin\left(\dfrac{\pi}{4m+6}\right)} \tag{7}$$

For $\alpha > \alpha_c(m)$ the endemic equilibrium is unstable, while at $\alpha = \alpha_c(m)$ it is nonhyperbolic with the conjugate unit-modulus roots in (5).

Indeed, substituting $c = (\alpha - 1)/\alpha$ into $0 < c < c_m$ yields (6); solving it for α gives (7). Moreover, $2\sin(\pi/(4m+6))$ strictly decreases to zero as $m \to \infty$. Consequently,

$$\alpha_c(m) \text{ is strictly decreasing}, \qquad \lim_{m\to\infty} \alpha_c(m) = 1.$$

Thus the interval $1 < \alpha < \alpha_c(m)$ becomes narrower as the delay grows. For every fixed $\alpha > 1$, a sufficiently large delay destabilizes the endemic equilibrium. Although β and γ determine the position of the endemic state, they cancel from its local stability condition; the stability boundary is governed by the transmission parameter α and the delay m.

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UDK 539.3

G‘OVAK-ELASTIK FAZONI IKKI QATTIQ SHAR BILAN NOSTATSIONAR KO‘NDALANG TEBRANISHI

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Ushbu tadqiqot ishi suyuqlik to’yingan g‘ovak-elastik fazoni ikki qattiq shar bilan nostatsionar ko‘ndalang tebranishi haqidagi masalani o‘rganishga va yechish algoritmini ishlab chiqishga, hamda nostatsionar to‘lqin jarayonlarini tadqiqotiga bag‘ishlangan.

Aytaylik, suyuqlik bilan to‘yingan cheksiz chiziqli g‘ovak-izotropik muhitda birinchi R_1 radiusli qattiq shar ikkinchi R_2 radiusli qattiq shar yaqinida joylashgan bo‘lsin. Ularning markazlari orasidagi o’zgarmas masofa l ga teng bo‘lib, u uchun $l > R_1 + R_2$ tengsizlik o‘rinli. Bu masalada muhit harakati boshlang‘ich O_i nuqtalari mos ravishda sferik qo‘yilmalar markazlarida bo‘lgan ikki turli $(r_i,\theta_i,\vartheta_i)$, $(i = 1, 2)$ sferik koordinatalar sistemasida o‘rganiladi.

Muhit boshlang‘ich $\tau = 0$ vaqtgacha muhit tinch holatda bo‘ladi.

Boshlang‘ich $\tau = 0$ vaqtda absolyut qattiq shar muhit bilan to‘liq mahkamlangan holda sharning berilgan $V_1(\tau, \theta_1)$ qonun bo‘yicha sharlar markazidan o’tuvchi o’q atrofida nostatsionar aylanma harakati yuzaga keltiriladi

$$u_{\vartheta_1}|_{r_1=R_1} = V_1(\tau, \theta_1). \tag{1}$$

Natijada g‘ovak-elastik muhitning nostatsionar ko‘ndalang tebranishi paydo bo’ladi.

Ikkinchi qattiq shar sirtida ko‘chish nolga teng bo‘lsin

$$u_{\vartheta_2}|_{r=R_2} = 0. \tag{2}$$

Masalaning o‘qsimmetrikligini e’tiborga olib, muhit harakati potensial ψ ga nisbatan to‘lqin tenglamasi bilan tasvirlanadi.

$$\gamma^2\ddot{\psi} = \Delta_i\psi - \frac{\psi}{r_i^2\sin^2\theta_i}, \quad \Delta_i = \frac{1}{r_i^2}\frac{\partial}{\partial r_i}\left(r_i^2\frac{\partial}{\partial r_i}\right) + \frac{1}{r_i^2\sin\theta_i}\frac{\partial}{\partial\theta_i}\left(\sin\theta_i\frac{\partial}{\partial\theta_i}\right). \tag{3}$$

Boshlang‘ich shartlar – bir jinsli

$$\psi|_{\tau=0} = \dot{\psi}\Big|_{\tau=0} = 0. \tag{4}$$

Cheksizlikda notinchlanish mavjud emas

$$\lim_{r_i\to\infty}\psi = 0. \tag{5}$$

(1) - (5) boshlang‘ich – chegaraviy masalani yechish uchun ham o‘lchovsiz vaqt τ bo‘yicha Laplas integral almashtirishlarini va o‘zgaruvchilarni to‘liq bo‘lmagan ajratish usulidan foydalaniladi. Tasvirlar fazosida ψ^L potensial ko‘chish vektori $u^L_{\vartheta_i}$ komponentasi va kuchlanish tenzorining $\sigma^L_{r_i\vartheta_i}$ komponentasi hamda berilgan $V_1^L(s,\,\theta_1)$ funksiyalarni $C^{3/2}_{n-1}(\cos\theta)$ Gegenbauer ko‘phadlari [1,4] bo‘yicha cheksiz qatorlar shaklida tasvirlanadi.

Bu holda, (1) va (2) chegaraviy shartlar qatorlarning koeffitsiyentlariga nisbatan quyidagi ko‘rinishni oladi:

$$u_{\vartheta_1 n}|_{r_1=R_1} = V^L_{1n}(s), \quad u^L_{\vartheta_2 n}\big|_{r_2=R_2} = 0. \tag{7}$$

Tasvirlar fazosida (3) tenglamaning umumiy yechimni Gegenbauer $C^{3/2}_{n-1}(\cos\theta)$ [1] ko‘phadlari bo‘yicha cheksiz qator ko‘rinishida izlanadi.

Ikkinchi tur Bessel funksiyalari $K_{n+1/2}(x)$ uchun qo‘shish [2] teoremasidan hamda ularning elementar funksiyalar [4] orqali ifodalaridan foydalanib, yechimning tasvirini i- chi sferik koordinata-lar sistemasida quyidagi cheksiz qator ko‘rinishida tasvirlaymiz.

Tasvirlar fazosida masala матрицали chiziqli algebraik tenglamalar sistemasini yechishga keltiriladi:

$$\begin{cases} \mathbf{M}\mathbf{A}_1y^2 + \mathbf{F}_1\mathbf{A}_2x - \mathbf{F}_2\mathbf{A}_2xy^2 = \mathbf{q}y \\ \mathbf{N}\mathbf{A}_2z^2 + \mathbf{T}_1\mathbf{A}_1x - \mathbf{T}_2\mathbf{A}_1xz^2 = 0 \end{cases}. \tag{8}$$

(8) matritsali tenglamalar sistemasining yechimni quyidagi cheksiz qatorlar shaklida tasvirlay-miz. Qatorlarni matritsali tenglamalar sistemasiga qo‘yib, chap va o‘ng tomondagi x, y va z (bu sistemaning birinchi tenglamasining o‘ng tomonda faqat bitta noldan farqli had bor) larning oldidagi koeffitsiyentlarni tenglab, qator koeffitsiyentlariga nisbatan quyidagi rekurrent munosabat-larni va ularga boshlang‘ich shartlarni olamiz. Rekurrent munosabatlar izlanayotgan funksiyalarning tasvirlarini tenglamalar sistemasining reduksiyalarisiz aniqlash imkonini beradi. Rekurrent munosabat-larning tahlili ko‘rsatadiki, tasvirlar Laplas almashtirishlari parametrining ratsional funksiyalaridan iboratdir. Bu esa yana o‘z navbatida ularning, hamda muhitdagi ko‘chish va kuchlanish originallarini qoldiqlar nazariyasi yordamida topish imkonini beradi [3].

Laplas almashtirishlarining tasvirlar fazosida ko’chish vektori va kuchlanish tenzorining kompo-nentalari uchun formulalar olingan. Formulalar asosida sonli eksperimentlar o’tkazilgan bo’lib, ularning natijalari gra klar shaklida tasvirlangan. Grafiklar qattiq shar sirtidan qaytgan to’lqinlar-ning ta’siri natijasida muhit kuchlanish-deformatsiya holatining o’zgarishini ko’rsatadi. Olingan natijalarni geozika, seysmologiya va yerosti inshoatlarini loyihalashtirishda foydalanish mumkin.

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Optimal quadrature formulas with the weight x^α on non-uniform nodes in the space $L_2^{(2)}(a,b)$

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Let $0 \le a < b$, $\alpha > -1$, and let the nodes $a = x_0 < x_1 < \cdots < x_N = b$, $h_k = x_k - x_{k-1}$, be given. We consider the quadrature formula

$$\int_a^b x^\alpha f(x)\,dx \approx \sum_{k=0}^{N} C_k f(x_k) \tag{1}$$

in the Sobolev space $L_2^{(2)}(a,b)$ with the norm $\|f\| = \|f''\|_{L_2(a,b)}$. Such integrals arise in problems with cylindrical ($\alpha = 1$) and spherical ($\alpha = 2$) symmetry and in fractional calculus; for non-integer α the weight has to be absorbed into the coefficients, since $x^\alpha f(x)$ is not smooth at $x = 0$.

The error functional $\ell(f) = \int_a^b x^\alpha f(x)\,dx - \sum_{k=0}^{N} C_k f(x_k)$ is bounded on $L_2^{(2)}(a,b)$ only if (1) is exact for $f = 1$ and $f = x$, i.e.

$$\sum_{k=0}^{N} C_k = \frac{b^{\alpha+1} - a^{\alpha+1}}{\alpha+1}, \qquad \sum_{k=0}^{N} C_k x_k = \frac{b^{\alpha+2} - a^{\alpha+2}}{\alpha+2}. \tag{2}$$

Following Sard and Sobolev [1–3], formula (1) is called *optimal* if, for fixed nodes, its coefficients minimize the quantity $\|\ell\| = \sup_{\|f''\|=1} |\ell(f)|$ under the constraints (2). For equally spaced nodes on $[0,1]$ the optimal coefficients with the weight x^α were found in [4] by Sobolev's method of discrete operators. The case of arbitrary (non-uniform) nodes on an arbitrary interval $[a,b]$ — natural when the values of f are known only at prescribed measurement points — remained open. Here it is solved for $N = 2$ and $N = 3$ by a variational method, and explicit formulas for the coefficients are obtained.

Method. Put $q(x) = x^{\alpha+2}/[(\alpha+1)(\alpha+2)]$, so that $q'' = x^\alpha$, and on each $[x_{k-1}, x_k]$ take $\varphi_k(x) = q(x) + A_k x + B_k$ with $\varphi_1(x_0) = 0$, $\varphi_k(x_k) = \varphi_{k+1}(x_k)$, $\varphi_N(x_N) = 0$. Integrating by parts twice one finds that if

$$C_0 = -\varphi_1'(x_0), \qquad C_k = A_k - A_{k+1} \quad (k = 1, \dots, N-1), \qquad C_N = \varphi_N'(x_N), \tag{3}$$

then (2) holds automatically, $\ell(f) = \int_a^b \varphi(x) f''(x)\,dx$, where φ is the piecewise function composed of the φ_k (the Peano kernel of ℓ), and $\|\ell\| = \|\varphi\|_{L_2}$. Hence the optimal coefficients are obtained by minimizing the strictly convex quadratic functional $F = \sum_{k=1}^N \int_{x_{k-1}}^{x_k} \varphi_k^2(x)\,dx$ under the above conditions; the minimizer is unique.

Theorem 1 (three nodes)**.** For $N = 2$ the optimal coefficients of (1) are given by (3) with

$$A_1 = \frac{h_1^2 U_1 - h_2^2 U_2 + h_2 d}{h_1(h_1+h_2)}, \qquad A_2 = \frac{h_2^2 U_2 - h_1^2 U_1 + h_1 d}{h_2(h_1+h_2)},$$

where $d = q(x_0) - q(x_2)$, $U_1 = -\frac{3}{h_1^3}\int_{x_0}^{x_1} \big(q(x) - q(x_0)\big)(x - x_0)\,dx$, $U_2 = -\frac{3}{h_2^3}\int_{x_1}^{x_2} \big(q(x) - q(x_2)\big)(x - x_2)\,dx$. For $x_k = kh$ these formulas reduce to the coefficients of [4].

Proof. The boundary conditions give $\varphi_1(x) = q(x) - q(x_0) + A_1(x - x_0)$, $\varphi_2(x) = q(x) - q(x_2) + A_2(x - x_2)$, and continuity at x_1 yields the single constraint $h_1 A_1 + h_2 A_2 = d$. Minimizing F under this constraint by the Lagrange method gives $A_i = U_i - 3\lambda/(2h_i^2)$, and λ is determined from the constraint. ■

Theorem 2 (four nodes)**.** For $N = 3$ the optimal coefficients of (1) are given by (3) with

$$A_1 = \frac{a_{12} r_2 - a_{22} r_1}{\Delta}, \qquad A_2 = \frac{a_{12} r_1 - a_{11} r_2}{\Delta}, \qquad A_3 = \frac{d - h_1 A_1 - h_2 A_2}{h_3}, \qquad \Delta = a_{11} a_{22} - a_{12}^2,$$

$$a_{11} = \tfrac{h_1^2}{3}(h_1 + 3h_2 + h_3), \quad a_{12} = \tfrac{h_1 h_2}{6}(3h_2 + 2h_3), \quad a_{22} = \tfrac{h_2^2}{3}(h_2 + h_3), \quad d = q(x_0) - q(x_3),$$

$$r_1 = J_1 + h_1 J_2 + \tfrac{h_1}{h_3} J_4 - \tfrac{h_1 h_3}{3}\, d, \qquad r_2 = J_3 + \tfrac{h_2}{h_3} J_4 - \tfrac{h_2 h_3}{3}\, d,$$

where $J_1 = \int_{x_0}^{x_1} \big(q - q(x_0)\big)(x - x_0)\,dx$, $J_2 = \int_{x_1}^{x_2} \big(q - q(x_0)\big)\,dx$, $J_3 = \int_{x_1}^{x_2} \big(q - q(x_0)\big)(x - x_1)\,dx$, $J_4 = \int_{x_2}^{x_3} \big(q - q(x_3)\big)(x_3 - x)\,dx$. For $x_k = k/3$ on $[0, 1]$ the coefficients of [4] are recovered.

Proof. The functions φ_1, φ_3 are fixed by the boundary conditions, B_2 is eliminated by the continuity conditions, and the remaining constraint $h_1 A_1 + h_2 A_2 + h_3 A_3 = d$ expresses A_3 through A_1, A_2. Expanding the normal equations $\partial F/\partial A_1 = \partial F/\partial A_2 = 0$ with the elementary integrals $\int (x - x_{k-1})^2 dx = h_k^3/3$, $\int (x - x_{k-1})(x_k - x)\,dx = h_k^3/6$ and $J_1, \dots, J_4$ gives the 2×2 system; its matrix is positive definite, so the solution is unique. ■

Example ($\alpha = 1$, $[0, 1]$)**.** For the equally spaced nodes $0, \frac{1}{2}, 1$ Theorem 1 gives the coefficients $\frac{1}{96}, \frac{5}{16}, \frac{17}{96}$ of [4] with $\|\ell\| = 9.05 \cdot 10^{-3}$, whereas for the nodes $0, \frac{3}{5}, 1$ it gives $\frac{79}{2250}, \frac{74}{225}, \frac{17}{125}$ with $\|\ell\| = 6.29 \cdot 10^{-3}$, i.e. 30% smaller. Similarly, for $N = 3$ the nodes $0, \frac{2}{5}, \frac{4}{5}, 1$ give $C = \big(\frac{151}{13200}, \frac{377}{2200}, \frac{1153}{4400}, \frac{91}{1650}\big)$ and $\|\ell\| = 2.94 \cdot 10^{-3}$ against $3.61 \cdot 10^{-3}$ for $x_k = k/3$. Since the weight x^α concentrates its mass near the right endpoint, shifting the nodes to the right reduces the worst-case error. For prescribed irregular nodes $0, \frac{1}{5}, \frac{1}{2}, 1$ the optimal formula has $\|\ell\| = 9.02 \cdot 10^{-3}$ against $1.22 \cdot 10^{-2}$ for the weighted trapezoidal rule on the same nodes; note that when nodes cluster, some optimal coefficients may become negative (here $C_1 = -\frac{343}{21744}$). For arbitrary N the problem reduces to an $(N-1) \times (N-1)$ symmetric tridiagonal system, which is solved analytically in a separate paper.

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An Optimal Quadrature Formula for Fourier Integrals in a Hilbert Space with a Derivative of Arbitrary Order

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In this work, we develop a Sard-type optimal quadrature formula based on a functional approach for the numerical computation of integrals involving highly oscillatory functions. To solve this problem, we employ the Sobolev method. First, we solve the boundary value problem for the extremal function. To solve the boundary value problem, we determine the fundamental solution of the given differential operator using the direct and inverse Fourier transforms. Using the extremal function, we determine the norm of the error functional. For the given set of nodes, we then compute the minimum value of the norm of the error functional with respect to the coefficients.

This quadrature formula is exact for the hyperbolic functions $\sinh(x)$ and $\cosh(x)$, as well as for polynomials of degree $m-3$. In this paper, we consider the cases $\omega h \notin \mathbb{Z}$ with $\omega \in \mathbb{R}$ and $\omega h \in \mathbb{Z}$ with $\omega \neq 0$ in the Hilbert space $\mathrm{K}_2^{(m)}(0,1)$.

For particular cases, numerical results are obtained using the constructed quadrature formula, and its application to image reconstruction in computed tomography is demonstrated.

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Construction of optimal quadrature formulas with derivatives for highly oscillatory integrals with trigonometric weights.

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This work is devoted to the construction of optimal quadrature formulas for highly oscillatory integrals with trigonometric weights in the Sobolev space $L_2^{(2)}(a,b)$. We consider quadrature formulas involving the values of a function and its first derivative at prescribed nodes. The optimal coefficients are determined by minimizing the corresponding error functionals using the φ-function method.

In this work, we consider quadrature formulas for the numerical evaluation of the following integrals in the space $L_2^{(2)}(a,b)$ and $\omega \neq 0$, $\omega \in \mathbb{R}$

$$I_c(f) = \int_a^b \cos(2\pi\omega x) f(x) dx, \tag{1}$$

$$I_s(f) = \int_a^b \sin(2\pi\omega x) f(x) dx, \tag{2}$$

where $f \in L_2^{(2)}(a,b)$, and its norm is defined as follows:

$$\|f\|^2_{L_2^{(2)}(a,b)} = \int_a^b \left(f''(x)\right)^2 dx.$$

For the integrals (1) and (2), we consider the following quadrature formulas:

$$I_c(f) = \sum_{k=0}^{n} A_{0k}^c f(x_k) + A_{1k}^c f'(x_k) + R_{n,2}^c(f), \tag{3}$$

$$I_s(f) = \sum_{k=0}^{n} A_{0k}^s f(x_k) + A_{1k}^s f'(x_k) + R_{n,2}^s(f). \tag{4}$$

Using the φ-function method [2], we determine the coefficients by minimizing the corresponding error functionals. Thus, the optimal quadrature formulas of the form (3) and (4) are obtained.

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φ-funksiya metodi asosida eksponensial kvadratur formula

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Aniq integrallarni hisoblash fizika, muhandislik, iqtisodiyot va boshqa sohalardagi ko‘plab amaliy masalalarda muhim ahamiyatga ega. Biroq ularni analitik usullar bilan hisoblash har doim ham imkoni yo‘q. Shu sababli sonli integrallash usullari, xususan, kvadratur formulalardan keng foydalaniladi. Kvadratur formulaning aniqligi va samaradorligi uning tugun nuqtalarini hamda koeffitsiyentlarini tanlanishiga bog‘liq.

Optimal kvadratur formulalarni qurishda splaynlar usuli, φ-funksiya usuli va Sobolev usuli kabi yondashuvlardan foydalaniladi. A. Sard yondashuvida tugunlar qat'iy belgilangan holda xatolik funksionalining normasini minimallashtiruvchi optimal koeffitsiyentlar izlanadi. Bunday formulalarni funksiyalarning turli sinflarida o‘rganish sonli integrallash usullarini rivojlantirishda muhim ahamiyat kasb etadi.

Mazkur ishda 1, e^x va e^{2x} funksiyalariga aniq bo‘ladigan optimal kvadratur formulani qurish masalasi qaraladi. Shuningdek, kvadratur formulaning koeffitsiyentlarini aniqlash hamda xatoligini baholash masalalari qo‘yiladi.

Shu sababli, biz ushbu ishda quyidagi ko‘rinishdagi kvadratur formulani qaraymiz:

$$\int_a^b f(x)g(x)\,dx = \sum_{k=0}^{n} A_{0,k} f\left(x_k\right) + \sum_{k=0}^{n} A_{1,k} f'\left(x_k\right) + \sum_{k=0}^{n} A_{2,k} f''\left(x_k\right) + R_n(f),$$

bunda $g(x)$ — vazn funksiyasi, $A_{0,k}$, $A_{1,k}$ va $A_{2,k}$ — kvadratur formulaning koeffitsiyentlari, x_k lar esa uning tugunlari.

Tugun nuqtalari quyidagicha joylashgan bo‘lsin:

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b.$$

f'' ikkinchi tartibli hosilasi $[a,b]$ kesmada absolyut uzluksiz va $f''' \in L_2(a,b)$ bo‘lgan funksiyalar sinfini qaraymiz. Ushbu sinf asosida faktorizatsiya qilingan Gilbert fazosini qaraymiz va uni $H_2^{(3)}(a,b)$ bilan belgilaymiz:

$$H_2^{(3)}(a,b) = \left\{ f : [a,b] \to \mathbb{R} \;\middle|\; f'' \text{ — absolyut uzluksiz, } f''' \in L_2(a,b) \right\}.$$

Ushbu faktorizatsiya qilingan Gilbert fazosida skalyar ko‘paytma quyidagicha aniqlanadi:

$$\langle f, h \rangle_{H_2^{(3)}} = \int_a^b \left(f'''(x) - 3f''(x) + 2f'(x)\right)\left(h'''(x) - 3h''(x) + 2h'(x)\right) dx.$$

Mos norma quyidagicha kiritiladi:

$$\|f\|_{H_2^{(3)}} = \left(\int_a^b \left(f'''(x) - 3f''(x) + 2f'(x) \right)^2 dx \right)^{1/2}.$$

Ushbu ishda quyidagi masalalar o‘rganiladi va yechiladi:

a) (1)-ko‘rinishdagi optimal kvadratur formulaning koeffitsiyentlarini aniqlash;

b) (1)-ko‘rinishdagi optimal kvadratur formulaning xatoligini baholash.

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CONSTRUCTION OF A QUADRATURE FORMULA INVOLVING DERIVATIVES OF ARBITRARY ORDER USING THE METHOD OF φ-FUNCTIONS

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The definite integral is one of the fundamental concepts in mathematical analysis, widely used for applications such as determining the area under a curve and the volume of various solids. However, in many practical and theoretical problems, finding the antiderivative of the integrand is complex or sometimes entirely impossible. In cases where finding the antiderivative is difficult or its elementary form does not exist, the problem of approximating the value of the definite integral is resolved using quadrature and cubature formulas.

In this work, using the method of φ-functions, we investigate the problem of constructing optimal quadrature formulas in the Hilbert space $K_2^{(m,m-2)}$ for a general natural number $m \geq 2$. When the nodes of the quadrature formula are arbitrarily preassigned, the principle of minimizing the norm of the error functional can be applied to determine its optimal coefficients.

We consider a quadrature formula of the following form:

$$\int_a^b f(x)dx = \sum_{k=0}^{n} A_{0k} f(x_k) + \sum_{j=1}^{n-1} \sum_{k=0}^{n} A_{jk} f^{(j)}(x_k) + R_n(f), \tag{1}$$

where A_{0k}, A_{1k}, ..., $A_{n-1,k}$ and x_k are the coefficients and nodes of the quadrature formula (1), respectively. Let the nodes be distributed on the interval $[a, b]$ as follows: $a = x_0 < x_1 < \ldots < x_n = b$.

Assume that the integrand $f(x)$ belongs to the space $K_2^{(m,m-2)}$, defined as:

$$K_2^{(m,m-2)} := \{ f : [a, b] \to \mathbb{R} |\, f^{(m-1)} \text{ is absolutely continuous and } f^{(m)} \in L_2(a, b) \}.$$

For arbitrary functions $f(x)$ and $g(x)$, the inner product and the norm in this space are respectively defined as:

$$\langle f,\, g\rangle_{K_2^{(m,m-2)}} = \int_a^b (f^{(m)}(x) + f^{(m-2)}(x))(g^{(m)}(x) + g^{(m-2)}(x))dx, \tag{2}$$

$$\|f\|_{K_2^{(m,m-2)}} = \left(\int_a^b \left(f^{(m)}(x) + f^{(m-2)}(x) \right)^2 dx \right)^{1/2}. \tag{3}$$

Definition 1. The quadrature formula of the form (1) is said to be optimal in the sense of Nikolskii in the space $K_2^{(m,m-2)}$, if the quantity $\sup\limits_{f\in K_2^{(m,m-2)}} |R_n(f)|$ attains its minimum value with respect to the coefficients and nodes.

Definition 2. The formula (1) is optimal in the sense of Sard in the space $K_2^{(m,m-2)}$, if the above supremum yields its minimum value with respect to the coefficients for fixed nodal points X.

We present the method of φ-functions for constructing the optimal quadrature formula (1) in the sense of Sard. For a given natural number n, let the nodal points be equally spaced. For each interval $[x_{k-1},\, x_k]$ $(k = 1,\, 2,\, \ldots,\, n)$, the function φ_k possesses the property:

$$\varphi_k^{(n)}(x) + \varphi_k^{(n-2)}(x) = (-1)^n, \;\; k = 1,\, 2,\, \ldots,\, n. \tag{4}$$

By utilizing the additivity property of definite integrals and taking into account equality (4), we obtain the following system for the coefficients:

$$A_{jk} = \begin{cases} (-1)^{n+j+1}[\varphi_1^{(n-1-j)}(x_0) + \varphi_1^{(n-3-j)}(x_0)],\ k = 0,\ j = 0, \ldots, n-3, \\ (-1)^{n+j}\left[\varphi_k^{(n-1-j)}(x_k) - \varphi_{k+1}^{(n-1-j)}(x_k) + \varphi_k^{(n-3-j)}(x_k) - \varphi_{k+1}^{(n-3-j)}(x_k)\right], \\ \qquad k = 1, \ldots, n-1,\ j = 0, \ldots, n-3, \\ (-1)^n[\varphi_n^{(n-1-j)}(x_n) + \varphi_n^{(n-3-j)}(x_n)],\ k = n,\ j = 0, \ldots, n-3, \\ -\varphi_1'(x_0), \quad k = 0,\ j = n-2, \\ \varphi_k'(x_k) - \varphi_{k+1}'(x_k), \quad k = 1, \ldots, n-1,\ j = n-2, \\ \varphi_1(x_0), \quad k = 0,\ j = n-1, \\ -\varphi_k(x_k) + \varphi_{k+1}(x_k), \quad k = 1, \ldots, n-1,\ j = n-1, \\ -\varphi_n(x_n), \quad k = n,\ j = n-1. \end{cases} \tag{5}$$

The error formula takes the form:

$$R_n(f) = \sum_{k=1}^{n} \int_{x_{k-1}}^{x_k} (f^{(n)}(x) + f^{(n-2)}(x))\varphi_k(x)dx. \tag{6}$$

Solving the non-homogeneous differential equation $\varphi_k{}^{(n)} + \varphi_k{}^{(n-2)} = (-1)^n$, the general solution is:

$$\varphi_k(x) = C_1^{(k)}\cos(x) + C_2^{(k)}\sin(x) + \sum_{\lambda=0}^{n-3} C_{\lambda+3}^{(k)} x^\lambda + (-1)^n \frac{x^{n-2}}{(n-2)!}. \tag{7}$$

Applying the Cauchy-Schwarz inequality, the absolute value of the error (6) is bounded by:

$$|R_n(f)| \leq \|f(x)\|_{K_2^{(m,m-2)}} \cdot \|\varphi(x)\|_{L_2(a,b)}. \tag{8}$$

To find the optimal coefficients, we aim to minimize the integral of the square of the function $\varphi_k(x)$:

$$\mathcal{F}_k(C_1^{(k)}, \ldots, C_n^{(k)}) = \int\limits_{x_{k-1}}^{x_k} (\varphi_k(x))^2 dx. \tag{9}$$

By taking the first-order partial derivatives of $\mathcal{F}_k$ with respect to $C_i^{(k)}$ and equating them to zero, we form a system of equations. Introducing the notations for the integrals of the basis functions $\psi_1(x) = \cos x$, $\psi_2(x) = \sin x$, $\psi_3(x) = 1$, $\ldots$, $\psi_n(x) = x^{n-3}$:

$$a_{ij} = \int\limits_{x_{k-1}}^{x_k} \psi_i(x)\psi_j(x)dx, \quad i,j = 1,2,\ldots,n, \tag{10}$$

we rewrite the system as:

$$\begin{cases} a_{11}C_1^{(k)} + a_{12}C_2^{(k)} + a_{13}C_3^{(k)} + \ldots + a_{1n}C_n^{(k)} = b_1^{(k)}, \\ a_{21}C_1^{(k)} + a_{22}C_2^{(k)} + a_{23}C_3^{(k)} + \ldots + a_{2n}C_n^{(k)} = b_2^{(k)}, \\ \qquad\qquad\qquad\qquad \vdots \\ a_{n1}C_1^{(k)} + a_{n2}C_2^{(k)} + a_{n3}C_3^{(k)} + \ldots + a_{nn}C_n^{(k)} = b_n^{(k)}. \end{cases} \tag{11}$$

The principal determinant composed of the elements a_{ij} is recognized as the Gram determinant: $\Delta_k = \det(a_{ij})_{i,j=1}^n$. Since the system of functions is linearly independent, $\Delta_k > 0$, ensuring $\Delta_k \neq 0$.

Theorem 1. Because $\Delta_k \neq 0$, the system (11) has a unique solution. The unknown coefficients are determined using Cramer's rule:

$$C_i^{(k)} = \frac{\Delta_i^{(k)}}{\Delta_k}, \quad i = 1, 2, \ldots, n. \tag{12}$$

By substituting the obtained coefficients into the solution (7), we write its general form, and using it, we express the optimal coefficients A_{0k}, A_{1k}, $\ldots$, $A_{n-1,k}$. The obtained results generalize previous studies and create the possibility to construct a uniformly structured quadrature formula for the case of an arbitrary derivative order m.

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Approximate Computation of Fourier Integrals by Periodicization of Functions

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In this work, we consider the approximation of Fourier integrals by periodicizing the given function. The main idea is to restrict the function to a finite interval, periodically extend it, and approximate the resulting periodic function by a finite Fourier series. This approach makes it possible to replace an integral over an infinite interval by a finite sum of Fourier coefficients. The accuracy of the method depends on the truncation interval, the smoothness of the function, and the number of Fourier modes used in the approximation.

0.2 Introduction

Fourier integrals are widely used for representing non-periodic functions and studying their frequency characteristics. For a sufficiently well-behaved function $f(x)$, its Fourier transform can be written in the form

$$f(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}\,dx. \tag{1}$$

Although this representation is theoretically useful, the direct numerical calculation of the integral may be difficult because the integration interval is unbounded and the integrand is generally oscillatory.

A convenient numerical approach is to replace the original function by a periodic function on a finite interval. The periodic function can then be represented by a Fourier series, allowing the Fourier integral to be approximated using a finite number of Fourier coefficients.

This technique is particularly useful when the function decreases sufficiently rapidly as $|x| \to \infty$.

The purpose of this work is to investigate the basic principle of periodicization and its application to the approximate computation of Fourier integrals.

0.3 Periodicization of a Function

Let $f(x)$ be a function defined on the real line. Choose a positive number L and consider the interval

$$[-L, L].$$

We define the periodic extension $f_L(x)$ of $f(x)$ with period $2L$ by

$$f_L(x+2L) = f_L(x),$$

where

$$f_L(x) = f(x), \qquad x \in [-L, L].$$

The periodic function $f_L(x)$ can be expanded into a Fourier series:

$$f_L(x) = \sum_{k=-\infty}^{\infty} c_k e^{i\pi kx/L}, \tag{2}$$

where the Fourier coefficients are given by

$$c_k = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-i\pi kx/L}\, dx. \tag{3}$$

Thus, instead of considering the function on the whole real axis, we work with its restriction to a finite interval and its periodic continuation.

0.4 Approximation of the Fourier Integral

The Fourier transform of $f(x)$ is

$$\widehat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx. \tag{4}$$

If $f(x)$ decreases sufficiently rapidly outside the interval $[-L, L]$, then

$$\widehat{f}(\omega) \approx \int_{-L}^{L} f(x) e^{-i\omega x}\, dx. \tag{5}$$

Using the Fourier coefficients of the periodicized function, we obtain

$$\int_{-L}^{L} f(x) e^{-i\omega x}\, dx = 2L \sum_{k=-\infty}^{\infty} c_k \frac{\sin(\omega L - \pi k)}{\omega L - \pi k}.$$

When $\omega L = \pi k$, the corresponding term is understood in the limiting sense.
In practical computations, the infinite series is truncated:

$$\widehat{f}(\omega) \approx 2L \sum_{k=-N}^{N} c_k \frac{\sin(\omega L - \pi k)}{\omega L - \pi k}. \tag{6}$$

Therefore, the original Fourier integral is reduced to a finite sum. The Fourier coefficients c_k themselves can be computed numerically, for example, by a quadrature formula or by the Fast Fourier Transform (FFT).

0.5 Numerical Scheme

The proposed procedure can be summarized as follows.

First, an interval $[-L, L]$ is selected so that the contribution of $f(x)$ outside this interval is sufficiently small. Next, the function is sampled at a set of equally spaced points

$$x_j = -L + \frac{2Lj}{M}, \qquad j = 0, 1, \ldots, M-1.$$

The discrete Fourier coefficients are then calculated from

$$c_k \approx \frac{1}{M} \sum_{j=0}^{M-1} f(x_j) e^{-2\pi i k j / M}.$$

These coefficients can be obtained efficiently using the FFT algorithm. Finally, the Fourier transform is approximated by a finite Fourier representation.

There are two main sources of error in this procedure. The first is the *truncation error*, which appears when the infinite interval $(-\infty, \infty)$ is replaced by $[-L, L]$. The second is the *Fourier truncation and discretization error*, which depends on the number of Fourier coefficients and sampling points.

Increasing L generally reduces the truncation error for rapidly decaying functions. Increasing N or M improves the resolution of the Fourier approximation, although excessive values may increase computational cost.

0.6 Error Analysis

The total approximation error can be considered as the sum of several components:

$$E \leq E_{\text{trunc}} + E_{\text{Fourier}} + E_{\text{disc}},$$

where E_{trunc} denotes the error caused by restricting the infinite integration interval, E_{Fourier} is the error associated with truncating the Fourier series, and E_{disc} represents the numerical discretization error.

For a rapidly decreasing function, the truncation error

$$E_{\text{trunc}} = \left| \int_{|x|>L} f(x) e^{-i\omega x}\, dx \right|$$

becomes small as L increases.

If the periodicized function is sufficiently smooth, its Fourier coefficients decay rapidly. Therefore, a relatively small number of terms may be sufficient to obtain an accurate approximation.

0.7 Conclusion

In this work, an approach for the approximate computation of Fourier integrals based on function periodicization has been considered. The main advantage of the method is that an integral over an infinite interval can be transformed into a finite computational problem involving Fourier coefficients.

The accuracy of the approximation is determined mainly by the choice of the truncation interval and the number of Fourier modes. For rapidly decreasing and sufficiently smooth functions, the method can achieve high accuracy with relatively few Fourier coefficients.

The use of FFT algorithms additionally makes the computation efficient for large sets of sampling points. Thus, periodicization provides a useful connection between Fourier integrals and Fourier series and can be effectively applied in numerical analysis, signal processing, and computational mathematics.

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Construction of an exponential-algebraic optimal quadrature formula in a space of differentiable functions

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Quadrature and cubature formulas approximate definite integrals and serve as one of the essential tools for solving differential and integral equations. The theory of cubature formulas comprises three mainapproaches: classic algebraic quadrature and cubature formulas, formulas based on functional-analytical methods, and probabilistic approaches. In the functional-analytical framework, the error between an integral and its approximation is modeled as a linear functional on a Banach space. Minimization of this error functional's norm leads to two problems:

a) Nikolskii's Problem: minimizing the norm of the error functional with respect to both nodes and coefficients.

b) Sard's Problem: minimizing the norm of the error functional with respect to coefficients while keeping nodes fixed.

Key approaches for constructing optimal formulas in the sense of Sard include the spline method, the φ-function method, and Sobolev's method, which utilizes discrete analogues of linear differential operators. Solving these problems across various differentiable function spaces yields optimal numerical integration formulas.

This paper is concerned with the construction of an exponential-algebraic optimal quadrature formula in a Hilbert space.

We consider a quadrature formula of the form

$$\int_0^1 \varphi(x)\,dx \approx \sum_{\beta=0}^{N} C_{\beta,0}\,\varphi(h\beta) + \sum_{\beta=0}^{N} C_{\beta,1}\big(\varphi'(h\beta) + \varphi(h\beta)\big), \tag{1}$$

where $C_{\beta,0}$ are coefficients of the optimal quadrature formula of the form

$$\int_0^1 \varphi(x)\,dx \approx \sum_{\beta=0}^{N} C_{\beta,0}\,\varphi(h\beta)$$

in the space $W_2^{(1,0)}(0,1)$ [1], i.e.

$$C_{0,0} = \frac{e^h-1}{e^h+1}, \qquad C_{\beta,0} = 2\cdot\frac{e^h-1}{e^h+1},\ \beta = 1,2,\dots,N-1, \qquad C_{N,0} = \frac{e^h-1}{e^h+1}. \tag{2}$$

$C_{\beta,1}$ are unknown coefficients of the formula (1) and they should be found, $h=\frac{1}{N}$, N is a natural number.

We assume that an integrand φ belongs to $W_2^{(2,1)}(0,1)$, where the class $W_2^{(2,1)}(0,1)$ under the pseudo-inner product

$$\langle\varphi,\psi\rangle = \int_0^1 (\varphi'(x)+\varphi(x))(\psi'(x)+\psi(x))dx$$

is a Hilbert space if we identify functions that differ by a linear combination of a constant and e^{-x}. Here, in the Hilbert space $W_2^{(2,1)}(0,1)$, we consider the corresponding norm

$$\|\varphi|W_2^{(2,1)}(0,1)\| = \left[\int_0^1 (\varphi'(x)+\varphi(x))^2 dx\right]^{1/2}. \tag{3}$$

The difference

$$(\ell,\varphi) = \int_0^1 \varphi(x)\,dx - \sum_{\beta=0}^{N} C_{\beta,0}\varphi(h\beta) - \sum_{\beta=0}^{N} C_{\beta,1}\big(\varphi'(h\beta)+\varphi(h\beta)\big) \tag{4}$$

is called *the error* and

$$\ell(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_{\beta,0}\,\delta(x-h\beta) + \sum_{\beta=0}^{N} C_{\beta,1}\big(\delta'(x-h\beta)-\delta(x-h\beta)\big), \tag{5}$$

is said to be *the error functional* of the quadrature formula (1), where $\varepsilon_{[0,1]}(x)$ is the indicator of the interval $[0,1]$ and δ is Dirac's delta function. The value (ℓ,φ) of the error functional ℓ at a function φ is defined as $(\ell,\varphi)=\int_{-\infty}^{\infty}\ell(x)\varphi(x)dx$.

In order that the error functional (5) is defined on the space $W_2^{(2,1)}(0,1)$ it is necessary to impose the following conditions for the functional ℓ

$$(\ell,1) := 1 - \sum_{\beta=0}^{N} C_{\beta,0} - \sum_{\beta=0}^{N} C_{\beta,1} = 0., \tag{6}$$

$$(\ell,e^{-x}) := \int_0^1 e^{-x}\,dx - \sum_{\beta=0}^{N} C_{\beta,0}e^{-h\beta} = 0. \tag{7}$$

The last two equations indicate that the quadrature formula (1) is exact for any constant and e^{-x}. We have chosen the coefficients $C_{\beta,0}$, $\beta = 0, 1, ..., N$ such that the equality (6) is fulfilled. Therefore we have only condition (7) for coefficients $C_{\beta,1}$, $\beta = 0, 1, ..., N$.

The error functional ℓ of the formula (1) is a linear functional in $W_2^{(2,1)*}(0,1)$, where $W_2^{(2,1)*}(0,1)$ is the conjugate space to the space $W_2^{(2,1)}(0,1)$.

By the Cauchy-Schwarz inequality we have the following

$$|(\ell, \varphi)| \leq \|\varphi|W_2^{(2,1)}(0,1)\| \cdot \|\ell|W_2^{(2,1)*}(0,1)\|.$$

Hence we infer that the error (4) of the formula (1) is controlled by the norm

$$\left\|\ell|W_2^{(2,1)*}(0,1)\right\| = \sup_{\left\|\varphi|W_2^{(2,1)}(0,1)\right\|=1} |(\ell, \varphi)| \tag{8}$$

of the error functional (5).

The main aim of this work is to determine the coefficients $C_{\beta,1}$ that minimize the absolute value of the error (4) for the given coefficients $C_{\beta,0}$ in the space $W_2^{(2,1)}$. That is the problem is to find the coefficients $C_{\beta,1}$ that attain the following quantity

$$\left\|\mathring{\ell}|W_2^{(2,1)*}\right\| = \inf_{C_{\beta,1}} \left\|\ell|W_2^{(2,1)*}\right\|. \tag{9}$$

The coefficients $C_{\beta,1}$ which attain the last quantity are called *optimal* and are denoted as $\mathring{C}_{\beta,1}$.

Thus, to obtain the optimal quadrature formula of the form (1) in the sense of Sard in the space $W_2^{(2,1)}(0,1)$, we need to solve the following problems.

Problem 1. Find the norm of the error functional (5) of the quadrature formula (1) in the space $W_2^{(2,1)*}$.

Problem 2. Find the coefficients $\mathring{C}_{\beta,1}$ that satisfy equality (9).

Below is the main result of our study

Theorem 1. *The coefficients of the optimal quadrature formula of the form (1) in the sense of Sard for the functions of the space $W_2^{(2,1)}(0,1)$ have the following form*

$$\begin{aligned} \mathring{C}_{0,1} &= -(h+2)F, \\ \mathring{C}_{\beta,1} &= -2hF, \quad \beta = 1, 2, ..., N-1, \\ \mathring{C}_{N,1} &= (2-h)F, \end{aligned}$$

where $F = \left(\frac{e^h-1)}{h(e^h+1)} - \frac{1}{2}\right)$.

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YUKLANISHLAR OSTIDAGI STERJENNING GEOMETRIK CHIZIQSIZLIKNI VA HARORATNI HISOBGA OLGAN HOLDA MATEMATIK MODELLASHTIRISH

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Annotatsiya. Mazkur maqolada yuklanishlar ostida ishlovchi sterjenlarning kuchlanganlik-deformatsiyalanganlik holatini aniqlash uchun geometrik chiziqsizlik va harorat ta'sirlarini hisobga olgan holda matematik model ishlab chiqilgan. Tadqiqotda sterjenning bo'ylama, ko'ndalang hamda buralish deformatsiyalari o'zaro bog'liq holda qaralib, Von Karman tipidagi geometrik nochiziqlilik tenglamalari asosida umumlashtirilgan differensial tenglamalar sistemasini hosil qilishga erishilgan. Modelda haroratning bir tekis va notekis taqsimlanishi natijasida yuzaga keladigan issiqlik deformatsiyalari va ichki kuchlar tahlil qilinib, ular konstruktsiya barqarorligiga va umumiy deformatsiya holatiga ta'siri baholangan. Shuningdek, chekli elementlar usuli asosida sonli yechim algoritmi ishlab chiqilib, Newton–Raphson iteratsion usuli yordamida nochiziqli tenglamalar sistemasi yechilgan.

Kalit so'zlar: geometrik chiziqsizlik, harorat ta'siri, sterjen, chekli elementlar usuli, FEM, deformatsiya, kuchlanganlik holati, termik deformatsiya, Von Karman modeli, Newton–Raphson usuli, barqarorlik, egilish, buralish, bo'ylama ko'chish, matematik modellashtirish.

Hozirgi zamonaviy muhandislik amaliyotida konstruksiyalarning ishonchliligi va xavfsizligini ta'minlash eng muhim vazifalardan biri hisoblanadi. Ayniqsa, yuqori yuklanishlar va harorat ta'sirida ishlovchi sterjen va balkali tizimlarning mexanik xatti-harakatini to'g'ri baholash dolzarb masalalardan biridir. Amaliyotda ko'plab hisoblashlar hali ham chiziqli nazariyaga asoslanadi, biroq real sharoitlarda katta deformatsiyalar va geometrik chiziqsizlik ta'siri sezilarli darajada namoyon bo'ladi. Bu esa hisoblash natijalarining aniqligiga salbiy ta'sir ko'rsatishi mumkin.

Sterjen va balkali tizimlarning geometrik chiziqsizligi hamda harorat ta'sirini hisobga olgan holda modellashtirish masalasi ko'plab xorijiy olimlar tomonidan chuqur o'rganilgan. Klassik mexanika asoslari Stephen Timoshenko tomonidan ishlab chiqilgan bo'lib, uning ishlanmalarida balkalar nazariyasi va deformatsiyalar tahlili keng yoritilgan. Xususan, Timoshenko va uning izdoshlari egilish va kesish deformatsiyalarini hisobga oluvchi modellarni taklif qilganlar [1-2].

Geometrik chiziqsizlikni hisobga olish masalasi esa Theodore Von Karman tomonidan taklif etilgan tenglamalar asosida rivojlantirilgan bo'lib, bu model katta deformatsiyalar sharoitida aniqlikni sezilarli oshiradi. Von Karman tenglamalari hozirgi kunda ham nochiziqli elastiklik nazariyasining asosiy tayanchlaridan biri hisoblanadi [3].

Chekli elementlar usulini rivojlantirish va uni muhandislik masalalariga qo'llashda Olgierd Zienkiewicz va Robert L. Taylor ning ilmiy ishlari muhim ahamiyatga ega. Ularning tadqiqotlarida FEM yordamida murakkab konstruksiyalarning kuchlanganlik-deformatsiyalanganlik holatini aniqlash usullari batafsil bayon etilgan [4].

Termik ta'sirlarni hisobga olish yo'nalishida J. N. Reddy tomonidan elastik va termik maydonlarning o'zaro bog'liqligini ifodalovchi nazariyalar ishlab chiqilgan. U o'z ishlarida harorat ta'siridagi deformatsiyalarni FEM asosida tahlil qilish usullarini taklif etadi [5-6].

Shuningdek, nochiziqli tenglamalarni yechishda Newton–Raphson usuli keng qo'llanilib, u iteratsion yondashuv orqali murakkab tizimlarning yechimini topishga imkon beradi. Zamonaviy tadqiqotlarda bu usul geometrik chiziqsiz va termo-mexanik tizimlarni hisoblashda asosiy vositalardan biri hisoblanadi [7]-[12].

Yuqoridagi tadqiqotlar mazkur yo'nalishda katta ilmiy asos yaratgan bo'lsa-da, geometrik chiziqsizlik va harorat ta'sirini kompleks holda, ayniqsa fazoviy yuklanishlarda birgalikda tahlil qilish masalasi hali ham dolzarbligicha qolmoqda.

Egilish masalasining **geometrik nochiziqli matematik modeli** odatda katta egilish yoki katta burilish holatini hisobga oladi. Asosiy farq shuki, deformatsiya faqat u' bilan emas, balki egilishdan keladigan kvadratik had bilan ham ifodalanadi.

Masalaning qo'yilishi. Zamonaviy muhandislik amaliyotida sterjen tipidagi konstruksiyalar turli xil mexanik yuklanishlar va harorat ta'siri ostida ishlaydi. Bunday sharoitlarda ularning kuchlanganlik-deformatsiyalanganlik holatini aniqlashda chiziqli nazariya yetarli aniqlik bermaydi, chunki katta deformatsiyalar natijasida geometrik chiziqsizlik sezilarli darajada namoyon bo'ladi. Shu sababli, yuklanishlar ostidagi sterjenning xatti-harakatini to'g'ri tavsiflash uchun geometrik nochiziqlilik va termik ta'sirlarni birgalikda hisobga oluvchi matematik model ishlab chiqish zarur.

Ko'rib chiqilayotgan masalada uzunligi L, ko'ndalang kesimi ixtiyoriy bo'lgan sterjen fazoviy yuklanishlar ostida qaraladi. Sterjen bo'ylab taqsimlangan bo'ylama kuchlar, ko'ndalang yuklar hamda buralish momentlari ta'sir qiladi. Shu bilan birga, sterjen uzunligi va kesimi bo'yicha haroratning bir tekis yoki notekis taqsimlanishi natijasida yuzaga keladigan termik deformatsiyalar ham hisobga olinadi. Natijada sterjenning bo'ylama ko'chishi $u(x)$, ko'ndalang siljishi $w(x)$, burilish burchagi $\theta(x)$ va buralish burchagi $\varphi(x)$ aniqlanishi talab etiladi.

Masalaning matematik qo'yilishida deformatsiyalarni aniqlashda geometrik nochiziqlilikni hisobga olish talab etiladi.

Modelning geometrik qo'yilishi: uzunligi L bo'lgan sterjen qaraladi. Koordinata o'qi: $x \in [0, L]$. Sterjenning ko'chishlari: $u(x)$ – bo'ylama ko'chish, $w(x)$ – ko'ndalang egilish, $\theta(x)$ – kesim burilishi, $\varphi(x)$ – buralish burchagi.

Geometrik nochiziqlilikni Von Karman modelini hisobga olib, deformatsiya quyidagicha ifodalanadi:

$$\varepsilon = u'(x) + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \alpha T(x), \tag{1}$$

Egilish egri chiziqligi:

$$k = -w''(x) - \alpha\frac{T_g(x)}{h}, \tag{2}$$

Buralish deformatsiyasi:

$$\gamma = -\varphi'(x) - \alpha\frac{T_{tor}(x)}{h}. \tag{3}$$

Yuqoridagi (1)–(3) tenglamalarni Lagranj variatsion tenglamaga qo'yib, quyidagi sterjenning nochiziqli va haroratni hisobga olingan tenglamalar sistemasi hosil bo'ladi.

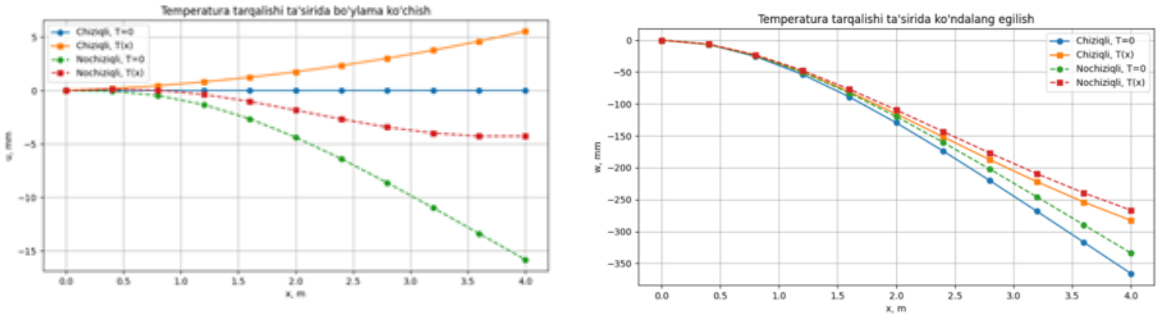
To'liq differensial tenglamalar sistemasi:

$$\begin{gathered}
\frac{d}{dx}\left[EA\left(u' + \frac{1}{2}(w')^2 - \alpha T(x)\right)\right] + q_x = 0, \\
EIw'''' - \frac{d}{dx}\left[\left(EA\left(u' + \frac{1}{2}(w')^2 - \alpha T(x)\right)\right)w'\right] = q_z, \\
\frac{d}{dx}\left[GJ\left(\varphi' - \alpha\frac{T_{tor}(x)}{h}\right)\right] + m_t = 0.
\end{gathered} \tag{4}$$

Chegaraviy shartlar:

$$\left[N\,\delta u\right]_0^L = 0,$$
$$\left[EIw''\delta w' + (-EIw''' + Nw')\delta w\right]_0^L = 0, \tag{5}$$
$$\left[T\,\delta\varphi\right]_0^L = 0.$$

Sonli tahlil chekli elementlar usuli asosida amalga oshirilib, nochiziqli tenglamalar sistemasi Newton–Raphson iteratsion usuli yordamida yechildi. Chiziqli va nochiziqli modellar, shuningdek haroratsiz ($T = 0$) va harorat taqsimoti $T(x)$ hisobga olingan holatlar taqqoslandi. 1-rasmda sterjenning bo'ylama va ko'ndalang ko'chishlarining koordinata bo'yicha o'zgarishi keltirilgan.



1-rasm. Harorat ta'sirida sterjenning bo'ylama va ko'ndalang ko'chishi

Olingan natijalar shuni ko'rsatadiki, geometrik chiziqsizlikni hisobga olish sterjenning bo'ylama ko'chishida chiziqli modelga nisbatan sezilarli farqni keltirib chiqaradi, harorat ta'siri esa bo'ylama ko'chishning ishorasi va qiymatiga bevosita ta'sir ko'rsatadi. Ko'ndalang egilishda nochiziqli va chiziqli modellar orasidagi farq katta deformatsiyalar sohasida yanada aniq namoyon bo'ladi.

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AKUSTIK YARIM FAZODA SFERIK ELASTIK QATLAMNING NOSTATSIONAR TEBRANISHI

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Matematik modellashtirish muammolari bog‘liq masalalar va tutash muhitdagi nostatsionar to‘lqin jarayonlarini o‘rganish, shuningdek, defor-matsiyalanuvchi jismlarning atrof-muhit bilan o‘zaro ta’siri bilan bog‘liq muammolar dolzarb bo‘lib, ular matematik hamda mexaniklarda amaliy va nazariy qiziqish hosil qiladi. Tutash muhitlar dinamikasidagi muammolarning dolzarbligi fan va texnikaning turli sohalarining rivojlanishi, dinamik kuchlanishlar ostida ishlaydigan yangi qurilmalar-ning yaratilishi, shuningdek, samolyotsozlik, kemasozlik va boshqa ko‘plab sohalardagi muammolar bilan bog‘liqdir. Deformatsiyalanuvchi jismlarning atrof-muhit bilan o‘zaro ta’siri bilan bog‘liq turli masalalar ilmiy maqolalarda [1, 2, 6] va boshqa joylarda tasvirlangan.

Ushbu ilmiy tadqiqot ishida akustik yarim fazoda o‘zaro konsentrik sferik sirtlar bilan chegara-langan elastik qatlamdan nostatsionar bo‘ylama to‘lqinlarning tarqalishi o‘rganiladi. Maqolaning maqsadi muammoni yechish algoritm ishlab chiqish va ideal suyuqlikning sferik elastik qatlamda va yarim fazosida nostatsionar bo‘ylama to‘lqin jarayonlarini o‘rganishdir.

Aytaylik, ideal suyuqliklar yarim fazosining $z = 0$ tekis sirtidan h masofada ichki va tashqi radiuslari mos ravishda r_1 va r_2 ($r_1 < r_2 < h$) bo‘lgan bir jinsli chiziqli izotropik muhitli sferik qatlamning O markazi joylashgan bo‘lsin. Ideal suyuqliklar va sferik qamlam muhitlarining harakati boshlang‘ich nuqtasi qatlam markazida bo‘lgan (r, θ, ϑ) sferik koordinatalar sistemasida qaraladi.

$\tau = 0$ boshlang‘ich vaqtda sferik elastik qatlamning ichki sirtiga perpendikulyar ma’lum sirt $p_s(\tau, \theta)$ kuchi ta’sir etadi

$$\sigma_{rr}|_{r=r_1} = p_s(\tau, \theta). \tag{1}$$

Akustik muhitning izotropik elastik qatlam bilan kontakt shartlari quyidagi tengliklar orqali berilgan

$$\sigma_{rr}|_{r=r_2} = -\eta_0 \frac{\partial \varphi_0}{\partial \tau}\bigg|_{r=r_2}, \quad \frac{\partial w}{\partial \tau}\bigg|_{r=r_2} = \frac{\partial \varphi_0}{\partial r}\bigg|_{r=r_2}. \tag{2}$$

Yarim fazoning tekis sirtini yoki erkin sirt, yoki qattiq sirt sifatida olish mumkin:

$$\frac{\partial \varphi_0}{\partial \tau}\bigg|_{z=0} = 0 \ yoki \ \frac{\partial \varphi_0}{\partial r}\bigg|_{z=0} = 0. \tag{3}$$

Atrof muhit va sferik qatlam harakati masalaning o‘q simmetrikligiga ko‘ra φ_l potensiallarga nisbatan to‘lqin tenglamalari bilan tasvirlanadi ($l = 0, 1$; индекс «0» bilan atrof muhit, indeks «1» - bilan esa qatlam belgilangan)

$$\gamma_l^2 \frac{\partial^2 \varphi_l}{\partial \tau^2} = \Delta \varphi_l, \tag{4}$$

bu yerda Δ - (r, θ, ϑ) sferik koordinat sistemasida Laplas operatori.

Boshlang‘ich shartlar bir jinsli bo‘lib, ular muhit va qatlam tizimi $\tau = 0$ boshlang‘ich vaqtda tinch holatda bo‘lishini ifodalaydi

$$\varphi_l|_{\tau=0} = \left.\frac{\partial \varphi_l}{\partial \tau}_l\right|_{\tau=0} = 0. \tag{5}$$

Cheksizlikda notinchlik mavjud emas, ya’ni

$$\lim_{r\to\infty} \varphi_0 = 0. \tag{6}$$

(1)-(6) boshlang‘ich-chegaraviy masala τ vaqt bo‘yicha Laslas integral almashtirishlarini qo‘llash hamda to‘liq bo‘lmagan o‘zgaruvchilarni ajratish usullari bilan yechiladi. Tasvirlar fazosida φ_l^L potensiallar, u_1^L ko‘chish vektori va kuchlanish tenzorining $\sigma_{rr}^{(1)L}$ komponentalari, hamda berilgan $p_s(\tau, \theta)$ funksiyalarni $P_n(\cos\theta)$ Lejandr ortogonal ko‘phadlari [3] bo‘yicha yoyish orqali cheksiz qatorlar shaklida tasvirlaymiz.

Boshlang‘ich O_1 nuqtasi yarim fazoning tekis sirtiga nisbatan O nuqtaga simmetrik bo‘lgan yordamchi $(r_1, \theta_1, \vartheta_1)$ sferik koordinatalar sistemasi kiritilgan bo‘lib, bir sistemadan ikkinchisiga o‘tish Bessel funksiyalari uchun qo‘shish teoremasidan foydalaniladi.

Tasvirlar fazosida masala quyidagi matritsali tenglamalar sistemasiga keltirilgan

$$\begin{cases} \mathbf{MA}y^2 + \mathbf{F}_1\mathbf{B} - \mathbf{F}_2\mathbf{B}y^2 = \mathbf{p}y \\ \mathbf{N}_1\mathbf{A}z^2t + \mathbf{N}_2\mathbf{B}t - \mathbf{N}_3\mathbf{B}z^2t - \mathbf{N}_4\mathbf{D}zt^2 - \mathbf{G}_1\mathbf{D}xz + \mathbf{G}_2\mathbf{D}xzt^2 = 0 \\ \mathbf{L}_1\mathbf{A}z^2t + \mathbf{L}_2\mathbf{B}t - \mathbf{L}_3\mathbf{B}z^2t - \mathbf{L}_4\mathbf{D}zt^2 - \mathbf{T}_1\mathbf{D}xz + \mathbf{T}_2\mathbf{D}xzt^2 = 0 \end{cases}, \tag{7}$$

(7) sistemaning yechimi eksponentalar bo‘yicha cheksiz qatorlar ko‘rinishida tavsirlanib, ularning koeffitsiyentiga nisbatan rekurrent munosabat olingan. Bu rekurrent munosabatlar qatorlar koeffitsiyentini ratsional funksiyalar shaklida topishga imkon berishi munosabati bilan ularning originallarini topish kompleks sonlari nazariyasi [4] metodlaridan foydalanib, oson aniqlashga olib keladi hamda rekurrent munosabatlar reduksiya usulini qo‘llashni talab qilmaydi.

Natijada atrof muhitdagi gidrodinamik bosim, qatlamdagi ko‘chish va kuchlanish tenzori komponentlarining qatorlari koeffitsiyenti uchun aniq formulalar topilgan.

Sonli eksperimentlar o‘tkazilgan. Ularning natijalarining tahlili shuni ko‘rsatadiki, chegaraviy sirtdan qaytuvchi to‘lqinlar atrof muhitdagi gidrodinamik bosim xarakteriga va izotropik sferik qatlam kuchlanish-deformatsiya holatiga ta’sir qilishi kelib chiqadi. Aniq formulalar asosida olingan natijalarni sonli yechish usullari yordamida aniqlangan qiymatlarni baholashda, hamda kemasozlik, samolyotsozlik va suv osti inshoatlarini loyihalashtirish tashkilotlarida foydalanish mumkin.

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AKUSTIK FAZODA QATTIQ SHAR YAQINIDAGI IZOTROPIK SFERIK QATLAMDAN NOSTATSIONAR TO‘LQIN TARQALISHI

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Ushbu ilmiy tadqiqot akustik fazoda qattiq shar yaqinidagi izotropik sferik qatlamdan nosta-tsionar to‘lqin tarqalishi haqidagi masalasini o‘rganishga bag‘ishlangan. Ideal suyuqliklar fazosida nostatsionar to‘lqin tarqalishi va difraksiyasi jarayonlarini matematik modellashtirish, hamda ularning tadqiqoti muhim va ahamiyatli bo‘lib, ular nazariy va amaliy jihatdan to‘lqinlar dinamika-sining dolzarb muammolaridan biri hisoblanadi. Muammoning dolzarbligi ularning turli sohalarda dinamik kuchlanishlar ta’sirida ishlovchi yangi konstruksiyalarni yaratish, hamda kemasozlik, samolyotsozlik va suv osti inshoatlarni loyihalashtirishdagi amaliy masalalarini yechish bilan aniqlanadi.

Hozirgi vaqtda ideal suyuqliklar fazosida nostatsionar to‘lqin tarqalishi va difraksiyasi jarayon-larini o‘rganishga bag‘ishlangan ko‘plab ilmiy tadqiqot ishlari mavjud bo‘lib, masalan, ulardan [1-3] ilmiy maqolalarni keltirish mumkin.

Ushbu ilmiy tadqiqot ishida akustik yarim fazoda o‘zaro konsentrik sferik sirtlar bilan chegara-langan elastik qatlamdan nostatsionar bo‘ylama to‘lqinlarning tarqalishi masalasi qaralgan. Maqo-laning maqsadi muammoni yechish algoritm ishlab chiqish, hamda sferik elastik qatlam va ideal suyuqlikning fazosida nostatsionar bo‘ylama to‘lqin jarayonlarini o‘rganishdir.

Aytaylik, akustik fazoda R radiusli qattiq shar yaqinida ichki va tashqi radiuslari mos ravishda a va b $(a < b)$ bo‘lgan izotropik sferik qatlam joylashgan bo‘lsin. Ularning markazlari orasidagi masofa l ga teng $(l > R + b)$. Akustik muhit va qatlam harakati boshlang‘ich nuqtalari sferik qatlam va qattiq shar markazlarida bo‘lgan $(r_1,\ \theta_1,\ \vartheta_1)$ va $(r_2,\ \theta_2,\ \vartheta_2)$ sferik koordinatalar sistemalarida qaraladi.

Vaqtning boshlang‘ich $\tau = 0$ momentida sferik qatlamning ichki sirtiga $p_s(\tau, \theta_1)$ normal kuch ta’sir etadi. Natijada izotropik sferik qatlam va akustik muhitda nostatsionar bo‘ylama to‘lqin tarqaladi.

Izotropik sferik qatlam harakati ko‘chish potensiali φ_1 ga nisbatan, akustik muhit harakati esa tezlik potensiali φ_2 ga nisbatan to‘lqin tenglamalari bilan tasvirlanadi:

$$\gamma_\kappa^2 \frac{\partial^2 \varphi_\kappa}{\partial \tau^2} = \Delta_\kappa \varphi_\kappa, \quad \kappa = 1, 2 \tag{1}$$

$$\Delta_\kappa = \frac{1}{r_\kappa^2} \frac{\partial}{\partial r_\kappa} \left(r_\kappa^2 \frac{\partial}{\partial r_\kappa} \right) + \frac{1}{r_\kappa^2 \sin \theta_\kappa} \frac{\partial}{\partial \theta_\kappa} \left(\sin \theta_\kappa \frac{\partial}{\partial \theta_\kappa} \right).$$

Sferik qatlamning ichki sirtida chegaraviy quyidagi ko‘rinishga ega:

$$\sigma_{r_1 r_1}|_{r_1 = a} = p_s(\tau, \theta_1). \tag{2}$$

Qatlam va ideal suyuqlik o‘rtasidagi kontakt shartlar esa

$$\sigma_{r_1 r_1}|_{r_1=b} = -\eta_0 p_0|_{r_1=b}, \quad \left.\frac{\partial w}{\partial \tau}\right|_{r_1=b} = V_0|_{r_1=b} \tag{3}$$

ko‘rinishga ega bo‘ladi.

Qattiq shar sirtida chearaviy shart quyidagi ko‘rinishda bo‘ladi:

$$w|_{r_2=R} = 0 \tag{4}$$

Vaqtning $\tau = 0$ boshlang‘ich momentida qobiq va muhit tinch holatda bo‘ladi, ya’ni boshlang‘ich shartlar bir jinslidir

$$\varphi_k|_{\tau=0} = \left.\frac{\partial \varphi_k}{\partial \tau}\right|_{\tau=0} = 0. \tag{5}$$

Cheksizlikda to‘lqin so‘nadi

$$\lim_{r_2\to\infty} \varphi_2 = 0. \tag{6}$$

(1)-(6) boshlang‘ich-chegaraviy masalani yechish uchun vaqt bo‘yicha Laplas integral almashtir-ishlari [4] va o‘zgaruvchilarni to‘liq bo‘lmagan ajratish usuli foydalanilgan. Tasvirlar fazosida φ_k^L potensiallar, u_1^L ko‘chish vektori va kuchlanish tenzorining $\sigma_{rr}^{(1)L}$ komponentalari, hamda berilgan $p_s(\tau, \theta_1)$ funksiyalarni $P_n(\cos\theta)$ Lejandr ortogonal ko‘phadlari [5] bo‘yicha yoyish orqali cheksiz qatorlar shaklida tasvirlaymiz. Bir sistemadan ikkinchisiga o‘tish Bessel funksiyalari [6] uchun qo‘shish teoremasidan foydalaniladi. Laplas integral almashtirishlarining tasvirlar fazosida masala cheksiz algebraik matritsali tenglamalar sistemasini yechishga keltirilib, uning yechimi cheksiz qatorlar shaklida izlanishi rekurrent munosabatlar va ular uchun boshlang‘ich shartlarga olib keladi. Bu rekurrent munosabatlar qatorlar koeffitsiyentini ratsional funksiyalar shaklida topishga imkon berishi munosabati bilan ularning originallarini topish kompleks sonlari nazariyasi [7] metodlaridan foydalanib, oson aniqlashga olib keladi hamda rekurrent munosabatlar reduksiya usulini qo‘llashni talab qilmaydi.

Natijada atrof muhitdagi gidrodinamik bosim, qatlamdagi ko‘chish va kuchlanish tenzori komponentlarining qatorlari koeffitsiyenti uchun aniq formulalar topilgan. Bosim va kuchlanish komponentalarining vaqt bo‘yicha o‘zgarishining sonli eksperiment natijalari ko‘rsatadiki, suyuqlik bilan qalin sferik qobiq chegarasidan qaytuvchi to‘lqinlar atrof muhitning gidrodinamik va qobiq-ning kuchlanish deformatsiya holatiga ta’sir etadi. Olingan natijalarni dinamik kuchlanishlar ta’sirida ishlovchi yangi konstruksiyalarni yaratishda va suv osti inshoatlarni loyihalashtirishda foydalanish mumkin.

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Filtration of homogeneous liquid in an elastoplastic non-stable, non-homogeneous porous medium

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In deep oil and gas reservoirs an elastoplastic deformation can occur. Nonstable porous media at high effective pressure can be destroyed, as a result, some particles of the reservoir rock washed out from the reservoir. This phenomenon can lead to the considerable change of the filtration-capacity characteristics of the reservoir [1]. The difference between an elastoplastic medium and other usual media is that it does not fully restore its original properties after the unloading of the medium. In this study, the equations for fluid filtration in elastoplastic non-stable, non-homogeneous media are derived and numerically analyzed [2].

In the works [2,3] the permeability k and the porosity m under elastoplastic deformations are given by the following equations:

$$\downarrow k = k_0 \exp\left(-a_{k0}\left(p_0 - p\right)\right) + \theta\left(p_s - p\right) k_{s0}\left[1 - \exp\left(-a_{ks}\left(p_s - p\right)\right]\right.,$$

$$\downarrow m = m_0 \exp\left(-\beta_{m0}\left(p_0 - p\right)\right) + \theta\left(p_s - p\right) m_{s0}\left[1 - \exp\left(-\beta_{ms}\left(p_s - p\right)\right]\right., \tag{1}$$

where a_{k0}, β_{m0} are corresponding coefficients of parameter changes; k_{s0}, m_{s0}, a_{ks}, β_{ms} are change coefficients of k and m due to particle removal; p_0 is initial pressure; p_s is the pressure at which particle removal regime begins; в†“ is corresponds to the pressure decreasing regime; $\theta(\cdot)$ is the Heaviside's function.

In this work, the initial permeability and porosity are taken as variables:

$$k_0(x) = k_1 + (k_0 - k_1)\exp\left(-\alpha_1 x\right),$$

$$m_0(x) = m_1 + (m_0 - m_1)\exp\left(-\beta_1 x\right), \tag{2}$$

where $k_1, k_0, m_1, m_0, \alpha_1, \beta_1$ - constant coefficients.

If we substitute (2) into (1), it takes the following form:

$$\downarrow k = \left(k_1 + (k_0 - k_1)\exp\left(-\alpha_1 x\right)\right)\exp\left(-a_{k0}\left(p_0 - p\right)\right) + +\theta\left(p_s - p\right) k_{s0}\left[1 - \exp\left(-a_{ks}\left(p_s - p\right)\right]\right.$$

$$\downarrow m = \left(m_1 + (m_0 - m_1)\exp\left(-\beta_1 x\right)\right)\exp\left(-\beta_{m0}\left(p_0 - p\right)\right) + +\theta\left(p_s - p\right) m_{s0}\left[1 - \exp\left(-\beta_{ms}\left(p_s - p\right)\right]\right. . \tag{3}$$

We use the following continuity equation and Darcy's law:

$$\frac{\partial(m\rho)}{\partial t} + \operatorname{div}(\rho\vec{w}) = 0, \tag{4}$$

$$\vec{w} = -\frac{k(x)}{\mu}\nabla p, \tag{5}$$

where ρ is density, $\vec{w}$ is filtration rate, μ is viscosity, ∇ - gradient operator.

Density and viscosity of liquid are described by the exponential law:

$$\rho = \rho_0 \exp\left(-\beta_{\text{P¶}}(p - p_0)\right), \tag{6}$$

$$\mu = \mu_0 \exp\left(-a_\mu\,(p - p_0)\right), \tag{7}$$

where ρ_0 and μ_0 are density and viscosity of liquid at some standard pressure p_0, a_μ and $\beta_{\text{P¶}}$ are viscosity change factor and compressibility factor respectively.

By applying (3) and (6) to the left side of equation (4), we can express it in the following form:

$$\begin{aligned}\frac{\partial(m\rho)}{\partial t} = \frac{\partial}{\partial t}\Bigg(\Big(m_0(x)\exp\big(-\beta_{m0}(p_0 - p)\big) \\ + \theta(p_s - p)m_{s0}\big[1 - \exp\big(-\beta_{ms}(p_s - p)\big)\big]\Big)\cdot \rho_0 \exp\big(-\beta_{\text{P¶}}(p_0 - p)\big)\Bigg).\end{aligned} \tag{8}$$

We use the following notations:

$$\begin{aligned}&\beta = \beta_{m0} + \beta_{\text{P¶}}, \quad \varphi = \exp\left(-\beta\,(p_0 - p)\right), \\ &\delta_1 = \exp\left(-\beta_{ms}\,(p_s - p_0)\right), \sigma(\varphi) = p_s - p_0 - \frac{1}{\beta}\ln\varphi,\end{aligned}$$

then, the first term in the left side of equation (4) takes the form:

$$\frac{\partial(m\rho)}{\partial t} = \rho_0\frac{\partial}{\partial t}\left((m_1 + (m_0 - m_1)\exp(-\beta_1 x))\,\varphi + \theta(\sigma(\varphi))m_{s0}\left[1 - \delta_1\varphi^{\frac{\beta_{ms}}{\beta}}\right]\varphi^{\frac{\beta_{\text{P¶}}}{\beta}}\right). \tag{9}$$

We can simplify the right side of equation (4) by applying (3) and (7) as follows:

$$\begin{aligned}\operatorname{div}(\rho\vec{w}) = \frac{\partial}{\partial x}(\rho\vec{w}) = \frac{\partial}{\partial x}\Bigg(-\frac{\rho_0\exp\left(-\beta_{\text{P¶}}\ (p_0 - p)\right)}{\mu_0\exp\left(-a_\mu\,(p_0 - p)\right)} \\ \cdot\frac{(k_0(x)\exp\left(-a_{k0}\,(p_0 - p)\right) + \theta\,(p_s - p)\,k_{s0}\left[1 - \exp\left(-a_{ks}\,(p_s - p)\right)\right])}{\beta\varphi}\frac{\partial\varphi}{\partial x}\Bigg)\end{aligned} \tag{10}$$

The following notations are used:

$$\begin{aligned}&w = -\frac{k}{\mu}\frac{\partial p}{\partial x}, \quad \frac{\partial p}{\partial x} = \frac{1}{\beta\varphi}\frac{\partial\varphi}{\partial x}, \quad \delta_2 = \exp\left(a_{ks}\,(p_0 - p_s)\right), \\ &\alpha = \beta_{\text{P¶}} - a_\mu + a_{k0}, \quad \beta = \beta_{\text{P¶}} + \beta_{m0}, \gamma = \frac{\alpha}{\beta}\end{aligned}$$

then we have:

$$\begin{aligned}\operatorname{div}(\vec{w}) = -\frac{\rho_0}{\mu_0\beta}\frac{\partial}{\partial x}\Bigg((k_1 + (k_0 - k_1)\exp(-\alpha_1 x))\,\varphi^{\gamma-1} \\ + \theta(\sigma(\varphi))k_{s0}\left[1 - \delta_2\varphi^{\frac{a_{ks}}{\beta}}\right]\varphi^{\frac{\beta_{m0}-a_\mu}{\beta}}\Bigg)\frac{\partial\varphi}{\partial x}.\end{aligned} \tag{11}$$

Substituting (9) and (11) into equation (4), we write filtration equation for the pressure decreasing regime:

$$\downarrow \frac{\partial}{\partial t}\left(\left(m_0(x)\varphi+\theta(\sigma(\varphi))m_{s0}\left[1-\delta_1\varphi^{\frac{\beta_{ms}}{\beta}}\right]\varphi^{\frac{\beta_{\text{P}\P}}{\beta}}\right)=\right.$$
$$\left.=\frac{1}{\mu_0\beta}\frac{\partial}{\partial x}\left(\left(k_0(x)\varphi^{\gamma-1}+\theta(\sigma(\varphi))k_{s0}\left[1-\delta_2\varphi^{\frac{a_{ks}}{\beta}}\right]\varphi^{\frac{\beta_{m0}-a_\mu}{\beta}}\right)\frac{\partial\varphi}{\partial x}\right),\right. \tag{12}$$

Now, we derive the filtration equation for the pressure recovery process. The permeability and porosity for the pressure recovery process are determined using the following relationships:

$$\begin{aligned}\uparrow k =&k_0(x)\exp\left(-\left(a_{k0}-a_{k1}\right)\left(p_0-p_1\right)\right)\exp\left(-a_{k1}\left(p_0-p\right)\right)+\\&+\theta\left(p_s-p_1\right)k_{s0}\left[1-\exp\left(-a_{ks}\left(p_s-p_1\right)\right)\right]\\ \uparrow m =&m_0(x)\exp\left(-\left(\beta_{m0}-\beta_{m1}\right)\left(p_0-p_1\right)\right)\exp\left(-\beta_{m1}\left(p_0-p\right)\right)+\\&+\theta\left(p_s-p_1\right)m_{s0}\left[1-\exp\left(-\beta_{ms}\left(p_s-p_1\right)\right)\right].\end{aligned} \tag{13}$$

where в†‘ corresponds to the pressure recovery regime, p_1 is initial pressure at the start of the pressure recovery regime, a_{k1} and β_{m1} are change coefficients for the pressure recovery regime.

The following notations are used:

$$\begin{aligned}&\psi(x)=\exp\left(-\beta\left(p_0-p_1\right)\right),\quad p_0-p_1=-\frac{1}{\beta}\ln\psi(x)\\&\varphi_1(x)=a_{k0}\left[1-\psi(x)^{\frac{\eta_k}{\beta}}\right]\left[-\frac{\ln|\psi(x)|}{\beta}\right],\quad \psi_1(x)=\beta_{\text{P}\P}-a_\mu+a_{k0}\psi^{\frac{\eta_k}{\beta}},\\&\varphi_2(x)=\beta_{m0}\left[1-\psi(x)^{\frac{\eta_m}{\beta}}\right]\left[-\frac{\ln|\psi(x)|}{\beta}\right],\quad \psi_2(x)=\beta_{\text{P}\P}+\beta_{m0}\psi(x)^{\frac{\eta_m}{\beta}},\\&\beta_{m1}=\beta_{m0}\exp\left(-\eta_m\left(p_0-p_1\right)\right),\quad a_{k1}=a_{k0}\exp\left(-\eta_k\left(p_0-p_1\right)\right)\\&\xi_1(x)=\theta\left(p_s-p_1\right)k_{s0}\left[1-\delta_2\psi(x)^{\frac{a_{ks}}{\beta}}\right],\quad \xi_2(x)=\theta\left(p_s-p_1\right)m_{s0}\left[1-\delta_1\psi(x)^{\frac{\beta_{ms}}{\beta}}\right].\end{aligned} \tag{14}$$

We simplify the left side of the (4) equation for the pressure recovery process as follows:

$$\begin{aligned}\frac{\partial(m\rho)}{\partial t}=&\rho_0\frac{\partial}{\partial t}\left(m_0(x)\exp\left(-\psi_2(x)\left(p_0-p\right)\right)\exp\left(-\varphi_2(x)\right)+\right.\\&\left.+\xi_2(x)\exp\left(-\beta_{\text{P}\P}\left(p_0-p\right)\right)\right.\end{aligned} \tag{15}$$

The right side of equation (4) is written as follows:

$$\begin{aligned}\operatorname{div}(\rho\vec{w})=&\frac{\partial}{\partial x}(\rho\vec{w})=\frac{\partial}{\partial x}\left(-\frac{\rho_0\exp\left(-\beta_{\text{P}\P}\left(p_0-p\right)\right)}{\mu_0\exp\left(-a_\mu\left(p_0-p\right)\right)}\right.\\&\cdot\left(k_0(x)\exp\left(-\left(a_{k0}-a_{k1}\right)\left(p_0-p_1\right)\right)\exp\left(-a_{k1}\left(p_0-p\right)\right)+\right.\\&\left.\left.+\theta\left(p_s-p_1\right)k_{s0}\left[1-\exp\left(-a_{ks}\left(p_s-p_1\right)\right)\right]\right)\frac{\partial p}{\partial x}\end{aligned} \tag{16}$$

Using the notations (14), we can write (16) as follows:

$$\begin{aligned}\operatorname{div}(\rho\vec{w})=&-\frac{\rho_0}{\mu_0}\frac{\partial}{\partial x}\Big(\big(k_0(x)\exp(-\varphi_1(x))\exp(-\psi_1(x)(p_0-p))\\&+\xi_1(x)\exp\big(-(\beta_{\text{P}\P}-a_\mu)(p_0-p)\big)\big)\frac{\partial p}{\partial x}\Big).\end{aligned} \tag{17}$$

Now, by substituting (15) and (17) into equation (4), we obtain the following filtration equation:

$$\uparrow \frac{\partial}{\partial t}\left(m_0(x)\exp\left(-\psi_2(x)\left(p_0-p\right)\right)\exp\left(-\varphi_2(x)\right)+\right.$$
$$+\xi_2(x)\exp\left(-\beta_{\mathrm{P\P}}\left(p_0-p\right)\right)=\frac{1}{\mu_0}\frac{\partial}{\partial x}\left(\left(k_0(x)\exp\left(-\varphi_1(x)\right)\right)\right. \tag{18}$$
$$\left.\left.\cdot\exp\left(-\psi_1(x)\left(p_0-p\right)\right)+\xi_1(x)\exp\left(-\left(\beta_{\mathrm{P\P}}-a_\mu\right)\left(p_0-p\right)\right)\right)\frac{\partial p}{\partial x}\right).$$

Equations (12) and (18) are the filtration equations that describe the processes of pressure decrease and pressure recovery, respectively, in elastoplastic non-stable porous media. Equations (12) and (18) are numerically solved and the influence of different model parameters on pressure distribution fields is estimated.

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A Numerical Method for Solving a Boundary Value Problem Involving the Regularized Prabhakar Fractional Derivative

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In this paper we solving a boundary value problem involving the Regularized Prabhakar fractional derivative using numerical method. The regularized Prabhakar fractional derivative of order β ${}^{PC}D_{at}^{\alpha,\beta,\gamma,\delta}$ is defined as follows [1]:

$${}^{PC}D_{at}^{\alpha,\beta,\gamma,\delta}f(t)={}^{P}I_{at}^{\alpha,m-\beta,-\gamma,\delta}\frac{d^m}{dt^m}f(t), m-1<\beta<m,$$

where $\alpha>0,\ \gamma,\delta\in\mathbb{R}$. Let us consider the following equation:

$${}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta}u(x,t)-u_{xx}(x,t)=f(x,t), \tag{1}$$

where $(x,t)\in((0,1)\times(0,T))$, denote the regularized Prabhakar fractional derivatives of orders $\beta\in(0,1)$, as introduced in, with parameters $\alpha>0,\ \gamma,\delta\in\mathbb{R}$, $f(x,t)$ are given source functions.

Problem. Find a function $u(x,t)$ in the domain $(0,1)\times(0,T)$, that satisfies equation (1) and the following initial and boundary conditions:

$$u(0,t)=\mu_1(t),\quad u(1,t)=\mu_2(t),\quad t\in[0,T], \tag{2}$$

$$u(x,0)=\varphi(x), u_t(x,0)=\psi(x), \qquad x\in[0,1]. \tag{3}$$

Here $\varphi(x)$ and $\psi(t)$ are given sufficiently smooth functions.

Problem (1)-(3) is solved using the finite difference method [2].

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Efficient denosing algorithm for meshs

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In the immersive video system supporting 6 Degrees of Freedom (DoF), the mesh obtained from optically scanned point cloud data is often disturbed by extensive amount of noise which should be removed to enhance the quality of the generated data. We analyze recent anisotropic and isotropic methods removing the noise in mesh data for point cloud. And then, we propose the efficient two-stage algorithm to enhance the quality of the mesh for Windowed 6DoF data. In the first stage, it applies bilateral normal filtering to smooth triangles, in which Cosine function is used to construct weighting factors. In the second stage, a modified Laplacian operator is used with Cauchy edge weighting factors. The proposed method takes advantages of both isotropic and anisotropic methods and it provides the robustness preserving features of the original data. The experiments show that the proposed algorithm outperforms conventional algorithms for both simple and complex meshes.

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KASR TARTIBLI DIFFERENSIAL TENGLAMALARNI CHEKLI ELEMENTLAR USULI ASOSIDA SONLI YECHISH

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So'nggi yillarda kasr tartibli hisoblash sohasi bo'yicha ilmiy qiziqish sezilarli darajada ortib bormoqda. Kasr tartibli differensial tenglamalar ham tadqiqotchilar e'tiborini tobora ko'proq jalb

etmoqda. Ushbu ishning dolzarbligi shundan iboratki, kasr tartibli differensial tenglamalar fan va texnikaning turli sohalarida keng qo'llanilmoqda. Suyuqliklar mexanikasi, qovushqoq elastiklik, kimyo, fizika, moliya va boshqa fanlarda kuzatiladigan ko'plab jarayonlar kasr tartibli hisoblash nazariyasining matematik apparati yordamida qurilgan modellar orqali tavsiflanishi mumkin. Bunday masalalar, odatda, aniq yechimga ega bo'lmaydi. Shu sababli, ushbu tenglamalar uchun taqribiy yechim usullarini ishlab chiqish va ularni qo'llash, shuningdek, bu usullarning nazariy asoslarini yaratish masalalari dolzarb hisoblanadi. Ta'kidlash joizki, so'nggi yillarda ilmiy adabiyotlarda ayrim sinfdagi tenglamalar uchun sonli usullar taklif etilgan tadqiqotlar paydo bo'lmoqda. Biroq, bu yo'nalishda erishilgan natijalarga qaramay, shunga o'xshash masalalarning yanada umumiy sinflari uchun taqribiy usullarni qo'llashning nazariy asoslarini ishlab chiqish masalasi hanuz ochiq qolmoqda. Ushbu ishning maqsadi kasr tartibli differensial tenglamani sonli yechish uchun hisoblash sxemasini ishlab chiqishdan iborat. Ishda kasr tartibli differensial tenglamalarning taqribiy yechimini topish uchun umumlashtirilgan BubnovвЂ“Galerkin va Rits usullari taklif etiladi. Quyidagi tenglamani ko'rib chiqamiz:

$$D^{(\alpha)}u + Tu = f, \quad u,\ f \in L_2([0, 2\pi]), \tag{1}$$

bu yerda $D^{(\alpha)} = (2\cos(\pi\alpha/2))^{-1}(D_+^{(\alpha)} + D_-^{(\alpha)})$ operatori $D_\pm^{(\alpha)}$ kasr tartibli differensiallash operatorlari orqali aniqlanadi. $[a, b]$ kesmada berilgan $\varphi(x)$ funksiyalar uchun kasr tartibli hosilalar quyidagi formulalar orqali aniqlanadi:

$$\left(D_{a+}^{(\alpha)}\varphi\right)(x) := \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int\limits_a^x \frac{\varphi(t)dt}{(x-t)^\alpha}, \quad \left(D_{b-}^{(\alpha)}\varphi\right)(x) := \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int\limits_x^b \frac{\varphi(t)dt}{(t-x)^\alpha}, \tag{2}$$

bu yerda $-\infty < x < \infty,\ 0 < \alpha < 1$. (2) formulada aniqlangan hosilalar mos ravishda α-tartibli chap tomonli va o'ng tomonli Riman–Liuvill hosilalari deb ham yuritiladi. Bu yerda u - noma'lum f funksiya $L_2[0, 2\pi]$ fazosiga tegishli berilgan funksiya. T- shunday operator bo'lib, $(D^{(\alpha)} + T)$ chiziqli operatorni hosil qiladi va umumiy holda cheklanmagan hamda musbat aniqlangan bo'lmagan operator hisoblanadi. Yetarlicha silliq funksiyalar uchun $D^{(\alpha)}$ operatori kasr tartibli differensiallashning Veyl operatori bilan mos keladi va quyidagi qoida bo'yicha ta'sir etadi:

$$D^{(\alpha)}u \sim \sum_{k=-\infty}^{\infty} |k|^\alpha u_k e^{ikx}. \tag{3}$$

Bu yerda u_k - u funksiyaning Furye koeffitsiyentlari hisoblanadi.

Quyidagi masalani ko'rib chiqamiz:

$$D^{(\alpha)}u + qu = f(x), \quad x \in (0, 1),\ u(0) = u(1) = 0. \tag{4}$$

(4) masalaning yaqinlashgan yechimini Rits usuli yordamida izlaymiz. Bu usulga ko'ra, bazis funksiyalar sifatida aniqlanish sohasi $D(A)$ bo'lgan $A = D^{(\alpha)} + q$ operatorining xos funksiyalarini tanlaymiz. Bu yerda $D(A)$ quyidagi shartlarni qanoatlantiruvchi funksiyalar to'plamidan iborat:

1. $u(x)$ funksiyalar $[0, 1]$ oraliqda uzluksiz;
2. ikkinchi tartibli hosilagacha ega;
3. chegaraviy shartlarni qanoatlantiradi: $u(0) = 0,\ u(1) = 0$.

Ta'kidlash joizki, A operatorining xos qiymati λ_i va unga mos xos funksiya φ quyidagi ko'rinishga ega: $\lambda_i = i^2\pi^2 + q,\ \varphi_i = \sin i\pi x,\ i = 1, 2, ...$ Shunda (4) masalaning yaqinlashgan yechimini quyidagi ko'rinishda izlaymiz:

$$u_N(x) = \sum_{i=1}^{N} \alpha_i \sin i\pi x, \tag{5}$$

bu yerda (5) yoyilmaning noma'lum koeffitsiyentlari quyidagi ko'rinishda aniqlanadi:

$$\alpha_i = \frac{2}{i^2\pi^2 + q} \int_0^1 f(x) \sin i\pi x dx.$$

Taklif etilgan hisoblash sxemasining sonli realizatsiyasini (1) masala uchun ayrim misollarda ko'rib chiqamiz. (1) masala uchun $\alpha = 2.5,\ q = 1$ qiymatlarni olamiz. U holda tenglama quyidagi ko'rinishni oladi:

$$D^{(2.5)}u + u = f(x), \quad x \in (0, 1),\ u(0) = u(1) = 0. \tag{6}$$

bu yerda (6) tenglamaning o'ng tomoni $f(x) = (1 + b^{2.5}) \sin bx - (1 + (\pi - b)^{2.5}) \sin(\pi - b)x$ funksiya ko'rinishida aniq berilgan. Aniq yechim $u(x) = \sin bx - \sin(\pi - b)x$ ko'rinishga ega ekanligini oson ko'rsatish mumkin. Haqiqatan ham, aniq yechimni (6) tenglamaga qo'llab va $D^{(\alpha)}(\sin bx) = b^\alpha \sin bx$ tenglikdan foydalansak, quyidagi natijani olamiz:

$$b^{2.5} \sin bx - (\pi - b)^{2.5} \sin(\pi - b)x + \sin bx - \sin(\pi - b)x =$$

$$= (1 + b^{2.5}) \sin bx - (1 + (\pi - b)^{2.5}) \sin(\pi - b)x.$$

Usulni tasvirlash uchun (6) tenglamaning yaqinlashgan yechimini (5) trigonometrik ko'phad ko'rinishida izlaymiz. Bu yerda noma'lum koeffitsiyentlar quyidagi aniq integral yordamida aniqlanadi:

$$\alpha_i = \frac{2}{i^2\pi^2 + 1} \int_0^1 \left((1 + b^{2.5}) \sin bx - (1 + (\pi - b)^{2.5} \sin(\pi - b)x)\right) \sin i\pi x dx.$$

Formulaga $b = 1$ ni qo'yib, $N = 10; 15; 20$ uchun yaqinlashgan yechimlarni quyidagi jadvalda ifodalaymiz.

x	Aniq yechim	$N = 10$	$N = 15$	$N = 20$
0.0	0.0000	0.0000	0.0000	0.0000
0.1	-0.112693	-0.1099	-0.10981	-0.10979
0.2	-0.216672	-0.20113	-0.20122	-0.2012
0.3	-0.303631	-0.26901	-0.26896	-0.26895
0.4	-0.366226	-0.31041	-0.31046	-0.31046
0.5	-0.398157	-0.32448	-0.32442	-0.32443
0.6	-0.394782	-0.31041	-0.31046	-0.31046
0.7	-0.353214	-0.26901	-0.26896	-0.26895
0.8	-0.272511	-0.20113	-0.20122	-0.2012
0.9	-0.153749	-0.1099	-0.10981	-0.10979
1.0	0.0000	0.0000	0.0000	0.0000

$N = 10$ va $N = 15$ uchun olingan yaqinlashgan funksiyalar qiymatlarini taqqoslash natijasida $\varepsilon = 10^{-3}$ aniqlik bahosi olindi. Biroq, N ni yanada oshirish, ya'ni misol sifatida $N = 20$ da yaqinlashish aniqligining juda sekin ortishiga olib keladi. Kasr tartibli differensial tenglamalarni sonli yechish uchun Bubnov–Galerkin va Rits usullariga asoslangan hisoblash sxemasi ishlab chiqildi. Taklif etilgan usul test masala misolida qo'llanilib, olingan sonli natijalar aniq yechim bilan taqqoslandi. Taqqoslash natijalari yaqinlashgan yechimning yuqori aniqligini hamda tanlangan yondashuvning kasr tartibli differensial tenglamalarni yechishda samarali va ishonchli ekanligini tasdiqladi.

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UDK 539.3

IZOTROPIK YARIM FAZOSIDA YASSI TO'LQIN DIFRAKSIYASI TA'SIRIDA QATTIQ SHARNING NOSTATSIONAR ILGARILANMA HARAKATI

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Hozirgi paytda elektron hisoblash mashinalarning juda tez rivojlanishi, operativ xotiraning kengligi, ishlashning ishonchligi, hisoblash tezligining yuqoriligi va ixchamligi fan - texnikaning ko'p sohalarining murakkab muammolari yechimini olishga imkon bermoqda.

Jumladan, matematik modellashtirishni murakkab hodisa va jarayonlarni o'rganishda ilmiy amaliy masalalarni yechishga tadbiq qilinishiga asosiy sabab bo'lib xizmat qilmoqda.

Ishda izotropik yarim fazosida yassi to'lqin difraksiyasi ta'sirida qattiq sharning nostatsionar ilgarilanma harakati jarayoni matematik ifodalanib, bu jarayonga mos boshlang'ich-chegaraviy masalaning yechish algoritmi ishlab chiqilgan.

Aytaylik, izotropik ($z \geq 0$) yarim fazosida R radiusli qattiq shar joylashgan bo'lsin. Yarim fazoning tekis ($z = 0$) sirtidan shar markazigacha bo'lgan masofa h ga teng ($h > R$). $Oxyz$ to'g'ri burchakli Dekart koordinatlar sistemasining Ox va Oy o'qlari yarim fazo tekis sirt bo'ylab, Oz o'qi esa yarim fazoning ichiga yo'nalgan bo'lib, shar markazidan o'tadi. Izotropik muhit va shar harakati O boshlang'ich no'qtasi shar markazida bo'lgan (r,θ,ϑ) sferik koordinatalar sistemasida o'rganiladi.

$\tau = 0$ vaqtning boshlang'ich momentida qo'zgaluvchan shar sirtiga Oz o'qi yo'nalishga teskari yo'nalishda potensiali bo'lgan kuchlanishning yassi to'lqini fronti kelib urinadi. Natijada, qattiq shar yarim fazoning tekis sirti tomon ilgarilanma harakat qiladi.

Notinch izoropik muhitning harakati ko'chish vektorining φ skalyar potensiali va ψ vektor potensialning noldan farkli komponentasiga nisbatan to'lqin tenglamalari bilan tasvirlanadi.

Yarim fazoning tekis sirtida kuchlanish nolga yoki ko'chish nolga teng. Shar harakati

$M\ddot{z} = R_e + R_z$ tenglama bilan ifodalanadi. M- shar massasi; R_z – muhitning reaksiya kuchi. Shar sirtida chegaraviy shartlar

$$(u + u^*)|_{r=1} = z(\tau)\cos\theta,$$
$$(\sigma_{r\theta} + \sigma^*_{r\theta})|_{r=1} = k\left(z(\tau)\sin\theta - (v + v^*)|_{r=1}\right),$$
$$u^* = u_0 + u_s,$$
$$\sigma^*_{r\theta} = \sigma_{r\theta 0} + \sigma_{r\theta s}$$

ko‘rinishga ega. Bu yerda k shar sirtida chegaraviy shartning ikki turini qarash imkonini beradi: $k = 0$ da chegaraviy sirtlar erkin sirpanishga ega. $k = \infty$ bo‘lganda esa ular qattiq mahkamlangan. $z(\tau)$- shar markazi ko‘chishi; φ_0 - yarim fazo tekis sirtidan kaytuvchi to‘lqin potensiali. Boshlang‘ich shartlar – bir jinsli va cheksizlikda to‘lqinning so‘nishini e’tiborga olamiz.

Bunga mos boshlangich – chegaraviy masala vaqt bo‘yicha Laplas integral almashtirishlari qo‘llash va to‘liq bo‘lmagan o‘zgaruvchilarni ajratish usuli yordamida yechiladi. Ko‘chish vektori potensiallarini, ko‘chish vektori va kuchlanish tenzori komponentalarini Lejandr va Gegenbauer [1] ortogonal ko‘pxadlarining cheksiz qatorlari ko‘rinishida tasvirlaymiz.

$K_{n+1/2}(x)$ funksiya uchun [2] qo‘shish teoremasiga asoslanib, qatorlarni (r,θ,ϑ) sferik koordina-talar sistemasida yozib olib, muhitning kuchlanish-deformatsiya holatining komponentalarini Lejandr va Gegenbauer ko‘phadlari bo‘yicha cheksiz qatorlarga yoyib, hamda ko‘chish vektori komponentalarini potensial funksiyalar bilan bog‘lanishi va Guk qonunidan foydalanib, ularning koeffitsiyentlari uchun ifodalar olamiz.

Boshlag‘nich-chegaraviy masala Laplas integral almashtirshilarining tasvirlar fazosida cheksiz algebraik tenglamalar sistemasiga keltiriladi va sistema yechimi eksponentalarning cheksiz qatori ko‘rinishida izlanadi. Bu cheksiz qator koeffitsintlari uchun rekurrent munosabatlar kelib chiqadi. Laplas almashtirishlarining tasvirlar fazosida sharning ko‘chishi uchun formula olingan. Funksiyalar- ning tasvirlaridan originaliga o‘tish chegirmalar nazariyasi [3] yordamida amalga oshiriladi. Shar ko‘chishi uchun sonli natijalar olindi. Sonli natijalar tahlilidan shar harakatiga yarim fazoning tekis sirtida qaytuvchi to‘lqinning ta’siri aniqlandi.

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G‘OVAK-IZOTROPIK YARIMFAZODA QATTIQ SHARNING BERILGAN QONUN BO‘YICHA NOSTATSIONAR ILGARILANMA HARAKATI

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Mazkur tadqiqot ishida g‘ovak-izotropik muhitning cheksiz fazosida absolyut qattiq sharning berilgan qonun bo‘yicha nostatsionar ilgarilanma harakati masalasining yechish algoritmini ishlab chiqilgan. Shar ilgarilanma harakatiga atrof muhitning qarshilik kuchi uchun formula olingan.

Aytaylik, ideal suyuqlik to‘yingan cheksiz g‘ovak-izotropik muhitli fazoda uning ($z = 0$) tekis sirtidan h masofada o‘lchovsiz $R = 1$ ($h > 1$) radiusli absolyut qattiq sharning markazi joylashgan bo‘lsin va boshlang‘ich $\tau = 0$ vaqtda sharga markazidan o‘tuvchi O_2z o‘q bo‘ylab yarim fazo tekis sirti tomon sharni nostatsionar ilgarilanma harakatining $z(\tau)$ qonuni berilgan bo‘lsin. G‘ovak-izotropik muhitning harakati boshlang‘ich nuqtasi shar markazida bo‘lgan (r,θ,ϑ) sferik koordinatalar sistemasida hamda boshlang‘ich nuqtasi yarim fazo tekis sirtida joylashgan va O_2z o‘qi yarim fazo ichi tomon yo‘nalgan O_2xyz dekart koordinatalar sistemasida o‘rganiladi.

G‘ovak-izotropik muhitning harakati ko‘chish vektorining φ_1, φ_2 va φ_3 potensiallariga nisbatan

$$\gamma_1^2\frac{\partial^2\varphi_1}{\partial\tau^2} = \Delta\varphi_1, \gamma_2^2\frac{\partial\varphi_2}{\partial\tau} = \Delta\varphi_2, \gamma_3^2\frac{\partial^2\varphi_3}{\partial\tau^2} = \Delta\varphi_3 - \frac{\varphi_3}{r^2\sin^2\theta} \tag{1}$$

to‘lqin tenglamalari bilan tasvirlanadi.

Yarim fazoning $z = 0$ chegarasi - erkin sirt:

$$\sigma_{zz}|_{z=0} = 0,\ \sigma_{zx}|_{z=0} = 0,\ \sigma_m|_{z=0} = 0 \tag{2}$$

Shar sirtida chegaraviy shart

$$u_1|_{r=1} = z(\tau)\cos\theta,\ \sigma_{r\theta}|_{r=1} = 0,\ u_2|_{r=1} = z(\tau)\cos\theta \tag{3}$$

ko‘rinishga ega, ya’ni shar bilan muhit chegaraviy sirtlari erkin sirpanishiga ega.

Boshlang‘ich shartlar bir jinsli:

$$\varphi_1|_{\tau=0} = \dot\varphi_1|_{\tau=0} = \varphi_2|_{\tau=0} = \dot\varphi_2|_{\tau=0} = \varphi_3|_{\tau=0} = \dot\varphi_3|_{\tau=0} = 0. \tag{4}$$

Cheksizlikda to‘lqin so‘nadi:

$$\lim_{r\to\infty}\varphi_1 = 0,\ \lim_{r\to\infty}\varphi_2 = 0,\ \lim_{r\to\infty}\varphi_3 = 0. \tag{5}$$

(5) to‘lqinning so‘nishi shartlarini e’tiborga olib, Laplas integral almashtir-ishlarining tasvirlar fazosida (1) to‘lqin tenglamalarining yechimlarini

$$\varphi_1^L = \sum_{n=0}^{\infty}\frac{1}{\sqrt{r}}A_{1n}^L(s)K_{n+1/2}\left(r\gamma_1 s\right)P_n(\cos\theta) + \sum_{p=0}^{\infty}\frac{1}{\sqrt{r_1}}B_p^L(s)K_{p+1/2}\left(r_1\gamma_1 s\right)P_p(\cos\theta_1)$$

$$\varphi_2^L = \sum_{n=0}^{\infty}\frac{1}{\sqrt{r}}A_{2n}^L(s)K_{n+1/2}\left(r\gamma_2 s\right)P_n(\cos\theta) + \sum_{p=0}^{\infty}\frac{1}{\sqrt{r_1}}C_p^L(s)K_{p+1/2}\left(r_1\gamma_2 s\right)P_p(\cos\theta_1) \tag{6}$$

$$\varphi_3^L = -\sin\theta\sum_{n=1}^{\infty}\frac{1}{\sqrt{r}}A_{3n}^L(s)K_{n+1/2}\left(r\gamma_3 s\right)C_{n-1}^{3/2}(\cos\theta) - \sin\theta_1\sum_{p=1}^{\infty}\frac{1}{\sqrt{r_1}}D_p^L(s)K_{p+1/2}\left(r_1\gamma_3 s\right)C_{n-1}^{3/2}(\cos\theta_1)$$

ko‘rinishda tasvirlaymiz. Bu yerda $K_{n+1/2}(x)$ - 2- tur Besselning modifitsirlangan funk-siyasi; $P_n(x)$, $C_{n-1}^{3/2}(x)$ - Lejandr va Gegenbauer ko‘phadlari [1]; $A_{in}^L(s)$, $i = 1 \div 3$, $B_n^L(s)$, $C_n^L(s)$ va $D_n^L(s)$ - lar s parametrning noma’lum funksiyalari; r_1 va θ_1 - lar boshlang‘ich no‘qtasi $z = 0$ tekislikka nisbatan shar markaziga simmetrik nuqta bo‘lgan sferik sistemaning koordinatalari.

Lejandr va Gegenbauer ko‘phadlarining xossalaridan foydalanib hamda

$$r|_{z=0} = r_1|_{z=0}\,, \theta|_{z=0} + \theta_1|_{z=0} = \pi$$

munosabatlarni e'tiborga olib, (2) yoki (3) chegaraviy shartlarni qanoatlantirishdan

$$B_n^L(s) = (-1)^{n+1} A_{1n}^L(s), C_n^L(s) = (-1)^{n+1} A_{2n}^L(s), D_n^L(s) = (-1)^{n+1} A_{3n}^L(s) \tag{7}$$

tengliklarni olamiz. Bu yerda yuqori ishora qattiq sirtga, pastgi ishora esa erkin sirtga mos keladi.

$K_{n+1/2}\,(x)$ funksiya uchun [2] qo'shish teoremasiga asoslanib, (6) qatorlarni $(r,\ \theta,\ \vartheta)$ sferik koordinatalar sistemasida yozib olib, muhitning kuchlanish-deformatsiya holatining komponentalarini Lejandr va Gegenbauer ko'phadlari bo'yicha cheksiz qatorlarga yoyib, hamda ko'chish vektori komponentalarini potensial funksiyalar bilan bog'lanishi va Guk qonunidan foydalanib, ularning koeffitsiyentlari uchun ifodalar olamiz.

Qatorlar koefitsiyentlariga nisbatan (3) shar siridagi chegaraviy shart

$$u_{11}^L\big|_{r=1} = z^L(s),\ \sigma_{r\theta 1}^L\big|_{r=1} = 0,\ U_{21}^L\big|_{r=1} = z^L(s) \tag{8}$$

$$u_{1n}^L\big|_{r=1} = 0,\ \sigma_{r\theta n}^L\big|_{r=1} = 0, (n \neq 1),\ U_{2n}^L\big|_{r=1} = 0 \tag{9}$$

ko'rinishga keladi

$u_{1n}^L\,, v_{1n}^L, U_{2n}^L, \sigma_{r\theta n}^L$ lar uchun ifodalarni (8) va (9) chegaraviy shartlarga qo'yib, ba'zi elementar almashtirishlardan so'ng, noma'lum funksiyalarga nisbatan cheksiz chiziqli algebraik tenglamalar sistemasiga kelamiz. Sistemaning yechimini eksponentalar bo'yicha cheksiz qatorlar ko'rinishda yozib olamiz va cheksiz qatorlarni sistemaga qo'yib, uningkoeffitsiyentlari uchun boshlang'ich shartlar va rekurrent munosabatlarni olamiz. Boshlang'ich shart va rekurrent munosabatlardan funksiyalarni s parametrning ratsional funksiyalari kabi aniqlanadi. Bu esa ularning originallarini qoldiqlar nazariyasi yordamida topish mumkinligini beradi.

Yarim fazoning ixtiyoriy no'qtasidagi ko'chish vektori va kuchlanish tenzori komponentalar

$$\begin{aligned}
u_1^L(r,s) &= u_{11}^L(r,s)\cos\theta,\\
v_1^L(r,s) &= v_{11}^L(r,s)\cos\theta,\\
u_2^L(r,s) &= u_{21}^L(r,s)\cos\theta\\
\sigma_{rr}^L(r,s) &= \sigma_{rr1}^L(r,s)\cos\theta,\\
\sigma_{r\theta}^L(r,s) &= \sigma_{r\theta 1}^L(r,s)\cos\theta,\\
\sigma_m^L(r,s) &= \sigma_{m1}^L(r,s)\cos\theta
\end{aligned}$$

ifodalar yordamida topiladi.

Muhitda shar harakatining muhim xarakteristikalaridan biri shar ilgarilanma harakatiga atrof muhitning qarshilik kuchidir. Uni

$$R_z^L(s) = \frac{4\pi}{3s} \sum_{ijnklm=0}^{\infty} \left[\sum_{q=1}^{3} \left(Q_{1q1}(s) + Q_{3q1}(s) - 2Q_{2q1}(s)\right) a_{ijnklm}^{(q,1)}(s) + \right.$$

$$\left. + \sum_{q=1}^{2} \left(J_{1q1}(s) + J_{3q1}(s) - 2J_{2q1}(s)\right) C_1^{(q)}(s) e^{-2h\gamma_q s} + \left(J_{131}(s) + J_{331}(s) - 2J_{231}(s)\right) S_1(s) e^{-2h\gamma_3 s} \right]$$

ifodani olamiz.

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Piecewise constant argument method using Adams-Bashforth technique for the solution of higher-order nonlinear systems of ODEs

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In this work, we show a construction of an approximate solution for the mth-order nonlinear system of differential equation of the form

$$\begin{cases} y_1^{(m)}(t) = f_1(t, y_1(t), y_1'(t), ..., y_1^{(m-1)}(t), ..., y_q(t), y_q'(t), ..., y_q^{(m-1)}(t)), \\ y_2^{(m)}(t) = f_2(t, y_1(t), y_1'(t), ..., y_1^{(m-1)}(t), ..., y_q(t), y_q'(t), ..., y_q^{(m-1)}(t)), \\ \quad . \quad . \quad . \\ y_q^{(m)}(t) = f_q(t, y_1(t), y_1'(t), ..., y_1^{(m-1)}(t), ..., y_q(t), y_q'(t), ..., y_q^{(m-1)}(t)), \quad t \in [0,1], \end{cases} \tag{1}$$

with initial conditions

$$y_p(0) = y_{p0}, \quad y_p'(0) = y_{p1}, \quad \ldots, \quad y_p^{(m-1)}(0) = y_{p,m-1}, \quad p = 1, ..., q, \tag{2}$$

where f_p are real valued differentiable, continuous functions on $[0,1] \times R^{mq}$ and y_{pi}, $p = 1, ..., q$, $i = 0, 1, ..., m-1$ are real constants.

Let $\mathbf{Y}(t) = (Y_1(t), Y_2(t), ..., Y_q(t))$ be the solution of the initial value problem (1)-(2). Then there exists $A > 0$ such that $|Y_p(t)| \leq A$ for all $t \in [0,1]$ and $p = 1, 2, ..., q$.

We suppose that the function $f_p(t, x_1, ..., x_{mq})$ has the compact support $D = [0,1] \times [-A, A]^{mq}$, i.e., we assume that

$$f_p(t, x_1, ..., x_{mq}) = 0 \quad \text{for} \quad (t, x_1, ..., x_{mq}) \notin D. \tag{3}$$

Moreover, under this assumption on the functions $f_1, ..., f_q$, the solution $\mathbf{y}(t) = (y_1(t), y_2(t), ..., y_q(t))$ of the initial value problem (1)-(2) is preserved.

We remark that the assumption (3) ensures the boundedness of the functions $f_p(t, x_1, ..., x_{mq})$ in D. Thus, under these conditions, we can construct a numerical approximation algorithm for the solution.

Since f is a differentiable function it satisfies the Lipschitz condition

$$|f_p(t, x_1, ..., x_{mq}) - f_p(t, y_1, ..., y_{mq})| \leq |L_p| \sum_{i=1}^{mq} |x_i - y_i|, \quad p = 1, 2, ..., q,$$

for all $(t, x_1, ..., x_{mq}), (t, y_1, ..., y_{mq}) \in D$, where

$$L_p = \max_{(t, x_1, ..., x_{mq}) \in D} \sum_{i=1}^{mq} \frac{\partial f_p}{\partial x_i}(t, x_1, ..., x_{mq}).$$

Let

$$L = \max_{1\le p\le q} |L_p|. \tag{4}$$

Let n be a positive integer number and $t_k = \frac{k}{n}$, $k = 0, 1, ..., n$. Using the Adams-Bashforth technique for solving initial value problems, discussed in many references (for example, in [1]), we construct piecewise continuous functions $F_p(t) := F_{pn}(t)$ as follows

$$F_p(t) = f_{pk} \quad \text{for} \quad t \in [t_{k-1}, t_k), \quad k = 1, 2, ..., n,$$

where the numbers f_{pk} are chosen by using Adams-Bashforth technique (See pages 302-307 in [1]) in Table 1. For the starting procedure, first- and second-order Adams-Bashforth formulas are employed to generate the required initial values. Once sufficient previous function evaluations are available, the fourth-order Adams-Bashforth formula is used for all subsequent time steps. The fourth-order Adams-Bashforth formulas are adopted because they offer a favorable balance between computational efficiency and accuracy while preserving the explicit nature of the numerical scheme. Consider mth-order nonlinear system of differential equations with piecewise

k	f_{pk}
1	$f_{p1} = f_p(t_0)$
2	$f_{p2} = \frac{3}{2}f_p(t_1) - \frac{1}{2}f_p(t_0)$
3	$f_{p3} = \frac{3}{2}f_p(t_2) - \frac{1}{2}f_p(t_1)$
≥ 4	$f_{pk} = \frac{1}{24}\Big(55f_p(t_{k-1}) - 59f_p(t_{k-2}) + 37f_p(t_{k-3}) - 9f_p(t_{k-4})\Big)$

Таблица 4: The construction of f_{pk} is reported in the present method.

constant arguments of the form

$$\begin{cases} y_1^{(m)}(t) = F_1(t), \\ y_2^{(m)}(t) = F_2(t), \\ \quad . \quad . \quad . \\ y_q^{(m)}(t) = F_q(t), \\ y_p(0) = y_{p0}, \quad y_p'(0) = y_{p1}, \quad \ldots, \quad y_p^{(m-1)}(0) = y_{p,m-1}, \end{cases} \tag{5}$$

where

$$y_p^{(r-1)}(t_{k-1}) = \lim_{t\to t_{k-1}-0} y_p^{(r-1)}(t), \quad p = 1, 2, ..., q. \tag{6}$$

We introduce the definition of a solution to the systems of equations (5).

Definition. A vector function $\mathbf{y}(t) = (y_1(t), y_2(t), ..., y_q(t))$, $y_p(t) := y_{pn}(t)$, $p = 1, 2, ..., q$ is called a solution of the initial value problem (5) if the following conditions are satisfied

1) $y_p^{(r-1)}(t)$, $r = 1, 2, ..., m$, $p = 1, 2, ..., q$ is continuous on $[0, 1]$;

2) $y_p^{(m)}(t)$ exists and continuous in $[0, 1]$ with possible exception at points $\frac{k}{n}$, $k = 0, 1, ..., n$, where one-sided derivatives exist;

3) $\mathbf{y}(t)$ satisfies initial value problem (5) in [0,1), with the possible exception at the points $\frac{k}{n}$, $k = 0, 1, ..., n$.

The system of equations (5) is called the mth-order system of differential equations with piecewise constant arguments corresponding to the system of equations (1).

Theorem 1. For any positive integer number n the initial value problem (5) has a unique solution $\mathbf{y}(t) = (y_1(t), y_2(t), ..., y_q(t))$, defined as

$$y_p(t) = \frac{(t - t_{k-1})^m}{m!} f_{pk} + \frac{(t - t_{k-1})^{m-1}}{(m-1)!} y_p^{(m-1)}(t_{k-1}) +$$

$$+\frac{(t-t_{k-1})^{m-2}}{(m-2)!}y_p^{(m-2)}(t_{k-1})+\cdots+y_p(t_{k-1}),\quad t\in[t_{k-1},t_k),\tag{7}$$

where

$$y_p^{(r-1)}(t_{k-1})=\lim_{t\to t_{k-1}-0}y_p^{(r-1)}(t),\quad p=1,...,q,\quad r=1,2,...,m,\quad k=2,3,...,n.$$

The following lemma asserts that the sets

$$\{y_p^{(r)}(t_0),\, y_p^{(r)}(t_1),\ldots,y_p^{(r)}(t_{k-1})\},\ r=0,\ldots,m-1,\quad p=1,2,...,q$$

are uniformly bounded, i.e. their upper and lower bounds does not depend of n.

Lemma. Let $\mathbf{y}(t)=(y_1(t),y_2(t),...,y_q(t))$ be solution of (5). There exists a positive number C such that for any n and $k\in\{1,\ldots,n\}$ the inequalities

$$|y_p(t_k)|\le C,\quad |y_p'(t_k)|\le C,\quad \ldots,\quad |y_p^{(m-1)}(t_k)|\le C\tag{8}$$

hold.

Theorem 2. Let $\mathbf{y}(t)=(y_1(t),y_2(t),...,y_q(t))$ be solution of (5). For any $\varepsilon>0$ there exists a positive integer number $n_0=n(\varepsilon)$ such that for any $n>n_0$ the estimation

$$\sup_{1\le p\le q}\sup_{t,t\ne\frac{k}{n}}|y_p^{(m)}(t)-f_p(t,y_1(t),y_1'(t),...,y_1^{(m-1)}(t),...,y_q(t),y_q'(t),...,y_q^{(m-1)}(t))|<\varepsilon\tag{9}$$

holds, where $y_i(t):=y_{in}(t)$, $i=1,2,...,q$.

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Symmetry Analysis and Invariant Solutions of the Two-Dimensional Heat Equation

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This study focuses on the symmetry analysis and invariant solutions of the two-dimensional heat equation. Lie group methods are applied to identify symmetries and reduce the governing partial differential equation to simpler forms. The obtained invariant solutions provide valuable insights into heat distribution and diffusion processes. The results can be applied to mathematical modeling and analytical investigation of two-dimensional heat transfer problems[1-2].

Consider the two-dimensional heat equation

$$u_t=\sum_{i=1}^{2}\frac{\partial}{\partial x_i}(k_i(u)\frac{\partial u}{\partial x_i})\tag{1}$$

where $u=u(x_1,x_2,t)$-temperature function, $k_i(u)\ge 0$ function of the temperature. Symmetry groups and the Lie algebra of infinitesimal generators of the symmetry group for the two-dimensional and three-dimensional heat equation are found in The Lie algebra of infinitesimal

generators of the symmetry group for the one-dimensional heat-conduction equation is found in [1-2].

One of the Lie infinitesimal generators of the symmetry group for equation (2) is the vector field

$$X = x_1 \frac{\partial}{\partial x_1} + x_2 \frac{\partial}{\partial x_2} + 2t \frac{\partial}{\partial t}, \tag{2}$$

which generates the following group of symmetries of the space of variables (x_1, x_2, t)

$$(x_1, x_2, t) \to (x_1 e^S, x_2 e^S, te^{2S}),$$

with respect to which the solutions of equation (2) are invariant.It means if $u = u(x_1, x_2, t)$ -solution of equation (2),then for each s function $u = u(x_1 e^{-S}, x_2 e^{-S}, te^{-2S})$ are also solutions of equation (2).

The numerical integration of this equation shows that there is a point ξ_0, which depends on the initial values of the function $v(\xi)$ and its derivative $v'(\xi)$, such that the derivative of the function $v(\xi)$ is equal to zero: $v'(\xi) = 0$. When $\xi \to \xi_0$ function $v(\xi)$ gradually increases and tends to the value $v(\xi_0)$. Thus, when $\xi \to \xi_0$ the temperature is stabilized.

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IKKI OLDINGI QADAMGA BOG‘LIQ MODIFIKATSIYALASHGAN LOGISTIK AKSLANTIRISH DINAMIKASI VA UNING JULIA–FATOU TO‘PLAMLARI

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Kirish. Biologik populyatsiyalar sonining vaqt bo‘yicha o‘zgarish dinamikasini modellashtirishda klassik logistik akslantirishlar keng qo‘llaniladi. Ammo ko‘pgina real biologik sistemalarda (masalan, generatsiyasi ustma-ust keluvchi yoki resurslari cheklangan populyatsiyalarda) keyingi holat faqat joriy qadamga emas, balki undan oldingi ikkinchi qadamga ham bog‘liq bo‘ladi. Bunday sistemalarni ikkinchi tartibli ayirmali tenglamalar orqali modellashtirish va ularning Julia–Fatou to‘plamlari yordamida asimptotik xatti-harakatini klassifikatsiyalash usuli R.L. Devani tomonidan ishlab chiqilgan umumiy nazariyaga asoslanadi [1]. Xususan, ko‘p o‘lchovli kvadratik va kubik akslantirishlar uchun Julia va Mandelbrot to‘plamlarining analitik tavsifi, shuningdek ikki orollardagi bog‘liq populyatsiyalar evolyusiyasi R. Ganixo‘jayev va SH.J. Seytov ishlarida o‘rganilgan [2,3]. Mazkur tezis ushbu yo‘nalishdagi tadqiqotlarni davom ettirib, oldingi ikkita qadamga bog‘liq modifikatsiyalashgan logistik akslantirishning qo‘zg‘almas nuqtalari barqarorligini qayta hisoblash va natijalarni sonli bifurkatsiya taxlili bilan solishtirishga bag‘ishlangan.

Model. Modifikatsiyalashgan logistik akslantirish (ML) quyidagi ikkinchi tartibli ayirmali tenglama bilan beriladi:

$$x_n = \lambda \cdot x_{n-1}(1 - x_{n-2}), \tag{1}$$

bu yerda x_n – n-qadamda mavjud bo‘lgan populyatsiya sonining uning maksimal mumkin bo‘lgan qiymatiga nisbati, $\lambda > 0$ esa yashash sharoitini ifodalovchi boshqaruvchi parametr [3]. (1) ikkinchi tartibli tenglamani standart usulda $y_n = x_{n-1}$ almashtirish orqali quyidagi ikki o‘lchovli birinchi tartibli sistemaga keltiramiz:

$$\begin{cases} x_n = \lambda \cdot x_{n-1}(1 - y_{n-1}), \\ y_n = x_{n-1}. \end{cases} \tag{2}$$

(2) sistemani $F_\lambda(x, y) = (\lambda x(1 - y),\ x)$ akslantirish iteratsiyasi orqali o‘rganamiz.

Qo‘zg‘almas nuqtalar. $F_\lambda(x, y) = (x, y)$ shartidan $y = x$ va $x = \lambda x(1 - x)$, ya’ni $x\big(1 - \lambda + \lambda x\big) = 0$ tenglama kelib chiqadi. Bundan

$$fix(ML) = \Big\{(0, 0);\ \Big(1 - \frac{1}{\lambda},\ 1 - \frac{1}{\lambda}\Big)\Big\}$$

ekanligi topiladi, ikkinchi nuqta faqat $\lambda > 1$ da $(0, 1)$ oralig‘ida yotadi.

F_λ ning Yakobi matritsasi

$$J(F_\lambda)(x, y) = \begin{pmatrix} \lambda(1 - y) & 1 \\ -\lambda x & 0 \end{pmatrix}$$

ko‘rinishiga ega.

Tegishlilik nuqtasi $(0, 0)$ **dagi barqarorlik.** $(x, y) = (0, 0)$ da $J(F_\lambda)(0, 0) = \begin{pmatrix} \lambda & 1 \\ 0 & 0 \end{pmatrix}$ bo‘lib, xarakteristik tenglama $\mu(\mu - \lambda) = 0$ dan xos sonlar $\mu_1 = 0,\ \mu_2 = \lambda$ kelib chiqadi.

Teorema 1. $|\lambda| < 1$ bo‘lganda $(0, 0)$ qo‘zg‘almas nuqta barqaror (jalb qiluvchi), $|\lambda| > 1$ bo‘lganda esa noustuvor bo‘ladi.

Notrivial nuqta $\big(1 - \frac{1}{\lambda}, 1 - \frac{1}{\lambda}\big)$ **dagi barqarorlik.** $x = y = 1 - \frac{1}{\lambda}$ qiymatlarni Yakobi matritsasiga qo‘yib,

$$J = \begin{pmatrix} 1 & 1 \\ 1 - \lambda & 0 \end{pmatrix}$$

ni hosil qilamiz. Xarakteristik tenglama $\det(J - \mu I) = \mu^2 - \mu + (\lambda - 1) = 0$ ko‘rinishida bo‘lib, uning ildizlari

$$\mu_{1,2} = \frac{1}{2}\Big(1 \mp \sqrt{5 - 4\lambda}\Big) \tag{3}$$

dir. Yuqoridagi kvadrat tenglamaning izi $T = \mu_1 + \mu_2 = 1$ va aniqlovchisi $D = \mu_1\mu_2 = \lambda - 1$ ga teng. Diskret sistemalar uchun ikkinchi tartibli xarakteristik ko‘phadli ildizlarining birlik doira ichida yotishi uchun Shur–Kon (Jury) mezoni quyidagicha bo‘ladi: $|D| < 1$ va $|T| < 1 + D$. Bu shartlarni $T = 1,\ D = \lambda - 1$ uchun yozsak:

$$|\lambda - 1| < 1 \quad \Longrightarrow \quad 0 < \lambda < 2,$$

$$1 < 1 + (\lambda - 1) = \lambda \quad \Longrightarrow \quad \lambda > 1.$$

Ikkala shartning kesishmasi $1 < \lambda < 2$ ni beradi. Demak, notrivial qo‘zg‘almas nuqta aniq $1 < \lambda < 2$ da barqaror bo‘ladi. Bunda $1 < \lambda < 5/4$ da (3) diskriminanti $5 - 4\lambda > 0$ bo‘lib, xos sonlar haqiqiy (tugun tipidagi barqarorlik), $\lambda = 5/4$ da ular qo‘shilib $\mu_{1,2} = 1/2$ ga aylanadi,

$5/4 < \lambda < 2$ da esa diskriminant manfiy bo‘lib, xos sonlar kompleks qo‘shma ko‘rinishga o‘tadi va moduli

$$|\mu_{1,2}| = \sqrt{\lambda - 1}$$

formulasi bilan aniqlanadi (spiral tipidagi barqarorlik). Bu qiymat $\lambda = 2$ da aniq birga teng bo‘lib, aynan shu yerda Naimark–Saker tipidagi bifurkatsiya yuz beradi va nuqta barqarorligini yo‘qotadi. Shunday qilib, quyidagi aniqlashtirilgan natija o‘rinli:

Teorema 2*. Notrivial qo‘zg‘almas nuqta $1 < \lambda < 2$ da barqaror (bunda $1 < \lambda \leq 5/4$ da tugun, $5/4 < \lambda < 2$ da spiral tipida) va $\lambda > 2$ da noustuvor bo‘ladi.

Ushbu aniqlashtirilgan chegara $\lambda = 2$ – maqolada kompyuter yordamida qurilgan bifurkatsiya diagrammasidagi (1-rasm) tarmoqlanish nuqtasining joylashuvi bilan to‘liq mos keladi, chunki diagrammada $\lambda = 1$ dan $\lambda \approx 2$ gacha bo‘lgan oraliqda trayektoriya yagona egri chiziq bo‘ylab (ya’ni barqaror qo‘zg‘almas nuqtaga mos) o‘zgaradi va faqat $\lambda \approx 2$ dan keyin tarmoqlanish boshlanadi.

Davriy va xaotik rejimlar. $\lambda > 2$ da qo‘zg‘almas nuqta barqarorligini yo‘qotishi bilan period ikki karra ortish (period-doubling) kaskadi boshlanadi. Sonli tajribalar (2–6-rasmlar) shuni ko‘rsatadiki, $\lambda \in [2{,}18;\ 2{,}2]$ oralig‘ida akslantirish prim davri mos ravishda 6 va 7 ga teng barqaror davriy orbitalarga ega bo‘ladi, $\lambda \in [2{,}21;\ 2{,}211]$ oralig‘ida esa sistema xaotik trayektoriyalarga o‘tadi. Bu natija umumiy nazariy chegara $\lambda = 9/4 = 2{,}25$ atrofida xaosning boshlanishi haqidagi xulosaga yaqin bo‘lib, kaskadning tezlashgan (accumulation) xarakterini namoyon etadi va bir o‘lchovli logistik akslantirishlardagi period-ikkilantirish ssenariysiga o‘xshashligi [1] bilan izohlanadi.

Julia va Fatou to‘plamlari. $K \subset \mathbb{R}^2$ orqali (1) akslantirishning chegaralangan orbitaga ega barcha nuqtalaridan iborat to‘ldirilgan Julia to‘plami belgilanadi. Julia to‘plami $J = \partial K$ – K ning chegarasi, Fatou to‘plami esa $F = \mathbb{R}^2 \setminus K$ sifatida aniqlanadi [1]. Ushbu tushunchalar Koshi masalasi yechimlarining barqaror, noustuvor, davriy yoki xaotik xususiyatga ega ekanligini klassifikatsiyalash imkonini beradi: boshlang‘ich qiymat K ning ichki qismida bo‘lsa, trayektoriya chegaralangan bo‘lib, yuqorida topilgan barqaror qo‘zg‘almas yoki davriy nuqtaga intiladi; aks holda trayektoriya cheksizlikka ketadi yoki J atrofida xaotik xatti-harakat namoyon qiladi. Bunday yondashuv oldinroq ko‘p o‘lchovli kvadratik va kubik populyatsion modellar uchun [2,3] da muvaffaqiyatli qo‘llanilgan edi.

Xulosa. Mazkur tezisda oldingi ikkita qadamga bog‘liq modifikatsiyalashgan logistik model (1) ning qo‘zg‘almas nuqtalari barqarorligi Shur–Kon mezoni asosida qayta hisoblandi va notrivial qo‘zg‘almas nuqta uchun barqarorlik chegarasi aniq $\lambda = 2$ ekanligi ko‘rsatildi, bu natija bifurkatsiya diagrammasidagi sonli kuzatishlar bilan to‘liq mos keladi. λ parametri ortishi bilan model barqaror holatdan period-ikkilantirish orqali davriy (6 va 7 davrli orbitalar) va so‘ngra xaotik rejimga ($\lambda \gtrsim 2{,}21$) o‘tishi aniqlandi. Julia va Fatou to‘plamlari orqali olingan klassifikatsiya ekologiya va matematik biologiyada populyatsiya dinamikasini modellashtirishda, shuningdek Koshi masalasi yechimlarining uzoq muddatli xatti-harakatini bashorat qilishda amaliy ahamiyatga ega [2,3,1].

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Orr–Sommerfeld tenglamasi uchun chegaraviy masalani spektral-to‘r usuli bilan sonli modellashtirish

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Bir fazali gidrodinamik sistemaning barqarorligini tadqiq qilishda asosiy masala sifatida Orr–Sommerfeld tenglamasi uchun xos son masalasini qaraymiz. U quyidagi differensial tenglama va bir jinsli chegaraviy shartlar bilan ifodalanadi:

$$\frac{1}{ik\mathrm{Re}}D^2\psi\left[\big(U(\eta)-\lambda\big)D+\frac{d^2U}{d\eta^2}\right]\psi=0,\qquad \eta_0<\eta<\eta_1, \tag{1}$$

bu yerda $D=\frac{d^2}{d\eta^2}-k^2$– differensial operator.

$$\psi(\eta_0)=\frac{d\psi}{d\eta}(\eta_0)=0,\qquad \psi(\eta_1)=\frac{d\psi}{d\eta}(\eta_1)=0. \tag{2}$$

Tenglama (1) da $U(\eta)$ – asosiy oqimning tezlik profili, η – asosiy oqimga ko’ndalang yo’nalishdagi koordinata, k – to’lqin soni, Re – Reynolds soni, $\psi(\eta)$ – qo’zg’alish amplitudasini ifodalovchi oqim funksiyasi, $\lambda=\lambda_r+i\lambda_i$ esa (1)–(2) masalaning kompleks xos soni bo’lib, λ_r va λ_i mos ravishda xos sonning haqiqiy va mavhum qismlaridir. Xos sonning mavhum qismi gidrodinamik qo’zg’alishlarning vaqt bo’yicha o’zgarishini belgilaydi. Xususan, $\lambda_i<0$ bo’lganda qo’zg’alishlar so’nadi va sistema barqaror bo’ladi, $\lambda_i>0$ bo’lganda qo’zg’alishlar kuchayadi va sistema nobarqaror holatga keladi. $\lambda_i=0$ holat neytral barqarorlikka mos keladi.

Masala (1)–(2)ni sonli yechishda spektral-to’r metodidan foydalanamiz. Integrallash oralig’ini

$$[\eta_0,\eta_1]=\bigcup_{j=1}^{N}[\eta_{j-1},\eta_j],\qquad l_j=\eta_j-\eta_{j-1},$$

ko’rinishda N ta elementga ajratamiz. Har bir $[\eta_{j-1},\eta_j]$ elementni $[-1,1]$ oraliqqa quyidagi almashtirish yordamida o’tkazamiz:

$$\eta=\frac{\eta_j+\eta_{j-1}}{2}+\frac{l_j}{2}y,\qquad -1\le y\le 1.$$

Bunda

$$\frac{d}{d\eta}=\frac{2}{l_j}\frac{d}{dy},\qquad \frac{d^2}{d\eta^2}=\frac{4}{l_j^2}\frac{d^2}{dy^2}.$$

Quyidagi belgilashlarni kiritamiz:

$$D_j=\frac{d^2}{dy^2}-k_j^2,\qquad k_j=\frac{l_j}{2}k,\qquad Re_j=\frac{l_j}{2}Re.$$

Natijada har bir elementda (1) tenglama operator ko’rinishida quyidagicha ifodalanadi:

$$L_j\psi_j=0,\qquad j=1,2,\dots,N, \tag{3}$$

bu yerda

$$L_j = D_j^2 - ik_j Re_j \left[(U_j(y) - \lambda) D_j + \frac{d^2 U_j}{dy^2} \right].$$

Har bir elementdagi yechimni

$$\psi_j(y) = \sum_{n=0}^{p_j} a_n^{(j)} U_n(y), \qquad j = 1, 2, \ldots, N, \tag{4}$$

ko'rinishda izlaymiz, bu yerda $a_n^{(j)}$ noma'lum koeffitsientlar, $U_n(y)$ esa ikkinchi tur Chebyshev ko'phadlari bo'lib,

$$U_0(y) = 1, \qquad U_1(y) = 2y,$$
$$U_{n+1}(y) = 2yU_n(y) - U_{n-1}(y).$$

Ma'lumki,ikkinchi tur Chebyshev ko'phadlari $[-1, 1]$ oraliqda

$$w(y) = \sqrt{1 - y^2}$$

vazn funksiyasi bo'yicha ortogonal hisoblanadi. Shuning uchun qoldiqning bazis funksiyalarga ortogonalligi

$$(L_j \psi_j, U_n) = 0, \qquad n = 0, 1, \ldots, p_j - 4, \tag{5}$$

shart orqali talab qilinadi. To'rtinchi tartibli tenglama bo'lganligi sababli elementlar tutashgan nuqtalarda yechim va uning birinchi uchta hosilasi uzluksiz bo'lishi talab qilinadi:

$$\psi_j^{(t)}(+1) = \psi_{j+1}^{(t)}(-1), \qquad t = 0, 1, 2, 3, \tag{6}$$

$$\frac{d^t}{d\eta^t} = \left(\frac{2}{l_j} \right)^t \frac{d^t}{dy^t}. \tag{7}$$

Chegaraviy shartlar esa

$$\psi_1(-1) = 0, \qquad \frac{d\psi_1}{dy}(-1) = 0, \qquad \psi_N(+1) = 0, \qquad \frac{d\psi_N}{dy}(+1) = 0 \tag{8}$$

ko'rinishda yoziladi.

Galerkin shartlari, chegaraviy shartlar va elementlararo uzluksizlik shartlarini birlashtirish natijasida noma'lum spektral koeffitsientlarga nisbatan umumlashgan xos son masalasi hosil qilinadi:

$$(A - \lambda B)\mathbf{x} = 0, \tag{9}$$

bu yerda $\mathbf{x}$ barcha $a_n^{(j)}$ spektral koeffitsientlardan tuzilgan vektor, A va B esa sonli diskretlash natijasida hosil bo'lgan matritsalardir.

Shunday qilib,bazis funksiyalari sifatida ikkinchi tur Chebyshev ko'phadlari olingan spektral-to'r usuli gidrodinamik turg'unlikning xos son masalasini yuqori aniqlikda sonli tadqiq etish imkonini beradi. Hisoblangan xos sonning mavhum qismi λ_i oqim turg'unligini aniqlaydi: $\lambda_i > 0$ da oqim noturg'un, $\lambda_i < 0$ da turg'un, $\lambda_i = 0$ da esa neytral turg'unlik holati yuzaga keladi.

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Uch o‘lchamli Puasson tenglamasi uchun Dirixle masalasini sonli modellashtirish

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To‘g‘ri parallelepipedli $\overline{D} = \{-1 \le x \le 1, -1 \le y \le 1, -1 \le z \le 1\}$ sohada Puasson tenglamasini

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -f(x,y,z), \tag{1}$$

Dirixle chegaraviy shartlari

$$\begin{gathered} u(-1,y,z) = 0, \quad u(1,y,z) = 0, \quad u(x,-1,z) = 0, \\ u(x,1,z) = 0, \quad u(x,y,-1) = 0, \quad u(x,y,1) = 0 \end{gathered} \tag{2}$$

bilan qaraymiz.

Ikki o‘lchamli Puasson tenglamasi uchun Dirixle chegaraviy masalasini yechishda dastlab integrallash metodining diskret variantidan foydalanib, samarali natijalarga erishilgan [2,3]. Ushbu natijalarni inobatga olgan holda, (1)–(2) differensial masalani sonli modellashtirishda ham dastlab integrallash metodining diskret variantidan foydalanamiz.

Tenglama (1) dagi xususiy hosilalar va uning o‘ng tomoni birinchi turdagi Chebishev ko‘phad -larining chekli qatori ko‘rinishida ifodalanadi:

$$\frac{\partial^2 u}{\partial x^2} = \sum_{i=0}^{N}{}' \sum_{j=0}^{M}{}' \sum_{q=0}^{R}{}' a_{ijq}^{(2x)} T_i(x) T_j(y) T_q(z), \qquad \frac{\partial^2 u}{\partial y^2} = \sum_{i=0}^{N}{}' \sum_{j=0}^{M}{}' \sum_{q=0}^{R}{}' a_{ijq}^{(2y)} T_i(x) T_j(y) T_q(y),$$

$$\frac{\partial^2 u}{\partial z^2} = \sum_{i=0}^{N}{}' \sum_{j=0}^{M}{}' \sum_{q=0}^{R}{}' a_{ijq}^{(2z)} T_i(x) T_j(y) T_q(y), \quad f(x,y,z) = \sum_{i=0}^{N}{}' \sum_{j=0}^{M}{}' \sum_{q=0}^{R}{}' g_{ijq} T_i(x) T_j(y) T_q(z). \tag{3}$$

bu yerda $T_i(x), T_j(y), T_q(z)$ - birinchi turdagi Chebishev ko‘phadlari, yig‘indining yuqorisidagi shtrix belgisi $i = j = q = 0$ bo‘lganda qator koeffitsiyentlari a_{ijq}, g_{ijq} o‘zgarmas $\frac{1}{8}$ ga, $i = j = 0$,

$j = q = 0$ yoki $i = q = 0$ bo‘lgan hollarda o‘zgarmas $\frac{1}{4}$ ga, $i = 0$, $j = 0$ yoki $q = 0$ bo‘lsa o‘zgarmas $\frac{1}{2}$ ga ko‘paytirilishini anglatadi.

Qatorlar (3) ni tenglama (1) ga qo‘yib, ushbu tenglamani hosil qilamiz:

$$\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}'\sum_{q=0}^{R}{}'\left(a_{ijq}^{(2x)}+a_{ijq}^{(2y)}+a_{ijq}^{(2z)}\right)T_i(x)T_j(y)T_q(z)=-\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}'\sum_{q=0}^{R}{}'g_{ijq}T_i(x)T_j(y)T_q(z).$$

Ushbu tenglamada bir xil darajali ko‘phadlar oldidagi koeffitsientlar tenglashtirib, ushbu algebraik sistema hosil qilinadi

$$a_{ijq}^{(2x)}+a_{ijq}^{(2y)}+a_{ijq}^{(2z)}=-g_{ijq},\quad i=0,1,2,\ldots,N,\quad j=0,1,2,\ldots,M,\quad q=0,1,2,\ldots,R. \tag{4}$$

So‘ngra “x”, “y” va “z” o‘zgaruvchilar bo‘yicha tenglama (4) dagi hosilalar tartibini pasaytirish imkonini beradigan quyidagi birinchi tur Chebishev ko‘phadlari rekurrent formulalarini qo‘llaymiz:

$$\begin{aligned} a_{ijq}^{((k-1)x)}&=\frac{a_{i-1,j,q}^{(kx)}-a_{i+1,j,q}^{(kx)}}{2i},\quad a_{ijq}^{((k-1)y)}=\frac{a_{i,j-1,q}^{(ky)}-a_{i,j+1,q}^{(ky)}}{2j},\\ a_{ijq}^{((k-1)z)}&=\frac{a_{i,j,q-1}^{(kz)}-a_{i,j,q+1}^{(kz)}}{2q}. \end{aligned} \tag{5}$$

va dastlab integrallash metodi diskret variantidagi asosiy algebraik tenglama shakllantiriladi. Chegaraviy shartlar ham birinchi tur Chebishev ko‘phadlari qatori orqali ifodalanadi. Natijada asosiy algebraik tenglama va chegaraviy shartlardan hosil qilinadigan tenglamalar birgalikda masalaning taqribiy yechimdagi noma’lum koeffitsientlarni aniqlash uchun ushbu chiziqli algebraik tenglamalar sistemasiga keltiriladi:

$$Ax=b, \tag{6}$$

bu yerda A matritsa- $K\times K$ tartibli kvadrat matritsadan iborat, bunda $K=(N+1)\cdot(M+1)\cdot(R+1)$ bo‘lib, u noma’lumlarning umumiy soniga teng.

Bundan tashqari, ma’lum $f(x,y,z)$ funksiyaga asosan tenglama (4) ning o‘ng tomonidagi g_{ijq} koeffitsiyentlarni $x_l=\cos\frac{\pi l}{N},(l=0,1,\ldots,N),\quad y_k=\cos\frac{\pi k}{M},$

$(k=0,1,\ldots,M)$ va $z_p=\cos\frac{\pi p}{R},\quad (p=0,1,\ldots,R)$ Chebishev ko‘phadlarining

kollokatsiya tugunlarida hisoblash uchun quyidagi diskret teskari almashtirish formulasi o‘rinli

$$g_{ijq}=\frac{4}{MNc_ic_jc_q}\sum_{l=0}^{N}\sum_{k=0}^{M}\sum_{p=0}^{R}\left(\frac{1}{c_lc_kc_p}f(x_l,y_k,z_p)T_i(x_l)T_j(y_k)T_q(z_p)\right), \tag{7}$$

$i=0,1,\ldots,N,\ j=0,1,\ldots,M,\ q=0,1,\ldots,R$

bu yerda $c_m=1,\ m=1,2,\ldots,N-1,\quad c_t=1,\ t=1,2,\ldots,M-1$ va $c_s=1,\ s=1,2,\ldots,R-1$

hamda $c_0=c_N=c_M=c_R=2$. Differensial masala (1)–(2) ning taqribiy yechimi Chebishev ko‘phadlarining chekli qatori ko‘rinishida izlanadi

$$u_a(x,y,z)=\sum_{i=0}^{N}\sum_{j=0}^{M}\sum_{q=0}^{R}a_{ijq}T_i(x)T_j(y)T_q(z), \tag{8}$$

bu yerda $a_{ijq}(i = 0,1,\dots,N; j = 0,1,\dots,M; q = 0,1,\dots,R)$ noma'lum koeffitsiyentlar bo'lib, ularning umumiy soni $(N+1)(M+1)(R+1)$ ga teng. Sistema (6) ni yechish orqali $a_{ijq}(i = 0,1,\dots,N; j = 0,1,\dots,M; q = 0,1,\dots,R)$ koeffitsiyentlar aniqlanadi, so'ngra formula (8) yordamida taqribiy yechim $u_a(x_l, y_k, z_p)$ ning qiymatlari Chebishev ko'phadlarining kollokatsiya tugunlari (x_l, y_k, z_p) da hisoblanadi.

Adabiyotlar

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CONSTRUCTION OF AN ALGEBRAIC-HYPERBOLIC NATURAL SPLINE IN $K_{2,v}^{(m)}$ AND ITS APPLICATION TO OPTIMAL QUADRATURE FORMULAS

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Abstract. In this work, an algebraic-hyperbolic natural interpolation spline of order $2m$ that minimizes a semi-norm in the Hilbert space $K_{2,v}^{(m)}(0,1)$ is investigated. By employing the discrete analog of the corresponding differential operator, the coefficients of the spline are determined explicitly. Furthermore, an optimal quadrature formula in the sense of Sard is derived by integrating the constructed natural spline. Numerical comparisons demonstrate that the error bounds of the derived quadrature formula coincide with those of the optimal quadrature formula in this space.

Keywords: Natural spline, discrete analog, optimal quadrature formula, Sobolev method, boundary conditions.

1. Introduction and problem statement. In numerical analysis and approximation theory, constructing interpolating splines and establishing their connection with optimal quadrature formulas is of fundamental importance. As established by I.J. Schoenberg [1], numerical integration of a natural spline that minimizes a prescribed semi-norm yields Sard's optimal quadrature formula.

In this work, we consider the Sobolev-type space associated with the differential operator $L = \frac{d^m}{dx^m} - v^2 \frac{d^{m-2}}{dx^{m-2}}$ $(v > 0)$:

$$K_{2,v}^{(m)}(0,1) = \left\{ \varphi : [0,1] \to \mathbb{R} \mid \varphi^{(m-1)} \text{ is abs. cont., } L\varphi \in L_2(0,1) \right\} \tag{1}$$

equipped with the following semi-norm:

$$\|f\|_{K_{2,v}^{(m)}} = \left(\int_0^1 (Lf(x))^2 dx \right)^{1/2} = \left(\int_0^1 \left(f^{(m)}(x) - v^2 f^{(m-2)}(x) \right)^2 dx \right)^{1/2}. \tag{2}$$

Let a uniform mesh $\Delta : 0 = x_0 < x_1 < \cdots < x_N = 1$ with step size $h = 1/N$ $(N \geq m-1)$ be given. We seek an interpolating function $S_{\Delta,m}(x) \in K_{2,v}^{(m)}(0,1)$ minimizing the functional (2) and satisfying interpolation conditions $S_{\Delta,m}(x_\beta) = f(x_\beta)$, $\beta = 0, 1, \ldots, N$.

2. Construction of the natural spline. The function that minimizes semi-norm (2) satisfies the Euler-Poisson differential equation $L^* L S_{\Delta,m}(x) = 0$:

$$\left(\frac{d^{2m}}{dx^{2m}} - 2v^2 \frac{d^{2m-2}}{dx^{2m-2}} + v^4 \frac{d^{2m-4}}{dx^{2m-4}} \right) S_{\Delta,m}(x) = 0, \quad x \in (x_i, x_{i+1}). \tag{3}$$

Theorem 1. *The algebraic-hyperbolic natural interpolation spline of order* $2m$ *that minimizes (2) has the explicit representation:*

$$S_{\Delta,m}(x) = \sum_{\gamma=0}^{N} C_\gamma G_m(x - x_\gamma) + d_1 \sinh(vx) + d_2 \cosh(vx) + \sum_{\alpha=0}^{m-3} p_\alpha x^\alpha, \tag{4}$$

where $G_m(x)$ *is the fundamental solution to the operator in (3):*

$$G_m(x) = \frac{\operatorname{sgn}(x)}{4v^{2m-1}} \left(vx \cosh(vx) - (2m-3) \sinh(vx) + 2 \sum_{k=1}^{m-2} \frac{(m-k-1)(vx)^{2k-1}}{(2k-1)!} \right), \tag{5}$$

and the unknown coefficients C_γ, d_1, d_2, p_α *are determined uniquely by the interpolation and orthogonality conditions:*

$$S_{\Delta,m}(x_\beta) = f(x_\beta), \quad \sum_{\gamma=0}^{N} C_\gamma \sinh(vx_\gamma) = 0, \quad \sum_{\gamma=0}^{N} C_\gamma \cosh(vx_\gamma) = 0, \quad \sum_{\gamma=0}^{N} C_\gamma x_\gamma^\alpha = 0. \tag{6}$$

The system (6) is of the discrete Wiener-Hopf type. Following S.L. Sobolev's method [2], the system is solved using the discrete analog $D_m(h\beta)$ of the differential operator (3) introduced in [3]. Convolving $D_m(h\beta)$ with the extension function $u_m(h\beta)$ directly yields the coefficients:

$$C_\beta = D_m(h\beta) * u_m(h\beta), \quad \beta = 0, 1, \ldots, N.$$

3. Optimal quadrature formula and numerical analysis. Integrating the constructed spline $S_{\Delta,m}(x)$ over the interval $[0,1]$ generates an optimal quadrature formula for computing definite integrals $\int_0^1 f(x)dx$.

Theorem 2. *The quadrature formula generated by the* $2m$*-th order algebraic-hyperbolic natural spline has the following form:*

$$\int_0^1 f(x)dx \approx \sum_{\beta=0}^{N} C_\beta \Gamma_m(x_\beta) + d_1 \frac{\cosh(v) - 1}{v} + d_2 \frac{\sinh(v)}{v} + \sum_{\alpha=0}^{m-3} \frac{p_\alpha}{\alpha + 1}, \tag{7}$$

where $C_\beta, d_1, d_2, p_\alpha$ are the spline coefficients in (4), and the weight function is given by:

$$\Gamma_m(x_\beta) = \frac{1}{4v^{2m}}\Big(vx_\beta \sinh(vx_\beta) + v(1-x_\beta)\sinh(v(1-x_\beta)) - (2m-2)\big(\cosh(vx_\beta)+ \\ + \cosh(v(1-x_\beta)) - 2\big) + \sum_{k=1}^{m-2} \frac{(m-k-1)v^{2k}}{k\cdot(2k-1)!}\left(x_\beta^{2k} + (1-x_\beta)^{2k}\right)\Big). \tag{8}$$

For $m = 2$, the formula simplifies significantly:

$$\int_0^1 f(x)dx \approx \sum_{\beta=0}^{N} C_\beta \Gamma_2(x_\beta) + d_1\frac{\cosh(v)-1}{v} + d_2\frac{\sinh(v)}{v},$$

where
$\Gamma_2(x_\beta) = \frac{1}{4v^4}\left(vx_\beta\sinh(vx_\beta) + v(1-x_\beta)\sinh(v(1-x_\beta)) - 2\cosh(vx_\beta) - 2\cosh(v(1-x_\beta)) + 2\right)$.

To validate the efficiency of the quadrature formula (7), numerical computations were carried out for the test function $f(x) = \sinh(vx) + x^3$ for $m = 2, 3, 4$ across various tension parameters v and nodes N.

Table 1. Comparison of absolute errors for $m = 2$ and $m = 3$.

Tension	$m=2,\ N=11$		$m=3,\ N=41$	
v	Our Spline Method	Optimal Formula [3]	Our Spline Method	Optimal Formula [3]
10	$1.40\cdot10^{-3}$	$1.40\cdot10^{-3}$	$1.96\cdot10^{-6}$	$1.96\cdot10^{-6}$
20	$7.35\cdot10^{-3}$	$7.35\cdot10^{-3}$	$8.62\cdot10^{-6}$	$8.62\cdot10^{-6}$
30	$1.82\cdot10^{-2}$	$1.82\cdot10^{-2}$	$1.95\cdot10^{-5}$	$1.95\cdot10^{-5}$
40	$3.27\cdot10^{-2}$	$3.27\cdot10^{-2}$	$3.44\cdot10^{-5}$	$3.44\cdot10^{-5}$
50	$4.85\cdot10^{-2}$	$4.85\cdot10^{-2}$	$5.29\cdot10^{-5}$	$5.29\cdot10^{-5}$

The numerical experiments demonstrate that the absolute errors of the spline-generated quadrature formula (7) are completely identical to those of the Sard-optimal quadrature formula. This fully confirms Schoenberg's theorem for the space $K_{2,v}^{(m)}(0,1)$.

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Construction optimal quadrature formula for the approximate calculation of Fourier integrals in the space $L_2^{(m)}(0,1)$.

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The quadrature of highly oscillatory integrals plays a crucial role in numerous research domains, including numerical analysis, electromagnetic analysis, acoustic scattering, and quantum chemistry. Historically regarded as a formidable challenge that demanded a large number of function evaluations (scaling approximately with the frequency), the field has undergone a significant revolution over the past fifteen years.

By employing asymptotic expansions as the primary analytical tool, a substantial number of efficient numerical methods have been developed for highly oscillatory integrals, such as asymptotic expansion, Filon-type methods, Levin's method, complex Gaussian quadrature, and various combinations thereof. Among these techniques, Filon-type methods exhibit several distinct advantages: they are easily implemented, generalizable to multidimensional settings, and demonstrate superior accuracy compared to standard asymptotic expansion methods. Specifically, they yield a reliable approximation of highly oscillatory integrals for all $\omega \geqslant 0$.

Prior to formulating the method, we first define the subject of our analysis and describe its asymptotic expansion. In this paper, we focus on the construction of optimal quadrature formulas involving derivatives for the approximate computation of rapidly oscillating integrals.

It is important to construct quadrature and cubature formulas in Hilbert spaces for the approximate calculation of integrals. The results obtained in the construction of quadrature formulas make it possible to develop effective mathematical models of natural processes. Therefore, carrying out scientific research in the following areas is one of the important problems of computational mathematics:

1) Numerical approximation of strongly oscillating integrals;

2) Construction of optimal quadrature formulas in different Hilbert spaces and estimation of their errors.

For the construction of optimal quadrature formulas for the approximate calculation of definite integrals, the Sobolev method, the method of spline functions and method of φ functions can be used.

This work is also devoted to the construction of the optimal quadrature formula for the approximate calculation of Fourier integrals. In this regard, we will consider the following quadrature formula

$$I[\varphi] = \int_0^1 \varphi(x) e^{2\pi i\omega x} dx \cong \sum_{\beta=0}^{N} d_0[\beta]\varphi(h\beta) + \sum_{\beta=0}^{N} d_1[\beta]\varphi'(h\beta), \tag{1}$$

where $\varphi(x)$ is a non-oscillating function,

$$d_0[\beta] = \begin{cases} h\left(\frac{K_{\omega,1}e^{2\pi i\omega h}}{e^{2\pi i\omega h}-1} - \frac{1}{2\pi i\omega h}\right), & \beta = 0 \\ hK_{\omega,1}e^{2\pi i\omega h\beta}, & \beta = \overline{1, N-1}, \\ h\left(\frac{K_{\omega,1}e^{2\pi i\omega h}}{1-e^{2\pi i\omega h}} + \frac{e^{2\pi i\omega}}{2\pi i\omega h}\right), & \beta = N, \end{cases}$$

$$K_{\omega,1} = \left(\frac{\sin \pi\omega h}{\pi\omega h}\right)^2, \quad \omega \in \mathbb{R}, \quad \omega \neq 0,$$

$d_1[\beta]$ are the coefficients of the quadrature formula (1), $h\beta$ ($\beta = \overline{0, N}$, $h = \frac{1}{N}$) nodes.

This space forms a Hilbert space with the following inner product after certain factorization

$$\langle \varphi, \psi \rangle_{L_2^{(m)}} = \int_0^1 \varphi^{(m)}(x)\overline{\psi^{(m)}(x)}dx. \tag{2}$$

This Hilbert space is provided by the following norm based on scalar multiplication

$$\|\varphi\|_{L_2^{(m)}} = \sqrt{\langle\varphi,\varphi\rangle} = \sqrt{\int_0^1 \varphi^{(m)}(x)\overline{\varphi^{(m)}(x)}dx}. \tag{3}$$

Naturally, when constructing quadrature formulas in a certain space, the convergence of the quadrature sum to the integral is studied. It is important to evaluate the resulting error. The following difference between the integral and the quadrature sum

$$(\ell,\varphi) = \int_0^1 e^{2\pi i\omega x}\varphi(x)dx - \sum_{\beta=0}^{N} d_0[\beta]\varphi(h\beta) - \sum_{\beta=0}^{N} d_1[\beta]\varphi'(h\beta) \tag{4}$$

is called the error of the quadrature formula (1).

The following difference

$$\ell(x) = E_{[0,1]}(x)e^{2\pi i\omega x} - \sum_{\beta=0}^{N} d_0[\beta]\delta(x-h\beta) + \sum_{\beta=0}^{N} d_1[\beta]\delta'(x-h\beta) \tag{5}$$

is the corresponding error functional, where E is the characteristic function of the segment $[0,1]$, $\delta(x)$ is Dirac's delta-function.

Based on the above, in order to have an optimal quadrature of the form (1) in the Hilbert space $L_2^{(m)}(0,1)$, we solve the following problems.

Problems.

a) Calculate the norm of the error functional of the form (5);

b) Find the coefficients $d_1[\beta] = \mathring{d}_1[\beta]$ that give the minimum value to the norm $\|\ell|L_2^{(m)*}\|$.

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COEFFICIENTS OF THE OPTIMAL QUADRATURE FORMULAS WITH DERIVATIVES IN THE SPACE $L_2^{(m)}(0,1)$

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In modern computational mathematics, the approximate calculation of definite integrals in given functional spaces and the minimization of their errors is considered one of the fundamental problems. Traditional quadrature formulas primarily rely solely on the values of the function at the nodes. However, in practical problems, specifically in the numerical solution of equations in mathematical physics and mechanics, various derivatives of the function under the integral are also known. The effective use of such data allows for a sharp increase in integration accuracy without increasing the number of nodes. From this perspective, minimizing the norm of the error functional under given constraints in Hilbert or Sobolev spaces that is, constructing optimal quadrature formulas is extremely relevant from both theoretical and practical perspectives.

In certain scenarios, classical quadrature rules are insufficient for accurately reflecting the specific behaviors of functions being integrated. This highlights the ongoing necessity to optimize these formulas, boost their accuracy, and reduce computational errors. One effective alternative is the use of derivative quadrature formulas, which rely on both the function's nodal values and its derivatives of various orders. These formulas provide superior results when integrating smooth functions, solving differential equations numerically, or addressing variational calculus problems.

We consider the problem of constructing an Hermitian-type quadrature formula in the space $L_2^{(m)}(0,1)$ as follows

$$\int_0^1 u(x)dx \cong \sum_{\beta=0}^{N} k_0[\beta]u\left(h\beta\right) + \frac{h^2}{12}\left(u'(0) - u'(1)\right) + \sum_{\beta=0}^{N} k_1[\beta]u'''\left(h\beta\right). \tag{1}$$

The error functional corresponding to this quadrature formula is as follows

$$l_N(x) = \chi_{[0,1]}(x) - \sum_{\beta=0}^{N} k_0[\beta]\delta\left(x - h\beta\right) + \frac{h^2}{12}\left(\delta'(x) - \delta'(x-1)\right) + \sum_{\beta=0}^{N} k_1[\beta]\delta'''\left(x - h\beta\right), \tag{2}$$

where $k_0[\beta] = \begin{cases} h/2\,; & \beta = 0, N, \\ h; & \beta = \overline{1, N-1} \end{cases}$ are known coefficients of quadrature formula(1), $k_1[\beta]$, $\beta = \overline{0, N}$ are unknown coefficients, $u(x)$ is an element of space $L_2^{(m)}(0,1)$, $\chi_{[0,1]}(x)$ is the characteristic function of the interval $[0,1]$, $\delta(x)$ is a Dirac delta function, N is a natural number and $h = \frac{1}{N}$.

The error functional $l_N(x)$ of the form (2), defined in the space $L_2^{(m)}(0,1)$, satisfies the following conditions:

$$(l_N, x^\alpha) = 0, \quad \alpha = 0, 1, 2, ..., m-1. \tag{3}$$

From the following Cauchy-Schwarz inequality

$$|(l_N, u)| \leq ||u||_{L_2^{(m)}} \cdot \|l_N\|_{L_2^{(m)*}}$$

the error of quadrature formula (1) is estimated by the norm of the error functional (2) in the space $L_2^{(m)*}(0,1)$

$$||l_N||_{L_2^{(m)*}} = \sup_{||u||_{L_2^{(m)}}=1} |(l_N, u)|\,. \tag{5}$$

The error functional depends on the $k_1[\beta]$, $\beta = 0, 1, ..., N$ coefficients and $h\beta$ nodal points. We consider solving the Sard problem when the nodal points are fixed. We find the minimum of the norm of the error functional with respect to the $k_1[\beta]$ coefficients.

$$\left\|\bar{l}_N\right\|_{L_2^{*(m)}} = \inf_{k_1[\beta]} \|l_N\|_{L_2^{*(m)}}. \tag{6}$$

Moreover, the norm of the error functional (2) depends on $k_1[\beta]$ coefficients, and we minimize the norm of the error function by $k_1[\beta]$ coefficients. $k_1,\ \beta = 0, 1, ..., N[\beta]$ coefficients that satisfy equality (5) are called optimal coefficients, and the corresponding quadrature formula is called the optimal quadrature formula. Thus, in order to construct the optimal quadrature formula with derivatives in the form (1), we first need to find the general form of the norm of the error functional $l_N(x)$ in the form (2). Following that, we determine the coefficients $k_1[\beta],\ \beta = 0, 1, ..., N$ that minimize the error functional (2).

To find the general form of the norm the error functional (2), we use the Riess theorem about the general form of a linear continuous functional and the concept of an extremal function [1], [2]. The norm of the error function of the quadrature formula can be written as follows [3]:

$$\|l_N\|^2 = (l_N, (-1)^m l_N(x) * G_m(x)) = \int\limits_{-\infty}^{\infty} l_N(x) \left((-1)^m \int\limits_{-\infty}^{\infty} l_N(y) G_m(x-y)dy \right) dx =$$

$$= (-1)^m \left\{ -\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} k_1[\beta]k_1[\gamma] \frac{(h\gamma - h\beta)^{2m-7}}{2(2m-7)!}\ sign(h\gamma - h\beta) - \right.$$

$$+2\sum_{\beta=0}^{N} k_1[\beta] \sum_{\gamma=0}^{N} k_0[\gamma] \frac{(h\beta - h\gamma)^{2m-4}}{2(2m-4)!} sign(h\beta - h\gamma) +$$

$$+2\sum_{\beta=0}^{N} k_1[\beta] \int\limits_0^1 \frac{(x-h\beta)^{2m-4}}{2(2m-4)!} sign(x-h\beta)dx - 2\cdot\frac{h^2}{12}\sum_{\beta=0}^{N} k_1[\beta] \left(\frac{(h\beta)^{2m-5}}{2(2m-5)!} - \right.$$

$$\left. - \frac{(1-h\beta)^{2m-5}}{2(2m-5)!} \right) - \frac{h^2}{6}\sum_{\beta=0}^{N} k_0[\beta] \left(\frac{(h\beta)^{2m-2}}{2(2m-2)!} + \frac{(1-h\beta)^{2m-2}}{2(2m-2)!} \right) +$$

$$+\sum_{\gamma=0}^{N}\sum_{\beta=0}^{N} k_0[\gamma]k_0[\beta] \frac{(h\gamma - h\beta)^{2m-1}}{2(2m-1)!} sign(h\gamma - h\beta) -$$

$$\left. -2\sum_{\beta=0}^{N} k_0[\beta] \int\limits_0^1 \frac{(x-h\beta)^{2m-1}}{2(2m-1)!} sign(x-h\beta)dx + \frac{h^2}{6(2m-1)!} + \frac{1}{(2m+1)!} \right\}$$

The following is true.

Theorem In the Sobolev space $L_2^{(m)}(0,1)$, $m \geq 4$, the coefficients of the optimal quadrature formula with the derivative of the form (1) are as follows.

$$k_1[0] = -\frac{h^4}{720} + \sum_{k=1}^{m-4} a_k \frac{q_k + q_k^N}{q_k - 1},$$

$$k_1[\beta] = h^4 \sum_{k=1}^{m-4} a_k \left(q_k^{\beta} - q_k^{N-\beta} \right),\ \ \beta = \overline{1, N-1},$$

$$k_1[N] = \frac{h^4}{720} - \sum_{k=1}^{m-4} a_k \frac{q_k^N + q_k}{q_k - 1},$$

where $a_k$are solutions of the following equation

$$\sum_{k=1}^{m-4} a_k \frac{q_k^{N+1} - q_k(-1)^{\alpha+1}}{(1-q_k)^{\alpha+1}} E_{\alpha-1}(q_k) = \frac{B_{\alpha+4}}{(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)}, \quad \alpha = \overline{1, m-4}.$$

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HARORAT TA'SIRIDA FAZOVIY STERJENNING EGILISH, SILJISH VA BURALISH JARAYONLARINI MODELLASHTIRISH

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Zamonaviy texnika va energetika sohalarida (issiqlik-energetika qurilmalari, dvigatel va reaktor elementlari, aerokosmik konstruksiyalar) ishlaydigan yuk ko'taruvchi elementlar ko'pincha bir vaqtning o'zida ham kuch, ham issiqlik ta'siriga uchraydi. Harorat ko'tarilishi materialning mexanik xususiyatlarini – elastik modul va oqish chegarasini – pasaytiradi, natijada oddiy sharoitda xavfsiz hisoblangan yuklanish ham konstruksiya elementini elastik-plastik holatga o'tkazishi mumkin. Shu sababli kuch va harorat birgalikda ta'sir etganda konstruksiya elementlarining kuchlanish-deformatsiya holatini ishonchli baholash qattiq deformatsiyalanuvchi jismlar mexanikasining dolzarb masalalaridan biri hisoblanadi.

Qaralayotgan masalada sterjenga tashqi mexanik yuk bir marta statik tarzda qo'yilib, keyin o'zgarmas holda saqlanadi; harorat maydoni esa vaqt davomida nostatsionar ravishda, quyida (5)-ifodada keltirilgan qonuniyat asosida ortib boradi.

MODELNING MATEMATIK ASOSLARI

Sterjenning deformatsiyalangan holati klassik Koshi geometrik munosabatlariga ko'ra, ya'ni deformatsiya tenzorining komponentlari quyidagi tarzda ko'chish maydoni hosilalari orqali aniqlanadi:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{1}$$

bunda u_i – ko'chish vektorining tashkil etuvchilari; deformatsiyalar kichik va chiziqli deb qaraladi.

Chiziqli bo'lmaganlikning yagona manbai jismning fizik xossasi – material zarrachasining elastik holatdan plastik holatga o'tishi – bo'lib, u plastiklik funksiyasi orqali hisobga olinadi.

Kuchlanish bilan deformatsiya orasidagi bog'lanish A.A.Ilyushinning o'zgaruvchan elastik parametrlar usuliga asoslanadi [7]: plastik deformatsiyalanish jarayonidagi chiziqli bo'lmagan

bog'lanish formal ravishda chiziqli (Guk turdagi) tenglama ko'rinishida saqlanadi, lekin doimiy elastik modul o'rniga kuchlanish intensivligiga bog'liq o'zgaruvchan kesuvchi modul – materialning qattiqlanish egri chizig'idan aniqlanadigan plastiklik funksiyasi – kiritiladi. Harorat ta'siridagi qo'shimcha deformatsiya esa Dyugamel-Neyman termoelastiklik qonuniga muvofiq hisobga olinadi. Shunday qilib, kuchlanish quyidagi ko'rinishni oladi:

$$\sigma_{11} = E(x,t)\big[1-\omega(x,z)\big]\big[\varepsilon_{11}(x,z)-\varepsilon^T(x,t)\big], \tag{2}$$

bunda $E(x,t)$ – haroratga bog'liq elastik modul (6)-ifoda, $\omega(x,z)$ – plastiklik funksiyasi, $\varepsilon^T(x,t)$ – (6)-ifodadagi termik deformatsiya; (1) va (2)-ifodalar sterjenning umumiy fizik-geometrik modelini tashkil etadi.

Yuklanishning takroriy, nostatsionar yoki siklik xususiyatga ega bo'lgan holatlari uchun esa V.V.Moskvitinning o'zgaruvchan yuklanishlardagi plastiklik nazariyasi qo'llaniladi [8]: u Ilyushin maktabining g'oyalarini rivojlantirib, yuklash – yukni olish – qayta yuklash sikllarida elastik parametrlarning o'zgarib borish jarayonini hisobga olish imkonini beradi. Shu ikki nazariyaning uyg'unlashtirilgan qo'llanilishi modelning fizik asoslanganligini ta'minlaydi.

$$\varepsilon_{11} = -z\alpha_1'(x) + a_1(z)\beta_1'(x) + \varepsilon^T(x,t), \quad \varepsilon_{12}=0, \quad \varepsilon_{13}=W'(x). \tag{3}$$

bunda $a_1(z)$ – kesim bo'ylab siljish taqsimotining shakl funksiyasi bo'lib, kesim balandligi h bo'yicha quyidagicha aniqlanadi [4]:

$$a_1(z) = \frac{3}{2}\int_{-h/2}^{h/2}\left(z-\frac{4z^3}{3h^2}\right). \tag{4}$$

Sterjen uchida eng issiq bo'lgan o'q bo'ylab koordinata ortishi bilan eksponensial ravishda pasayuvchi va vaqt bo'yicha statsionar holatga astalik bilan yaqinlashuvchi harorat maydoni quyidagi qonuniyat orqali olinadi:

$$T(x,t) = T_0 + (T_m - T_0)e^{-x/L_t}\left(1-e^{-t/\tau}\right). \tag{5}$$

bu yerda T_0 – boshlang'ich (muhit) harorati, T_m – sterjen uchining maksimal qizdirilish harorati, L_t – issiqlik tarqalish uzunligi (harorat maydonining o'q bo'ylab so'nish masshtabi), τ – issiqlik o'tish jarayonining vaqt doimiysi.

Harorat ko'tarilishi yuqorida ta'kidlanganidek, elastik modulni pasaytiradi – bu bog'lanish quyi chegara bilan cheklangan chiziqli qonun orqali ifodalanadi – hamda Dyugamel-Neyman nazariyasiga muvofiq qo'shimcha issiqlik (termik) deformatsiyasini hosil qiladi va harorat oshishi bilan chiziqli ravishda ortadi:

$$E(x,t) = E_0\max\big[1-k_E\big(T(x,t)-T_0\big),\,E_{\min}\big], \qquad \varepsilon^T(x,t) = \alpha_T\big(T(x,t)-T_0\big). \tag{6}$$

bunda E_0 – boshlang'ich elastik modul, k_E – haroratga sezgirlik koeffitsienti, $E_{\min}$ – quyi chegara, α_T – chiziqli issiqlik kengayish koeffitsienti.

Sterjenning egilishi, siljishi va buralishi natijasidagi ko'chish maydoni esa quyidagi ko'rinishda ifodalanadi:

$$u_1 = z\alpha_1(x) + a_1(z)\beta_1(x), \quad u_2=0, \quad u_3=W(x). \tag{7}$$

Bu yerda u_1, u_2, u_3 – ko'chish vektorining tashkil etuvchilari, $\alpha_1(x)$ va $\beta_1(x)$ – yuqorida kiritilgan egilish va siljish burchaklari, $W(x)$ – ko'ndalang ko'chish funksiyasi. Shu (7)-ifoda va (2)-ifodadagi elastik-plastik bog'lanishdan foydalangan holda, sterjenning kinetik va potensial

energiyasi, shuningdek tashqi kuchlarning bajargan ishi asosida Lagranj variatsion prinsipi yordamida muvozanat tenglamalari sistemasi keltirib chiqarildi.

Olingan sistema chekli ayirmalar va progonka (haydash) usullariga asoslangan algoritm yordamida *Python* dasturlash tilida yechildi.

Shunday qilib, harorat ta'siri hisoblash modelida ikkita yo'l orqali hisobga olinadi: birinchidan, materialning elastik qattiqligining (modulining) haroratga bog'liq holda pasayishi orqali (6)-ifoda, ikkinchidan, termik deformatsiya tashkil etuvchisi orqali. Natijada harorat sterjenning umumiy kuchlanish-deformatsiya holatiga hamda uning elastik-plastik deformatsiya darajasiga bevosita ta'sir ko'rsatadi.

HISOBLASH NATIJALARI VA ULARNING TAHLILI

Modelning samaradorligini baholash maqsadida sterjenning turli harorat va mexanik yuklanish sharoitlaridagi kuchlanish-deformatsiya holati sonli jihatdan tadqiq qilindi. Ikkita holat qiyoslandi: harorat ta'siri hisobga olinmagan boshlang'ich holat ($t = 0$) va sterjen uchining $T_{\max} = 200°\mathrm{C}$ haroratgacha qizdirilishi natijasidagi termomexanik holat ($t = 0{,}1$). Olingan natijalar sterjenning o'rta kesimida ($x = l/2$) aniqlanib, 1-jadvalda keltirilgan.

1-jadval. Sterjenning o'rta kesimidagi hisoblash natijalari

Holat	**Iter. soni**	**Plastik zona, %**	W^{el}	W^{pl}	α_1^{el}	α_1^{pl}	β_1^{el}	β_1^{pl}
$t = 0$	8	9,3	0,000830	0,000758	0,001301	0,001200	0,00000115	0,00000107
$t = 0{,}1$ ($T \to 200°\mathrm{C}$)	10	13,8	0,000892	0,000806	0,001376	0,0101255	0,00000119	0,00000110

Jadvaldagi natijalar bir qator qonuniyatlarni ko'rsatadi. Iteratsiyalar sonining 8 dan 10 gacha ortishi harorat ta'sirida materialning nochiziqlilik kuchayganini bildiradi, ammo algoritm baribir barqaror va tez (≤ 10 iteratsiya) yakunlanadi. Plastik zona ulushi $t = 0$ holatda 9,3% dan $t = 0{,}1$ ($T \to 200°\mathrm{C}$) holatda 13,8% gacha, ya'ni 1,5 barobar ortadi – bu (6)-ifodaga muvofiq harorat ortishi bilan $E(x,t)$ pasayib, oqish chegarasiga yetish uchun zarur kuchlanish kamayishi bilan izohlanadi. Elastik yechim (W^{el}) har doim elastik-plastik yechimdan (W^{pl}) katta chiqadi, chunki elastik hisob kesimning plastik yumshashini inobatga olmaydi; bu farq harorat bilan ortib, mutlaq ayirma $t = 0{,}1$ holatda 20% ga kattalashadi (0,000072→0,000086 mm) – demak, yuqori haroratda faqat elastik modelga tayanish aniqlikni pasaytiradi. α_1 burchaklari kutilganidek, mahkamlangan uchdan uzoqda elastik qiymat plastikdan katta bo'ladi; ammo β_1 buning aksini ko'rsatadi – mahkamlangan uch yaqinida, ayniqsa $t = 0{,}1$ da ($\alpha_1^{pl} = 0{,}0101255 \gg \beta_1^{el} = 0{,}00000119$), elastik-plastik qiymat keskin ortadi, chunki bu zonada kuchlanish konsentratsiyasi kuchli va material tezroq oqish rejimiga o'tadi. Xulosa qilib, harorat ta'siri aynan chegaraviy shartga yaqin zonalarda eng sezilarli namoyon bo'ladi.

XULOSA

Fazoviy sterjenning reduksiyalangan umumlashgan noma'lumlari asosida qurilgan model statsionar bo'lmagan harorat maydoni, haroratga bog'liq elastik modul va termik deformatsiyani o'z ichiga oladigan tarzda kengaytirildi. Kuchlanish-deformatsiya bog'lanishi Ilyushinning o'zgaruvchan elastik parametrlar usuli va Moskvitinning o'zgaruvchan yuklanish nazariyasi asosida, Dyugamel-Neyman termoelastiklik qonuni bilan birgalikda qurildi. Chekli ayirmalar va haydash usuliga asoslangan sonli algoritm amalga oshirildi, uning yaqinlashishi tasdiqlandi.

Natijalar shuni ko'rsatdiki, sterjen uchining 200°C gacha qizdirilishi plastik zona ulushini 9,3% dan 13,8% gacha oshiradi hamda elastik va elastik-plastik yechimlar orasidagi farqni, ayniqsa mahkamlangan uch atrofidagi siljish burchaklari bo'yicha, sezilarli kattalashtiradi – bu harorat ta'sirini loyihalash hisoblarida inobatga olish zarurligini tasdiqlaydi.

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COMPARISON OF MOMENT APPROXIMATIONS WITH A DIRECT NUMERICAL METHOD FOR A KINETIC HALF-SPACE PROBLEM

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The study of kinetic equations is one of the important directions in modern mathematical physics and is closely related to the analysis of transport processes and boundary-value problems. In this work, we consider a kinetic boundary-value problem and compare the results obtained by the Maxwell and half-moment methods with those obtained by a direct computational approach.

Consider the following kinetic boundary-value problem:

$$\vartheta \frac{\partial \varphi}{\partial x} + \varphi = \widetilde{Q}(\rho, q, S).$$

Here, the source term is defined by

$$\widetilde{Q} = \left(\rho + \vartheta q + \frac{1}{2}(\vartheta^2 - 1)(S - \rho) \right) M(\vartheta),$$

where ρ, q and S denote the corresponding macroscopic moments, and $M(\vartheta)$ is the Maxwellian distribution.

The boundary condition is given by

$$\varphi(0, \vartheta) = k(\vartheta), \qquad \vartheta \geq 0.$$

Since the original problem is posed in the half-space, for numerical computation we replace the infinite spatial domain by the finite interval

$$0 \leq x \leq L.$$

At the artificial boundary $x = L$, we impose the symmetry condition

$$\varphi(L,\vartheta)=\varphi(L,-\vartheta),\qquad \vartheta_{\min}<\vartheta<0.$$

For the numerical treatment of the problem, we consider the direct computation as an independent reference procedure. In contrast to the Maxwell and half-moment methods, the direct approach is applied to the original kinetic equation without reducing the problem to a finite system of macroscopic moments.

The spatial domain is truncated to the finite interval $0\le x\le L$, while the velocity variable is discretized on a suitable finite interval. The resulting discrete problem is used to compute the distribution function $\varphi(x,\vartheta)$ and the corresponding macroscopic quantities

$$\rho(x)=\int_{\mathbb{R}}\varphi(x,\vartheta)M(\vartheta)\,d\vartheta,$$

$$q(x)=\int_{\mathbb{R}}\vartheta\varphi(x,\vartheta)M(\vartheta)\,d\vartheta,$$

$$S(x)=\int_{\mathbb{R}}(\vartheta^2-1)\varphi(x,\vartheta)M(\vartheta)\,d\vartheta.$$

The obtained numerical solution is compared with the solutions constructed by the Maxwell and half-moment methods. Such a comparison makes it possible to evaluate the accuracy of the moment approximations and to determine their agreement with the direct numerical computation. The comparison is performed for the same boundary conditions and the same computational parameters.

Numerical computation.

To verify the accuracy of the direct numerical approach, we consider the following half-space problem:

$$\vartheta\frac{\partial f}{\partial x}=-\left[f-\left(\rho+\vartheta q+\frac{1}{2}(\vartheta^2-1)(S-\rho)\right)M(\vartheta)\right].$$

with

$$q=\int_{\mathbb{R}}\vartheta f\,d\vartheta=0,\qquad f(0,\vartheta)=\vartheta(\vartheta^2-a^2)M(\vartheta),\qquad \vartheta>0.$$

For the numerical computation, the velocity interval $[-5,5]$ is divided into $M=350$ grid points. The spatial discretization is chosen as

$$\Delta x=\frac{L}{N_x},\qquad N_x=500,\qquad L=5.$$

The direct numerical computation gives the asymptotic values

$$\rho_\infty=-2.4156055,\qquad S_\infty=1.71757512.$$

Maxwell approximation.

For comparison, the Maxwell approximation gives the following relations:

$$\frac{\rho_\infty+S_\infty}{2\sqrt{2\pi}}=0,\qquad \frac{S_\infty}{2}=\frac{2}{\sqrt{2\pi}}.$$

Hence,

$$S_\infty = \frac{4}{\sqrt{2\pi}}, \qquad \rho_\infty = -S_\infty = -\frac{4}{\sqrt{2\pi}}.$$

Consequently,

$$\rho_\infty \approx -1.596173769, \qquad S_\infty \approx 1.596173769.$$

Half-moment approximation.

The half-moment approximation leads to the following system:

$$\left(\frac{\gamma\beta-\alpha}{\alpha\gamma} - \lambda - \sqrt{2\pi}\right)\rho_\infty + \left(\frac{\gamma\beta-\alpha}{\alpha\gamma} - \lambda\right) S_\infty = 2,$$

$$\left(\sqrt{2\pi} + \frac{\lambda\alpha}{\gamma}\right) S_\infty + \frac{\lambda\alpha}{\gamma}\rho_\infty = 4.$$

Solving this system gives

$$\rho_\infty = \frac{2(\lambda\alpha+2)(2\gamma+\alpha)}{\sqrt{2\pi}\,(2\gamma+\alpha+\lambda\alpha(\alpha+\gamma))} \approx -2.234254296,$$

and

$$S_\infty = \frac{2\left(4\gamma+2\alpha-\sqrt{2\pi}\alpha\gamma+\lambda\alpha(2\gamma+\alpha)\right)}{\sqrt{2\pi}\,(2\gamma+\alpha+\lambda\alpha(\alpha+\gamma))} \approx 1.687754247.$$

For clarity, the asymptotic values obtained by the three approaches are summarized below:

Method	$\boldsymbol{\rho_\infty}$	$\boldsymbol{S_\infty}$
Direct computation	-2.4156055	1.71757512
Maxwell approximation	-1.596173769	1.596173769
Half-moment approximation	-2.234254296	1.687754247

Table 1. Comparison of the asymptotic quantities.

The numerical results clearly show that the half-moment approximation is closer to the direct computation than the Maxwell approximation. For the asymptotic density ρ_∞, the direct computation gives -2.4156055, whereas the half-moment method gives -2.234254296. In contrast, the Maxwell approximation gives -1.596173769, which differs considerably from the direct result.

For S_∞, the direct numerical value is 1.71757512. The half-moment approximation gives 1.687754247, while the Maxwell approximation gives 1.596173769. Thus, both moment approximations provide values of the correct magnitude, but the half-moment result is again closer to the direct numerical computation.

Conclusion.

The direct numerical computation provides a reference solution for comparing the Maxwell and half-moment approximations. The obtained results show that the half-moment approximation gives a more accurate value of the asymptotic density ρ_∞ than the Maxwell approximation. For the asymptotic quantity S_∞, both approximations remain relatively close to the direct numerical result. Therefore, the half-moment approach provides a better approximation of the considered kinetic half-space problem.

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IKKINCHI TUR FREDGOLM INTEGRAL TENGLAMASINI UMUMIY YADRO HOLIDA POLINOMIAL VA CHEBYSHEV BAZISLARI YORDAMIDA TAQRIBIY YECHISHNING QIYOSIY TAHLILI

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Annotatsiya. Ushbu maqolada 2-tur Fredgolm integral tenglamasining umumlashtirilgan modeli uchun ikkita yaqinlashish bazisi - oddiy polinomial bazis va Chebyshev bazisi qiyosiy tahlil qilinadi. Integral operatorning degeneratsiyalangan yadrosidan foydalanib, integral qism analitik ko'rinishga keltiriladi. Qoldiq funksiyasi asosida eng kichik kvadratlar mezoni qurilib, umumiy normal tenglamalar sistemasi hosil qilinadi. Test misollar uchun koeffitsiyentlar, qoldiq va xatoliklar hisoblanadi. Tahlil shuni ko'rsatadiki, mazkur bazislar bir xil N - darajadan oshmaydigan polinomal fazoni hosil qilganligi sababli ideal arifmetikada ularning approksimatsion imkoniyati bir xil, farq asosan koeffitsiyentlarning parametrlanishi va sonli barqarorlikda namoyon bo'ladi.

Kalit so'zlar: Fredgolm integral tenglama, integral operator, degeneratsiyalangan yadro, polinomial bazis, Chebyshev polinomlari, eng kichik kvadratlar usuli, qoldiq, normal tenglamalar, sonli barqarorlik.

Kirish

Integral tenglamalarni sonli yechish masalalarida noma'lum funksiyani chekli o'lchamli funksiyanal fazodagi bazis orqali approksimatsiyalash muhim bosqich hisoblanadi. Bazisni to'g'ri tanlash koeffitsientlarni hisoblashning qulayligi, algebrayik sistemaning shartlanganligi va sonli

xatoliklarning tarqalishiga bevosita ta'sir qiladi. Mazkur ishda quyidagi 2-tur Fredgolm integral tenglamasi qaraladi:

$$u(x) - \int_0^1 P(x)y^m u(y)\,dy = f(x), \qquad 0 \le x \le 1. \tag{1}$$

Bu yerda

$$P_n(x) = \sum_{r=0}^{n} a_r x^r = a_0 + a_1 x + \cdots + a_n x^n, \qquad a_n \neq 0. \tag{2}$$

$K(x,y) = P(x)y^m$ separabelli yadro bo'lib, integral operatorning rangi bitta funksionalga tushiriladi. Shu sababli (1) tenglama

$$u(x) - P_n(x)\int_0^1 y^m u(y)\,dy = f(x) \tag{3}$$

ko'rinishga keladi.

$$A = \int_0^1 y^m u(y)\,dy.$$

U holda

$$u(x) = f(x) + AP_n(x). \tag{4}$$

(4) ni y^m ga ko'paytirib integrallashdan

$$A = \int_0^1 y^m f(y)\,dy + A\int_0^1 y^m P_n(y)\,dy, \qquad \int_0^1 y^m P_n(y)\,dy = \sum_{r=0}^{n}\frac{a_r}{m+r+1}.$$

Demak,

$$A = \frac{\displaystyle\int_0^1 y^m f(y)\,dy}{\displaystyle 1 - \sum_{r=0}^{n}\frac{a_r}{m+r+1}}, \qquad u(x) = f(x) + P_n(x)A. \tag{5}$$

Taqribiy yechim umumiy holda

$$u_N(x) = \sum_{j=0}^{N} c_j \phi_j(x). \tag{6}$$

Oddiy polinomial bazis:

$$\phi_j^O(x) = x^j, \qquad u_N^O(x) = \sum_{j=0}^{N} c_j x^j. \tag{7}$$

Bu holda

$$A^O = \sum_{j=0}^{N}\frac{c_j}{m+j+1}, \qquad R_N^O(x) = \sum_{j=0}^{N} c_j\left(x^j - \frac{P(x)}{m+j+1}\right) - f(x).$$

Qoldiqning kvadratlar integralini minimallashtiramiz:

$$J(c) = \int_0^1 R^2(x)\,dx \longrightarrow \min. \tag{8}$$

$r_j(x) = (I - K)\phi_j(x)$ deb olsak,

$$r_j(x) = x^j - \frac{P(x)}{m + j + 1}.$$

Klassik Chebyshev polinomlari:

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \quad n \geq 2. \tag{9}$$

Approksimatsiya:

$$u_N^C(x) = \sum_{j=0}^{N} d_j T_j(x).$$

Siljitilgan Chebyshev bazisi $[0, 1]$ intervali uchun:

$$\widetilde{T}_j(x) = T_j(2x - 1), \qquad u_N^S(x) = \sum_{j=0}^{N} e_j \widetilde{T}_j(x). \tag{10}$$

Normal tenglamalar sistemasi:

$$\sum_{j=0}^{N} A_{ij} c_j = F_i, \qquad A_{ij} = \int_0^1 r_i(x) r_j(x)\, dx, \qquad F_i = \int_0^1 f(x) r_i(x)\, dx.$$

Tajriba natijalari

Ushbu ishda

$$P_1(x) = x, \qquad P_2(x) = 1 + x, \qquad P_3(x) = 1 + x + x^2, \qquad K_{ij}(x, y) = P_i(x) y^{m_j}, \quad i, j = 1, 2, 3$$

tanlanib, jami 9 ta yadro qaraladi.

1-jadval. Yadro kombinatsiyalari jadvali

Ko'phad hadi	$m = 1$	$m = 2$	$m = 3$
$P_1(x) = x$	xy	xy^2	xy^3
$P_2(x) = 1 + x$	$(1 + x)y$	$(1 + x)y^2$	$(1 + x)y^3$
$P_3(x) = 1 + x + x^2$	$(1 + x + x^2)y$	$(1 + x + x^2)y^2$	$(1 + x + x^2)y^3$

Aniq yechim $u(x) = e^x$ ko'rinishida tanlanadi.

Test 1: $u(x) - \int_0^1 xyu(y)\, dy = e^x - x.$

Test 2: $u(x) - \int_0^1 (1 + x)y^2 u(y)\, dy = e^x - (e - 2)(1 + x).$

Test 3: $u(x) - \int_0^1 (1 + x + x^2)y^3 u(y)\, dy = e^x - (6 - 2e)(1 + x + x^2).$

Ushbu 3 ta bazis N-darajadan oshmaydigan polinomlar fazosini hosil qiladi:

$$\text{span}\{1, x, \ldots, x^N\} = \text{span}\{T_0, \ldots, T_N\} = \text{span}\{\widetilde{T}_0, \ldots, \widetilde{T}_N\} = P_N.$$

Shu sababli ularning nazariy approksimatsion imkoniyati bir xil bo'lib, farq asosan koordinata parametrizatsiyasi va sonli shartlanganlikda namoyon bo'ladi.

2-jadval. Uchta bazis yordamida Test1, Test2 va Test3 uchun taqribiy yechim natijalari

x	$u(x)=e^x$	$u_3^{(O,C,S)}(x)$	$e_3^{(O,C,S)}(x)$
0.00	1.000000000	0.9990600540	$+9.3994600\cdot 10^{-4}$
0.25	1.2840254167	1.2843164960	$-2.9107934\cdot 10^{-4}$
0.50	1.6487212707	1.6483498304	$+3.7144031\cdot 10^{-4}$
0.75	2.1170000166	2.1172811618	$-2.8114519\cdot 10^{-4}$
1.00	2.7182818285	2.7172315950	$+1.0502335\cdot 10^{-3}$

Shunday qilib, uchala bazis uchun $N=3$ bo'lganda approksimatsion fazo bir xil bo'lib, ideal arifmetikada eng kichik kvadratlar yechimi ham bir xil funksiyani ifodalaydi. Biroq siljitilgan Chebyshev bazisining asosiy afzalligi $[0,1]$ intervaliga tabiiy moslashuvi va algebraik sistemaning sonli shartlanganligi bilan bog'liq bo'lishi mumkin. Shu sababli bazislarning yakuniy qiyosiy bahosi faqat $e_N(x)$ qiymatlari orqali emas, balki $\kappa_2(A)$ ko'rsatkichi orqali ham amalga oshirilishi lozim.

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Extension of the Euler-Maclaurin Quadrature Formula in Hilbert Space

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Consider the following quadrature formula

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N}\Big(C_0\,[\beta]\varphi(h\beta)+C_1\,[\beta]\varphi'(h\beta)+C_3[\beta]\varphi'''(h\beta)+C_4[\beta]\varphi^{(4)}(h\beta)\Big) \tag{1}$$

where

$$\begin{array}{lcl} C_0[0] & = & \frac{h}{2}, \\ C_0[\beta] & = & h,\ \beta=1,2,...,N-1, \\ C_0[N] & = & \frac{h}{2}, \end{array} \tag{2}$$

$$\begin{array}{lcl} C_1[0] & = & \frac{h^2}{12}, \\ C_1[\beta] & = & 0,\ \beta = 1,2,...,N-1, \\ C_1[N] & = & -\frac{h^2}{12}, \end{array} \tag{3}$$

and

$$\begin{array}{lcl} C_3[0] & = & -\frac{h^4}{720}, \\ C_3[\beta] & = & 0,\ \beta = 1,2,...,N-1, \\ C_3[N] & = & \frac{h^4}{720}, \end{array} \tag{4}$$

$C_4[\beta]$ are the unknown coefficients of the quadrature formula (1), $h = \frac{1}{N}$, and N is a natural number.

Assume that the integrand functions φ belong to the space $W_2^{(5,4)}$, where $W_2^{(5,4)}$ is the class of all functions φ defined on $[0,1]$ that possess an absolutely continuous fourth derivative and whose fifth derivatives belong to $L_2(0,1)$. The class $W_2^{(5,4)}$ equipped with the inner product

$$\langle \varphi, \psi \rangle_{W_2^{(5,4)}} = \int_0^1 (\varphi^{(5)}(x) + \varphi^{(4)}(x))(\psi^{(5)}(x) + \psi^{(4)}(x))dx$$

is a Hilbert space if we identify functions that differ by a linear combination of 1, x, x^2, x^3, and e^{-x} (see, e.g., [1],[3]). The space $W_2^{(5,4)}$ is equipped with the corresponding norm

$$\|\varphi\|_{W_2^{(5,4)}} = \left[\int_0^1 (\varphi^{(5)}(x) + \varphi^{(4)}(x))^2 dx\right]^{1/2}.$$

The error of formula (1) is the following difference

$$(\ell, \varphi) = \int_0^1 \varphi(x)dx - \sum_{\beta=0}^{N} \Big(C_0[\beta]\varphi(h\beta) + C_1[\beta]\varphi'(h\beta) + C_3[\beta]\varphi'''(h\beta) + C_4[\beta]\varphi^{(4)}(h\beta)\Big) \tag{5}$$

and this defines the functional

$$\ell(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} \Big(C_0[\beta]\delta(x-h\beta) - C_1[\beta]\delta'(x-h\beta) - C_3[\beta]\delta'''(x-h\beta) + C_4[\beta]\delta^{(4)}(x-h\beta)\Big), \tag{6}$$

which is called the *error functional* of the quadrature formula (1), where $\varepsilon_{[0,1]}(x)$ is the indicator function of the interval $[0,1]$ and δ is the Dirac delta function.

The value of the error functional ℓ for the function φ is evaluated as $(\ell, \varphi) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx$ and ℓ is a linear functional in the space $W_2^{(5,4)*}$, where $W_2^{(5,4)*}$ is the dual space to the space $W_2^{(5,4)}$.

Since the error functional (6) is defined in the space $W_2^{(5,4)}$, it is necessary to impose the following conditions on the error functional ℓ:

$$(\ell, 1) := 1 - \sum_{\beta=0}^{N} C_0[\beta] = 0, \tag{7}$$

$$(\ell, x) := \frac{1}{2} - \sum_{\beta=0}^{N} (C_0[\beta](h\beta) + C_1[\beta]) = 0, \tag{8}$$

$$(\ell, x^2) := \frac{1}{3} - \sum_{\beta=0}^{N} \big(C_0[\beta](h\beta)^2 + 2\,C_1[\beta](h\beta)\big) = 0, \tag{9}$$

$$(\ell, x^3) := \frac{1}{4} - \sum_{\beta=0}^{N} \big(C_0[\beta](h\beta)^3 + 3\,C_1[\beta](h\beta)^2 + 6\,C_3[\beta]\big) = 0, \tag{10}$$

$$(\ell, e^{-x}) := \int_0^1 e^{-x}dx - \sum_{\beta=0}^{N} \Big(C_0[\beta]\,e^{-h\beta} - C_1[\beta]\,e^{-h\beta} - C_3[\beta]\,e^{-h\beta} + C_4[\beta]\,e^{-h\beta}\Big) = 0. \tag{11}$$

Equations (7)–(11) show that the quadrature formula (1) is exact for the functions 1, x, x^2, x^3, and e^{-x}. It can be seen that the coefficients $C_0[\beta]$, $C_1[\beta]$, and $C_3[\beta]$ defined by equalities (2), (3), and (4) satisfy equations (7), (8), (9), and (10). Therefore, for the unknown coefficients $C_4[\beta]$, $\beta = 0, 1, ..., N$, we only have equation (11).

The absolute value of the error (5) is bounded from above by the norm

$$\|\ell\|_{W_2^{(5,4)*}} = \sup_{\varphi, \|\varphi\|_{W_2^{(5,4)}} \neq 0} \frac{|(\ell, \varphi)|}{\|\varphi\|_{W_2^{(5,4)}}}$$

of the error functional ℓ as follows:

$$|(\ell, \varphi)| \leq \|\varphi\|_{W_2^{(5,4)}} \cdot \|\ell\|_{W_2^{(5,4)*}}.$$

Furthermore, it is clear from (5) that the norm of the error functional (6) depends on the coefficients $C_4[\beta]$.

Thus, to construct an *optimal quadrature formula* of the form (1) in the sense of Sard, we need to solve the following problem.

Problem 1. Find the minimum of the norm of the error functional (6) with respect to the coefficients $C_4[\beta]$, i.e.,

$$\left\|\mathring{\ell}\right\|_{W_2^{(5,4)*}} = \inf_{C_4[\beta]} \|\ell\|_{W_2^{(5,4)*}}. \tag{12}$$

The coefficients satisfying equality (12) are called the *optimal coefficients* and are denoted by $\mathring{C}_4[\beta]$, $\beta = 0, 1, ..., N$.

We obtain the main result of the present paper.

Theorem. *The coefficients of the optimal quadrature formula of the form (1) in the space* $W_2^{(5,4)}(0,1)$ *have the following form:*

$$\begin{aligned}
\mathring{C}_4[0] &= \tfrac{e^h-1}{e^h+1}\big(1 + \tfrac{h^2}{12} - \tfrac{h^4}{720}\big) - \tfrac{h}{2},\\
\mathring{C}_4[\beta] &= \tfrac{e^h-1}{e^h+1}\big(2 + \tfrac{h^2}{6} - \tfrac{h^4}{360}\big) - h, \quad \beta = 1, 2, ..., N-1,\\
\mathring{C}_4[N] &= \tfrac{e^h-1}{e^h+1}\big(1 + \tfrac{h^2}{12} - \tfrac{h^4}{720}\big) - \tfrac{h}{2}.
\end{aligned}$$

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KUCHLANISHLAR TA'SIRIDA YUZAGA KELADIGAN ELASTIK BOG'LIQ MASALALARNI SONLI MODELLASHTIRISH VA YECHISH

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Annotatsiy. Ushbu maqolada temperatura ta'sirini inobatga olgan holda parabolik-giperbolik holatdagi izotrop jismlarning deformatsiyalanish jarayonlarini matematik jihatdan tavsiflash masalalari ko'rib chiqilgan. Jarayonlarni ifodalovchi matematik model hamda unga mos hisoblash modellari ishlab chiqilgan. Tuzilgan matematik modellarni diskretlashtirish va tahlil qilish asosida hisoblash algoritmlari shakllantirilgan hamda ularning $C\sharp$ dasturlash tilidagi dasturiy ta'minoti yaratilgan. Ishlab chiqilgan dasturiy ta'minotdan foydalangan holda muayyan amaliy masalalar yechilgan va olingan natijalar tahlil qilingan.

Kalit so'zlar: parabolik, giperbolik, izotrop jism, deformatsiya, termoplastik jarayon, oshkor va oshkormas sxemalar, dinamik jarayon.

Kirish. Ma'lumki, $\Omega = \left\{a \le x \le b,\ t_0 \le t \le T^*\right\}$ mazkur sohada izotrop sterjen shaklidagi qattiq jismlarning temperatura ta'siridagi termoplastik dinamik holatini ifodalash masalalari o'zaro bog'langan chiziqsiz giperbolik turdagi harakat tenglamasi [4]

$$\frac{\partial^2 u}{\partial t^2} = K_{21}\frac{\partial}{\partial x}(K_{22}\frac{\partial u}{\partial x}) + K_{23}\frac{\partial u}{\partial x} + K_{24}u + K_{25}\frac{\partial T}{\partial x} + P \tag{1}$$

hamda parabolik turdagi issiqlik o'tkazuvchanlik tenglamasi

$$\frac{\partial T}{\partial t} = K_{11}\frac{\partial}{\partial x}(K_{12}\frac{\partial T}{\partial x}) + K_{13}\frac{\partial T}{\partial x} + K_{14}T + K_{15}\frac{\partial^2 u}{\partial x \partial t} + Q \tag{2}$$

orqali umumiy ko'rinishda ifodalanishi mumkin. Bu yerdagi K_{ij}, P va Q hadlar umuman olganda x, t, T va u o'zgaruvchilarning chiziqsiz funksiyalarini ifodalaydi. Ayrim xususiy holatlarda esa ushbu funksiyalar o'zgarmas koeffitsiyentlar ko'rinishida aniqlanishi mumkin [1-2]. Masalaning asosiy mazmuni (1)-(2) ko'rinishidagi giperbolo-parabolik tenglamalar sistemasini berilgan boshlang'ich

$$u|_{t=t_0} = \varphi(x), \qquad u_t|_{t=t_0} = \psi(x), \qquad T|_{t=t_0} = \eta_0(x) \tag{3}$$

va chegaraviy shartlar

$$u|_{x=a} = \chi_1(t), \qquad u|_{x=b} = \chi_2(t) \tag{4}$$

$$T|_{x=a} = \xi_1(t)\,, \qquad T|_{x=b} = \xi_2(t) \tag{5}$$

asosida sonli usulda yechish algoritmini ishlab chiqish, ushbu algoritm asosida dasturiy ta'minot yaratish hamda turli iteratsion sonli usullarning samaradorligini tadqiq etishdan iborat. Bunda

masala korrekt qo'yilgan deb qabul qilinadi. Mazkur masala umumiy holda ma'lum darajadagi chiziqsizliklarni ham qamrab olganligi sababli, uni sonli yechishda to'r usuliga asoslangan iteratsion yondashuvlardan foydalanish talab etiladi. Shu maqsadda, dastlab berilgan Ω soha Ox o'qi bo'yicha $h = (b-a)/(m-1)$ va Ot o'qi bo'yicha $\tau = (T^* - t_0)/(n-1)$ qadamlar bilan $x_i = a + (i-1)h$ va $t_j = t_0 + (j-1)\tau$ $(i = \overline{1,m};\ j = \overline{1,n})$ chiziqlar yordamida diskretlashtirib olinadi [3]. Masalada ishtirok etuvchi hadlarni chekli-ayirmali ifodalarga keltirish jarayonida tenglamalar tarkibidagi chiziqsiz funksiyalarni chiziqlashtirish zarurati paydo bo'ladi. Buning uchun Pikar yoki Nyuton usullariga asoslangan chiziqlashtirish yondashuvlaridan foydalanish mumkin. Odatda, tenglamalarda ishtirok etuvchi koeffitsiyentlarni chiziqsizlashtirishda vaqt bo'yicha avvalgi qatlamda olingan hisoblash natijalaridan yoki Pikar iteratsiya usulidan foydalanish mumkin. Q va P funksiyalarini chiziqlashtirishda esa bevosita Nyuton usulidan foydalanish maqsadga muvofiq hisoblanadi. Barcha $\left\{T_i^{j+1}; u_i^{j+2}\right\}$ juftliklar sistemasi ko'rinishidagi variantlar yuqorida berilgan (3)-(5) shartlar: $T_i^1 = \eta_0(x_i)$, $T_1^j = \xi_1(t_j)$, $T_m^j = \xi_2(t_j)$, $u_i^1 = \varphi(x_i)$, $\frac{u_i^2 - u_i^1}{\tau} = \psi(x_i)$, $u_1^j = \chi_1(t_j)$, $u_m^j = \chi_2(t_j)$ yordamida yechiladi. (1)–(5) masalada keltirilgan shartlar asosida

$$K_{11} = K_{12} = 1,$$

$K_{13} = K_{14} = 0$, $K_{15} = T^{n_1}$, $K_{21} = K_{22} = 1$, $K_{23} = K_{24} = 0$, $K_{25} = T^{n_2}$, $Q = T(T^{n_1-1}u - 2)$, $Q_T^{'} = n_1 T^{n_1-1}u - 2$, $P = T^{n_2+1}$, $P_u^{'} = 0$,$\chi_2(t) = \sin(1+t)$, $\xi_1(t) = e^{-t}$, $\xi_2(t) = e^{-(1+t)}$ qiymatlarida aniq yechimlari $T(x,t) = e^{-(x+t)}$ va $u(x,t) = \sin(x+t)$ ko'rinishda ma'lum bo'lgan hol uchun hisoblash eksperimentlari o'tkazildi. Ω sohani o'zgarmas qadamli diskret to'rga keltirish maqsadida a=0, b=1, $t_0 = 0$, $T^* = 1$, $m = n = 10$, $n_1 = 4$, $n_2 = 2$ va vaqt bo'yicha qatlamlar orasidagi chiziqlashtirish usullaridan foydalanishda iteratsion jarayonning yaqinlashishi uchun aniqlikni $\varepsilon = 0.001$ deb olindi. Nyuton usuli asosida chiziqlashtirish natijasida ($\mu_1 = \mu_2 = 1$) nisbiy xatoliklarning maksimal qiymati 1.58 dan oshmagan bo'lsa, Pikar usulida ($\mu_1 = \mu_2 = 0$) ushbu ko'rsatkich 2.08 dan yuqori bo'lmadi. Ushbu yondashuvda olingan natijalar Pikar usuli yordamida olingan natijalar bilan deyarli bir xil ekanligi kuzatildi. Shu bilan birga, Nyuton usuli asosida olingan natijalarning aniqligi yuqoriroq ekanligi aniqlandi. Shuningdek, aniq va taqribiy yechimlar o'rtasidagi absolyut hamda nisbiy xatoliklar qiymatlari ham taqdim etilgan.

Xulosa qilib aytganda, (1)–(5) ko'rinishida ifodalangan masalalarni sonli yechish uchun taklif etilgan algoritm, uning asosida ishlab chiqilgan dasturiy majmua hamda olingan hisoblash natijalari chiziqsizlikni hisobga oluvchi turli amaliy masalalarni yechishda samarali metodik asos sifatida qo'llanilishi mumkin. Ishlab chiqilgan yondashuv va dasturiy vositalar mazkur turdagi masalalarni tadqiq etish, sonli hisoblashlarni amalga oshirish hamda olingan natijalarni tahlil qilish uchun amaliy ahamiyat kasb etadi.

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SONLI USULLARNING QO‘LLANILISHIDAGI AYRIM MUAMMO VA YECHIMLAR

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Annotatsiya. Tabiiy jarayonlarni tavsiflovchi masalalarni yechishda ko‘pincha sonli usulni to‘g‘ri tanlash masala bo‘yicha olingan natijalarning to‘g‘riligini ta’minlaydi. Olingan yechimning aniqligi esa, albatta, o‘rganilayotgan jarayonning matematik modelining qanchalik adekvatligiga bog‘liq.

Tayanch iboralar: sonli usullar, differensial tenglama, sonli hisoblash, turg‘unlik, yaqinlashish, chegaraviy shart.

Kirish

Tabiiy jarayonlarni tadqiq qilishda ko‘pincha matematik modellashtirish usullariga murojaat qilinadi. Ma’lumki, matematik modellashtirish natijasida xususiy hosilali differensial tenglama yoki xususiy hosilali differensial tenglamalar sistemasini hosil bo‘ladi. Ko‘pchilik hollarda olingan differensial tenglamalar analitik usullar yordamida yechilmaydi va biz ularni kompyuter yordamida yechishga majbur bo‘lamiz. Kompyuter yordamida yechishda sonli usullarni to‘g‘ri tanlash masala yechimining asosiy qismini beradi.

Ushbu ishda xususiy hosilali differensial tenglamalarni sonli usullar yordamida yechishning ba’zi jihatlari, ya’ni qiyinchiliklari va ularning yechimlari ko‘rib chiqiladi.

Sonli usulni tanlash

Sonli usulni to‘g‘ri tanlash yechimning katta aniqlikda olinishini ta’minlab qolmasdan, balki mashina tiliga o‘tkazishni osonlashtiradi. Nostatsionar masalalarda shunday holatlar uchraydiki, ularda vaqt bo‘yicha bir o‘zgaruvchi tez, ikkinchisi esa juda sekin o‘zgaradi. Masalan, suyuqlikning kichik tezliklarida tub sirtida to‘lqin hosil bo‘lishi masalalarida suyuqlikning erkin sirti tez, tub sirti esa sekin o‘zgaradi. Bunday hollarda vaqt qadamini tanlash juda qiyin [1].

Ikki o‘lchovli va uch o‘lchovli masalalarda, agar masala imkon bersa, approksimatsiyalovchi funksiyalarni qo‘llash eng yaxshi usul hisoblanadi. Bu bizga hosilalarni qo‘lda hisoblash imkonini beradi. Agar chegaraviy shartlar davriy bo‘lsa, u holda approksimatsiyalovchi funksiyalarni tanlashning eng yaxshi usuli Furye qatorlari (trigonometrik qatorlar) hisoblanadi. Furye qatorlari teskari almashtirish bilan qo‘llanilganda juda qulay bo‘lib, hisob-kitoblarni sezilarli darajada kamaytirishga imkon beradi [2].

Sonli usulni tanlashda qo‘llaniladigan usulning turg‘unligi va yaqinlashishini hisobga olish kerak. Bu, ayniqsa, nostatsionar jarayonlarni ko‘rib chiqishda kerak. Sonli usulni tanlashning to‘g‘riligini tekshirish uchun bu qo‘llanilayotgan usulni aniq yechimi (test masalalari) mavjud bo‘lgan masalalarga qo‘llash va sonli usulning turg‘unligi hamda yaqinlashishi deb ataladigan

sifatlar bajarilishiga ishonch hosil qilish lozim. Test masalalari chiziqsiz hadlarga ega bo‘lishi maqsadga muvofiq.

Nostatsionar masalalarda vaqt bo‘yicha har bir qadamda chegaraviy shartlar, ya’ni Dirixle yoki Neyman sharti bajarilishi kerak. Bu yerda chegaraviy shartlarning chiziqli yoki nochiziqliligi muammo tug‘dirishi mumkin. Hamma vaqt ham bu muammoni muvaffaqiyatli yechib bo‘lavermaydi. Bu tadqiqotchidan mashaqqatli mehnat talab qiladi. Barchaga ma’lumki, chegaraviy shartlar vaqt bo‘yicha (iteratsiya) har bir qadamda hisoblash sohasi chegaralarida bajarilishi kerak.

Sonli hisoblashdagi muammolar

Endi sonli hisoblashlardagi navbatdagi qiyinchiliklarni ko‘rib chiqamiz. Bu qiyinchiliklar mashina nollari bilan bog‘liq. Ko‘pincha sonli hisob yuritishda hisoblash jarayoni uzilib qoladi. Buni tuzatish uchun, birinchidan, mos dasturlash tilini to‘g‘ri tanlash va ikkinchidan, integrallash sohasida tenglamalarning har bir hadining xatti-harakatini tekshirish kerak. Shunday qilib, biz sonli hisoblash qayerda uzilib qolganini aniqlashimiz va uni tuzatishimiz mumkin.

Nostatsionar jarayonlar o‘rganilganda, sonli hisoblash yetarlicha uzoq davom etadi. Bunday hollarda sonli usulning barqarorligiga e’tibor qaratish kerak, chunki hisoblash jarayonida xatolarning o‘sishi ularning cheksiz to‘planishiga va yakuniy natijaning to‘g‘riligini kamaytirishga olib keladi, bu esa matematik modelning adekvatligiga ta’sir qiladi. Usulning barqarorligi kirishdagi kichik buzilishlar chiqishdagi kichik buzilishlarga olib kelishini ta’minlaydi, ya’ni sonli yechimning ishonchliligini ta’minlaydi.

Xulosa

Sonli usulni tanlashda masalaning xususiyatlari, chegaraviy shartlar, usulning turg‘unligi va yaqinlashishi hamda hisoblash jarayonida yuzaga kelishi mumkin bo‘lgan xatolarni hisobga olish muhimdir. Ayniqsa, nostatsionar masalalarda vaqt qadamini to‘g‘ri tanlash va har bir iteratsiyada chegaraviy shartlarning bajarilishini ta’minlash yechimning ishonchliligini oshiradi. Shuningdek, test masalalari orqali sonli usulni tekshirish va hisoblash jarayonida xatolarning to‘planishini nazorat qilish zarur.

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FIVE-ELECTRON FIRST DOUBLET STATE IN THE IMPURITY HUBBARD MODEL

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We consider the energy operator of five-electron systems in the Impurity Hubbard model and describe the structure of the essential spectra and discrete spectrum of the system for first doublet states in the $\nu-$ dimensional lattice. The Hamiltonian of the model has the form

$$H = A\sum_{m,\gamma} a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m,\gamma}a_{m+\tau,\gamma} + U\sum_{m} a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow} + (A_0 - A)\sum_{\gamma} a^+_{0,\gamma}a_{0,\gamma} +$$

$$+(B_0 - B)\sum_{\tau,\gamma}(a^+_{0,\gamma}a_{\tau,\gamma} + a^+_{\tau,\gamma}a_{0,\gamma}) + (U_0 - U)a^+_{0,\uparrow}a_{0,\uparrow}a^+_{0,\downarrow}a_{0,\downarrow}. \quad (1)$$

Here A (A_0) is the electron energy at a regular (impurity) lattice site and B (B_0) is the transfer integral between electrons (between an electron and impurities) at neighboring sites; we assume for convenience that $B > 0$. Further, $\tau = \pm e_j$, $j = 1, 2, ..., \nu$, where e_j are pairwise orthogonal unit vectors, the summation is taken over the nearest neighbors, U (U_0) is the parameter of the on-site Coulomb interaction of two electrons in the regular (impurity) sites, γ is the spin index, $\gamma =\uparrow$ or $\gamma =\downarrow$, where $\uparrow$ and $\downarrow$ denote the spin values $\frac{1}{2}$ and $-\frac{1}{2}$, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $z \in Z^\nu$.

Hamiltonian (1) commutes with all components of the total spin operator $S = (S^+, S^-, S^z)$, and the structure of eigenfunctions and eigenvalues of the system therefore depends on S. The Hamiltonian H acts in the antisymmetric Fo'ck space $\mathcal{H}_{as}$.

Let φ_0 be the vacuum vector in the space $\mathcal{H}_{as}$. The first doublet state corresponds to the free motion of five electrons and their interactions over the lattice with the basic functions ${}^1d^{1/2}_{p,q,r,t,k\in Z^\nu} = a^+_{p,\downarrow}a^+_{q,\downarrow}a^+_{r,\uparrow}a^+_{t,\uparrow}a^+_{k,\uparrow}\varphi_0, p, q, r, t, k, \in Z^\nu$. The subspace ${}^1\mathcal{H}^d_{1/2}$ corresponding to the first doublet state is the set of all vectors of the form ${}^1\psi^d_{1/2}\sum_{p,q,r,t,k\in Z^\nu} f(p,q,r,t,k){}^1d^{1/2}_{p,q,r,t,k\in Z^\nu}, f \in l^{as}_2$, where l^{as}_2 is the subspace of antisymmetric functions in the space $l_2((Z^\nu)^5)$. We let ${}^1H^d_{1/2}$ denote the restriction of operator H to the subspace ${}^1\mathcal{H}^d_{1/2}$. We set $\varepsilon_1 = A_0 - A$, $\varepsilon_2 = B_0 - B$ and $\varepsilon_3 = U_0 - U$.

Theorem 1. (coordinate representation of the action of operator ${}^1H^d_{1/2}$). The subspace ${}^1\mathcal{H}^d_{1/2}$ is invariant under the operator H, and the restriction ${}^1H^d_{1/2}$ of operator H to the subspace ${}^1\mathcal{H}^d_{1/2}$ is a bounded self-adjoint operator. It generates a bounded self-adjoint operator ${}^1\overline{H}^d_{1/2}$ acting in the space l^{as}_2 as

$${}^1\overline{H}^d_{1/2}{}^1\psi^d_{1/2} = 5Af(p,q,r,t,k) + B\sum_{\tau}[f(p+\tau,q,r,t,k) + f(p,q+\tau,r,t,k) + f(p,q,r+\tau,t,k) +$$

$$+f(p,q,r,t+\tau,k) + f(p,q,r,t,k+\tau)] + U[\delta_{p,r} + \delta_{q,r} + \delta_{p,t} + \delta_{p,t} + \delta_{q,t} + \delta_{q,k}]f(p,q,r,t,k) +$$

$$+\varepsilon_1[\delta_{p,0} + \delta_{q,0} + \delta_{r,0} + \delta_{t,0} + \delta_{k,0}]f(p,q,r,t,k) + \varepsilon_2\sum_{\tau}[\delta_{p,0}f(\tau,q,r,t,k) + \delta_{q,0}f(p,\tau,r,t,k) + \delta_{r,0}\times$$

$$\times f(p,q,\tau,t,k) + \delta_{t,0}f(p,q,r,\tau,k) + \delta_{k,0}f(p,q,r,t,\tau) + \delta_{p,\tau}f(0,q,r,t,k) + \delta_{q,\tau}f(p,0,r,t,k) + \delta_{r,\tau}\times$$

$$\times f(p,q,0,t,k) + \delta_{t,\tau}f(p,q,r,0,k) + \delta_{k,\tau}f(p,q,r,t,0)] + \varepsilon_3[\delta_{p,0}\delta_{r,0} + \delta_{p,0}\delta_{t,0} + \delta_{p,0}\delta_{k,0} + \delta_{p,0}\delta_{r,0} +$$

$$+\delta_{q,0}\delta_{r,0} + \delta_{q,0}\delta_{t,0} + \delta_{q,0}\delta_{t,0} + \delta_{q,0}\delta_{k,0}]f(p,q,r,t,k). \quad (2)$$

The operator ${}^1H^d_{1/2}$ acts on a vector ${}^1\psi^d_{1/2} \in {}^1\mathcal{H}^d_{1/2}$ as

$${}^1\psi^d_{1/2}{}^1\psi^d_{1/2} = \sum_{p,q,r,t,k\in Z^\nu}({}^1\overline{H}^d_{1/2}f)(p,q,r,t,k){}^1d^{1/2}_{p,q,r,t,k\in Z^\nu}. \quad (3)$$

Lemma 1. The spectra of the operators ${}^1H^d_{1/2}$ and ${}^1\overline{H}^d_{1/2}$ coincide.

Let $\mathcal{F} : l_2((Z^\nu)^5) \to L_2((T^\nu)^5) \equiv {}^1\widetilde{\mathcal{H}}^d_{1/2}$ be the Fourier transformation, where T^ν is the $\nu-$dimensional torus endowed with the normalized Lebesgue measure $d\lambda$, i.e., $\lambda(T^\nu) = 1$. We set ${}^1\widetilde{H}^d_{1/2} = \mathcal{F}\,{}^1\overline{H}^d_{1/2}\mathcal{F}^{-1}$.

Theorem 2. (quasimomentum representation of the action of operator ${}^1H^d_{1/2}$). The Fourier transform of operator ${}^1\overline{H}^d_{1/2}$ is an operator ${}^1\widetilde{H}^d_{1/2}$ acting in the space $L^{as}_2((T^\nu)^5)$ be the formula

$$
\begin{aligned}
{}^1\widetilde{H}^d_{1/2}\,{}^1\psi^d_{1/2} &= \{5A + 2B\sum_{i=1}^{\nu}[\cos\lambda_i + \cos\mu_i + \cos\gamma_i + \cos\theta_i + \cos\eta_i]\}f(\lambda,\mu,\gamma,\theta,\eta) + \\
&+U[\int_{T^\nu} f(s,\mu,\lambda+\gamma-s,\theta,\eta)ds + \int_{T^\nu} f(\lambda,t,\mu+\gamma-t,\theta,\eta)dt + \int_{T^\nu} f(s,\mu,\gamma,\lambda+\theta-s,\eta)ds + \\
&+\int_{T^\nu} f(\lambda,t,\gamma,\mu+\theta-t,\eta)dt + \int_{T^\nu} f(s,\mu,\gamma,\theta,\lambda+\eta-s)ds + \int_{T^\nu} f(\lambda,t,\gamma,\theta,\mu+\eta-t)dt] + \\
&+\varepsilon_1[\int_{T^\nu} f(s,\mu,\gamma,\theta,\eta)ds + \int_{T^\nu} f(\lambda,t,\gamma,\theta,\eta)dt + \int_{T^\nu} f(\lambda,\mu,l,\theta,\eta)dl + \int_{T^\nu} f(\lambda,\mu,\gamma,u,\eta)du + \\
&+\int_{T^\nu} f(\lambda,\mu,\gamma,\theta,v)dv] + 2\varepsilon_2[\int_{T^\nu}\sum_{i=1}^{\nu}[\cos\lambda_i + \cos s_i]f(s,\mu,\gamma,\theta,\eta)ds + \int_{T^\nu}\sum_{i=1}^{\nu}[\cos\mu_i + \cos t_i]\times \\
&\times f(\lambda,t,\gamma,\theta,\eta)dt + \int_{T^\nu}\sum_{i=1}^{\nu}[\cos\gamma_i + \cos l_i]f(\lambda,\mu,l,\theta,\eta)dl + \int_{T^\nu}\sum_{i=1}^{\nu}[\cos\theta_i + \cos u_i]f(\lambda,\mu,\gamma,u,\eta)du + \\
&+\int_{T^\nu}\sum_{i=1}^{\nu}[\cos\eta_i + \cos v_i]f(\lambda,\mu,\gamma,\theta,v)dv] + \varepsilon_3\int_{T^\nu}\int_{T^\nu}[f(s,\mu,l,\theta,\eta)dsdl + f(s,\mu,\gamma,u,\eta)dsdu + \\
&+f(s,\mu,\gamma,\theta,v)dsdv + f(\lambda,t,l,\theta,\eta)dtdl + f(\lambda,t,\gamma,u,\eta)dtdu + f(\lambda,t,\gamma,\theta,v)dtdv]. \qquad (4)
\end{aligned}
$$

In the impurity Hubbard model, the spectral properties of the energy operator of five-electron systems are closely related to those of its two-particle subsystems (one-electron systems with impurity). We first study the spectrum and localized impurity electron states of one-electron impurity systems.

The change of the energy spectrum of operator one-electron system in the impurity Hubbard model in the case $\nu = 1, 2, 3$ given in [1] (Theorem 9), and we therefore do not give it here. According to the theorem 9 from [1], the spectrum of operator one-electron system in the impurity Hubbard model consists of a continuous spectrum and at most two eigenvalues. Now, using the obtained results, we describe the structure of the essential and discrete spectrum of the energy operator of five-electron systems in the impurity Hubbard model in first doublet state on the one-, two- and three-dimensional lattice $Z^\nu, \nu = 1, 2, 3$.

Here and hereafter We denote $E = \frac{B+\varepsilon_2)^2}{\varepsilon_2^2+2B\varepsilon_2}$.

Theorem 3. Let $\nu = 1$. When then have the following results.

1). If $\varepsilon_2 = -B$ and $\varepsilon_1 < -2B$ (respectively, $\varepsilon_2 = -B$ and $\varepsilon_1 > 2B$), then the essential spectrum of the operator ${}^1\widetilde{H}^d_{1/2}$ is the union of twelve segments (5):

$$
\begin{aligned}
&\sigma_{ess}({}^1\widetilde{H}^d_{1/2}) = [5A-10B, 5A+10B]\cup[4A-8B+z, 4A+8B+z]\cup[3A-6B+2z, 3A+6B+2z]\cup[2A- \\
&-4B+3z, 2A+4B+3z]\cup[A-2B+4z, A+2B+4z]\cup[3A-6B+z_4, 3A+6B+z_4]\cup[2A-4B+z+z_4, 2A+
\end{aligned}
$$

$+4B+z+z_4]\cup[A-2B+2z+z_4, A+2B+2z+z_4]\cup[3A-6B+z_3, 3A+6B+z_3]\cup[2A-4B+z+z_3, 2A+$
$+4B+z+z_3]\cup[A-2B+2z+z_3, A+2B+2z+z_3]\cup[A-2B+z_3+z_4, A+2B+z_3+z_4]$.
The discrete spectrum of operator ${}^1\widetilde{H}^d_{1/2}$ consists of four eigenvalues,

$$\sigma_{disc}({}^1\widetilde{H}^d_{1/2}) = \{5z, 3z+z_3, 3z+z_4, z+z_3+z_4,\} \tag{6}$$

where $z = A+\varepsilon_1$ and z_3 and z_4 are the additional eigenvalues of operator ${}^1\widetilde{H}^d_{1/2}$.

2). If $\varepsilon_2 = -2B$ or $\varepsilon_2 = 0$ and $\varepsilon_1 < 0$ (respectively, $\varepsilon_2 = -2B$ or $\varepsilon_2 = 0$ and $\varepsilon_1 > 0$), then the essential spectrum of the operator ${}^1\widetilde{H}^d_{1/2}$ is the union of twelve segments (see formula (5)). The discrete spectrum of operator ${}^1\widetilde{H}^d_{1/2}$ consists of four eigenvalues (see formula (6)), where $z = A-\sqrt{4B^2+\varepsilon_1^2}$ (respectively, $z = A+\sqrt{4B^2+\varepsilon_1^2}$).

3). If $\varepsilon_1 = 0$ and $\varepsilon_2 > 0$ or $\varepsilon_1 = 0$ and $\varepsilon_2 < -2B$, then the essential spectrum of the operator ${}^1\widetilde{H}^d_{1/2}$ is the union of twenty eight segments:

$\sigma_{ess}({}^1\widetilde{H}^d_{1/2}) = [5A-10B, 5A+10B]\cup[4A-8B+z_1, 4A+8B+z_1]\cup[4A-8B+z_2, 4A+8B+z_2]\cup[3A-6B+$
$+2z_1, 3A+6B+2z_1]\cup[3A-6B+2z_2, 3A+6B+2z_2]\cup[3A-3B+z_1+z_2, 3A+3B+z_1+z_2]\cup[2A-4B+3z_1,$
$2A+4B+3z_1]\cup[2A-4B+3z_2, 2A+4B+3z_2]\cup[3A-6B+z_4, 3A+6B+z_4]\cup[2A-4B+z_1+z_4, 2A+$
$+4B+z_1+z_4]\cup[2A-4B+z_2+z_4, 2A+4B+z_2+z_4]\cup[A-2B+4z_1, A+2B+4z_1]\cup[A-2B+4z_2, A+2B+4z_2]\cup$
$\cup[A-2B+z_1+3z_2, A+2B+z_1+3z_2]\cup[A-2B+2z_1+2z_2, A+2B+2z_1+2z_2]\cup[A-2B+3z_1+z_2, A+2B+3z_1+$
$+z_2]\cup[A-2B+2z_1+z_4, A+2B+2z_1+z_4]\cup[A-2B+z_1+z_2+z_4, A+2B+z_1+z_2+z_4]\cup[2A-4B+z_2+z_3, 2A+$
$+4B+z_2+z_3]\cup[A-2B+z_1+z_2+z_3, A+2B+z_1+z_2+z_3]\cup[A-2B+2z_2+z_3, A+2B+2z_2+z_3]\cup[3A-6B+z_3,$
$3A+6B+z_3]\cup[2A-4B+z_1+z_3, 2A+4B+z_1+z_3]\cup[A-2B+2z_1+z_3, A+2B+2z_1+z_3]\cup[A-2B+2z_2+z_4, A+$
$+2B+2z_2+z_4]\cup[A-2B+z_3+z_4, A+2B+z_3+z_4]$. The discrete spectrum of operator ${}^1\widetilde{H}^d_{1/2}$ consists of sixteen eigenvalues,

$\sigma_{disc}({}^1\widetilde{H}^d_{1/2}) = \{5z_1, 4z_1+z_2, z_1+4z_2, 5z_2, 3z_1+2z_2, 2z_1+3z_2, 3z_1+z_4, 2z_1+z_2+z_4, z_1+2z_2+z_4, 3z_1+z_3, 3z_2+$
$+z_4, z_1+2z_2+z_3, 2z_1+z_2+z_3, 3z_2+z_3, z_1+z_3+z_4, z_2+z_3+z_4\}$, where $z_1 = A-\frac{2BE}{\sqrt{E^2-1}}$, $z_2 = A+\frac{2BE}{\sqrt{E^2-1}}$.

4). If $\varepsilon_1 > 0$ and $-2B < \varepsilon_2 < 0$, then the essential spectrum of the operator ${}^1\widetilde{H}^d_{1/2}$ is the union of four segments, and the discrete spectrum of the operator ${}^1\widetilde{H}^d_{1/2}$ is the empty set, $\sigma_{disc}({}^1\widetilde{H}^d_{1/2} = \emptyset$.

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MATEMATIK MAYATNIK TEBRANISH DAVRINING AMPLITUDAGA BOG‘LIQLIGINI MAPLE’DA TAHLIL QILISH

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Chiziqsiz differensial tenglamalar fizika, mexanika, biologiya va muhandislikdagi ko‘plab real jarayonlarni matematik modellashtirishda muhim vosita hisoblanadi. Bunday modellar uchun analitik yechimni yopiq ko‘rinishda olish ko‘pincha murakkab bo‘lib, ayniqsa kichik parametr, kuchli chiziqsizlik, qattiqlik va xaotik dinamika mavjud bo‘lgan hollarda sonli hisoblash zarur bo‘ladi. Shu sababli analitik va sonli yondashuvlarni yagona hisoblash muhitida birlashtirish amaliy jihatdan muhimdir. Ushbu tezisda Maple kompyuter algebra tizimida chiziqsiz modellarni Puankare-Lindshted, ko‘p vaqt shkalasi va Lie guruh usullari asosida analitik tadqiq etish hamda Runge–Kutta, RKF45 va BDF usullari yordamida sonli hisoblash imkoniyatlari ko‘rib chiqiladi. Natijalar analitik yaqinlashuvlarni sonli yechimlar bilan taqqoslash, xatolikni baholash va dinamik xususiyatlarni aniqlash imkonini beradi [1-3].

Ma’lumki, kichik parametrli chiziqsiz model umumiy holda

$$L[x] + \varepsilon N[x] = 0, \qquad 0 < \varepsilon \ll 1, \tag{1}$$

ko‘rinishida qaraladi. Bunda Puankare–Lindshted usulida yechim va chastota parametr bo‘yicha qatorga yoyiladi. Umumiy xolda Duffing tipidagi

$$\ddot{x} + x + \varepsilon x^3 = 0, \qquad x(0) = A, \quad \dot{x}(0) = 0 \tag{2}$$

uchun Maple‘da yechim qatori avtomatik tuziladi va sekulyar hadlarni yo‘qotish sharti orqali chastota aniqlanadi. Birinchi yaqinlashuvda

$$x(t) = A\cos(\omega t) - \frac{\varepsilon A^3}{32}\cos(3\omega t) + O(\varepsilon^2), \tag{3}$$

$$\omega = 1 + \frac{3\varepsilon A^2}{8} - \frac{21\varepsilon^2 A^4}{256} + O(\varepsilon^3) \tag{4}$$

olinadi. Bu natija chiziqsizlik sababli tebranish chastotasining amplitudaga bog‘liqligini ko‘rsatadi.

Mazkur hisoblashni Maple‘da avtomatlashtirish uchun ‘series‘, ‘coeff‘, ‘solve‘ va ‘dsolve‘ funksiyalaridan foydalanish mumkin. Masalan, asosiy algoritmning qisqa ko‘rinishi quyidagicha beriladi:

Ko‘p vaqt shkalasi usulida esa $T_0 = t$, $T_1 = \varepsilon t$ kabi mustaqil o‘zgaruvchilar kiritilib, hosila zanjir qoidasi asosida yoyiladi:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + O(\varepsilon^2). \tag{5}$$

Van der Pol tenglamasi uchun sekulyar hadlarni yo‘qotish sharti amplituda tenglamasiga olib keladi:

$$\frac{da}{dT_1} = \frac{a}{2}\left(1 - \frac{a^2}{4}\right). \tag{6}$$

Shundan $a = 2$ barqaror limit sikl amplitudasi ekanligi kelib chiqadi. Maple‘dagi sonli integratsiya bilan ushbu xulosa turli boshlang‘ich amplitudalar uchun tekshirilishi mumkin.

Lie guruh tahlilida "DEtools[symgen]"funksiyasi differensial tenglamaning simmetriya generatorlarini topishga yordam beradi. Masalan,

$$y'' + y^3 = 0 \tag{7}$$

uchun simmetriyalar aniqlanib, tartibni kamaytirish va birinchi integralni topish imkoniyati yaratiladi. Demak, Maple analitik usullarni faqat hisoblash emas, balki algoritmlashtirish va natijani tekshirish vositasi sifatida ham qo‘llash imkonini beradi.

Analitik yechim mavjud bo‘lmagan yoki yaqinlashish faqat cheklangan vaqt oralig‘ida ishonchli bo‘lgan hollarda sonli usullar muhim ahamiyat kasb etadi. Birinchi navbatda Runge–Kutta usullari qo‘llaniladi.

Ma’lumki klassik to‘rtinchi tartibli RK4 sxemasi

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{8}$$

bu yerda

$$k_1 = f(t_n, y_n), \quad k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right),$$

$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right), \quad k_4 = f(t_n + h, y_n + hk_3),$$

ko‘rinishida bo‘lib, xatoligi $O(h^4)$ tartiba bo‘ladi [2].

Maple‘da "dsolve(...,type=numeric,method=rkf45)"orqali moslashuvchi bosqichli RKF45 algoritmidan foydalanish mumkin. RKF45 bir vaqtning o‘zida to‘rtinchi va beshinchi tartibli yechimlarni hisoblab, ularning farqi orqali xatolikni baholaydi:

$$e_{n+1} = y_{n+1}^{(5)} - y_{n+1}^{(4)}. \tag{9}$$

Bosqich o‘lchami esa berilgan aniqlik talabiga mos ravishda avtomatik o‘zgartiriladi.

Chiziqsiz tenglamalarda oddiy ochiq Runge–Kutta usullari juda kichik qadam talab qilishi mumkin. Bunday masalalarda BDF kabi yopiq usullar samaraliroqdir. Umumiy BDF sxemasi

$$\sum_{k=0}^{s} \alpha_k y_{n-k} = h\beta_0 f(t_n, y_n) \tag{10}$$

ko‘rinishida bo‘lib, har bir qadamda chiziqsiz algebraik tenglamani yechish talab etiladi. Maple‘da ‘stiff=true‘ parametri yordamida qattiq tizimlar uchun mos algoritm tanlanadi. Bu yondashuv tez va sekin jarayonlar bir vaqtda mavjud bo‘lgan kimyoviy kinetika kabi modellar uchun qulaydir.

Sonli hisoblashning muhim jihati – barqarorlikni nazorat qilishdir. $y' = \lambda y$ test tenglamasi uchun usulning barqarorlik $R(z)$, $z = h\lambda$ funksiyasi orqali $|R(z)| \leq 1$ shart aniqlanadi. RK4 uchun

$$R(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24}.$$

Maple yordamida ushbu barqarorlik sohasini kompleks tekislikda vizualizatsiya qilish va tanlangan qadamning maqbulligini tekshirish mumkin.

Maple muhitining asosiy afzalligi turli usullar natijalarini bir xil parametrlar ostida bevosita taqqoslash imkoniyatidir. Duffing tenglamasi uchun Puankare–Lindshted yaqinlashuvi RKF45 sonli yechim bilan bir grafikda tasvirlanib, mutlaq xatolik

$$E(t) = |x_{an}(t) - x_{num}(t)| \tag{11}$$

ko‘rinishida hisoblanadi. Bunday qiyoslash analitik formuladan foydalanish mumkin bo‘lgan vaqt oralig‘ini aniqlashga xizmat qiladi. Kichik ε da analitik yaqinlashuv yetarlicha aniq bo‘lsa, vaqt oralig‘i yoki chiziqsizlik kuchayganda sonli usulning afzalligi ortadi.

Van der Pol tizimida esa sonli hisoblash fazodagi traektoriyasi va limit siklni aniqlash imkonini beradi. Turli boshlang‘ich shartlardan boshlangan trayektoriyalar $a = 2$ atrofidagi barqaror limit siklga yaqinlashishi ko‘p vaqt shkalasi natijasini tasdiqlaydi. Parametrning o‘zgarishini ketma-ket hisoblash orqali limit sikl amplitudasi va bifurkatsion xususiyatlarni ham tadqiq qilish mumkin. Poincare kesimi, Lyapunov ko‘rsatkichlari va fraktal o‘lcham kabi vositalar esa murakkab va xaotik dinamikaning sonli tavsifini beradi [3].

Tadqiqot natijalari shuni ko‘rsatadiki, Maple muhitida chiziqsiz modellarni analitik va sonli usullar bilan kompleks tadqiq etish yagona hisoblash zanjiri sifatida tashkil qilinishi mumkin. Puankare–Lindshted usuli kichik parametrli tebranishlarda chastota va yechimning yaqinlashuvini aniqlashga, ko‘p vaqt shkalasi usuli uzoq vaqtli amplituda evolyutsiyasini tavsiflashga, Lie guruh tahlili esa tenglama simmetriyalaridan foydalanib tartibni kamaytirishga imkon beradi. Bu usullarni ‘series‘, ‘dsolve‘, ‘symgen‘, ‘firint‘ kabi Maple vositalari bilan avtomatlashtirish hisoblash jarayonini sezilarli soddalashtiradi [1].

Sonli bosqichda RK4 va RKF45 umumiy chiziqsiz oddiy differensial tenglamalar uchun, BDF esa chiziqsiz tizimlar uchun samarali vosita sifatida qaraladi. RKF45 ning moslashuvchi qadam mexanizmi aniqlik va hisoblash xarajati o‘rtasida muvozanat yaratadi. BDF usuli esa katta chiziqsizlik nisbatiga ega tizimlarda barqaror hisoblashni ta‘minlaydi. Shu bilan birga, barqarorlik funksiyasini tekshirish, mutlaq xatolikni hisoblash va turli qadam o‘lchamlarida konvergensiyani solishtirish sonli natijalarning ishonchliligini baholash uchun zarurdir.

Amaliy hisoblash nuqtai nazaridan usul tanlash modelning tuzilishi va maqsadiga bog‘liq. Agar kichik parametr mavjud bo‘lib, yechimning analitik xususiyatlari muhim bo‘lsa, Puankare–Lindshted yoki ko‘p vaqt shkalasi usuli maqsadga muvofiqdir. Agar asosiy maqsad uzoq vaqtli dinamika, parametrik o‘zgarish yoki murakkab boshlang‘ich shartlarni o‘rganish bo‘lsa, RKF45 kabi moslashuvchi sonli usul afzal hisoblanadi. Chiziqsiz tizimlarda esa BDF yoki Rosenbrock tipidagi usullar barqarorlik talabini yaxshiroq qanoatlantiradi.

Shunday qilib, analitik va sonli usullarni Maple muhitida birlashtirish chiziqsiz differensial tenglamalarni tadqiq etishning samarali metodologiyasini shakllantiradi. Analitik yechim modelning matematik mazmunini ochib bersa, sonli hisoblash real parametrlar, uzoq vaqtli dinamika, barqarorlik va xaotik jarayonlarni chuqurroq o‘rganish imkonini beradi. Ushbu yondashuv matematik fizika, mexanika, muhandislik va boshqa amaliy sohalardagi chiziqsiz modellarni hisoblash va tahlil qilishda qo‘llanishi mumkin.

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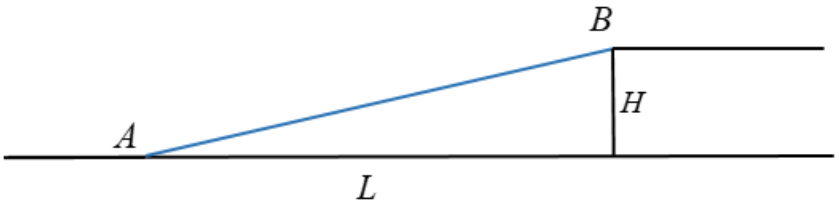
3. **Смирнов А.С., Булов С.А., Дегилевич Е.А.** *Построение и анализ нелинейных форм колебаний трехзвенного маятника асимптотическими методами.* Известия Саратовского университета. Новая серия. Серия: Математика. Механика. Информатика. — 2024. — Т. 24, №4. — С. 598–610.

UDK: 625.111 : 625.12

Temir yo‘l egriliklarining o‘zgarishida elastik deformatsiyasidan foydalanish

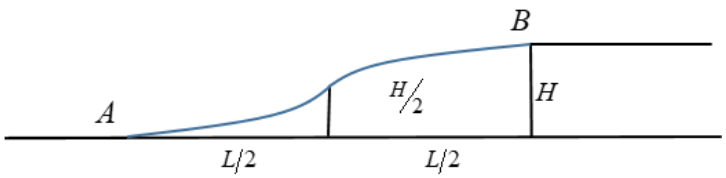
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Ma’lumki [1], temir yo‘l gorizontal tekislikka nisbatan ko‘tarilishini, texnik talablariga ko‘ra har bir kilometrda 240 mmdan katta bo‘lmasligi zarur. Ammo temir yo‘l geografik joylashishiga bog‘liq holda, bazan bir necha metrga ko‘tarilishi kerak bo‘ladi. Bu ko‘tarilish odatda quyida keltirilgan sxema shaklida ko‘rsatiladi:



Bu rasmdagi $L-$ masofada H balandlikka ko‘tarilishi ko‘rsatilgan.

Temir yo‘l izining matematik jihatdan silliq va texnik talablarga javob beruvchi bo‘lishi uchun uning grafigi ushbu shaklda bo‘lishi maqsadga muvofiq ekanligi ko‘rsatilgan [2].



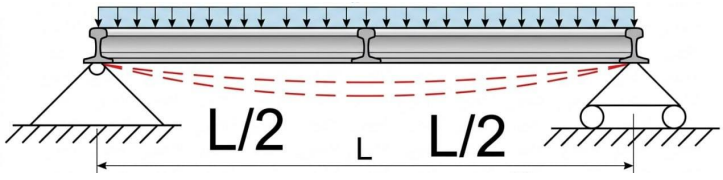
Bu holatda A, B nuqtalarda talab darajasidagi silliqlikka ega bo‘lish mumkin.

Biz $ACB-$ shaklni hosil qilishda temir yo‘l relsining tabiiy egriligi qobiliyatidan foydalanish imkonini o‘rganib chiqdik. Buning uchun, biz, avvalo bir uzunlik birligidagi (25m) rels bo‘lagini o‘z og‘irligi ta’sirida tabiiy egilish imkonini baholadik.

Tajriba uchun P65 tipdagi rels olindi. Bu relsni bir metrdagi massasi

$$m(q) = 64,9\ kg/m$$

elastiklik moduli $E = 2,1 \times 10^5 MPa$.



Bunda ushbu natijaga ega bo‘ldik. Relsning o‘z og‘irligi ta’sirida maksimal egilishi:

$$f_{\max} = \frac{5qL^4}{384 \cdot EI}$$

ga teng bo‘lar ekan [2].

Agar temir yo‘l o‘tish qismi klotoida shaklida bo‘lsa, klotoidaning uning uchlaridan o‘tuvchi to‘g‘ri chiziqdan chetlanishi $d(s)$

$$d = \sin\frac{\alpha}{2}\int\limits_0^t \cos(\frac{\pi s^2}{2})ds - \cos\frac{\alpha}{2}\int\limits_0^t sin(\frac{\pi s^2}{2})ds.$$

tenglik bilan aniqlanar edi [3].

Taklif etilayotgan egri chiziq shaklida ko‘tarilish chizig‘i klotoida shaklida bo‘lsa, bu egri chiziqqa relsning tabiiy og‘irligiga bog‘liq egish yo‘li bilan erishish mumkin bo‘lar ekan, bunda $|f_{\max}| > d$ bo‘lishi ta’minlanadi.

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УДК 519.6

To‘rtinchi tartibli yupqa plyonka tenglamalari uchun avtomodel yechimlar nazariyasining rivojlanishi

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Annotatsiya. Tezisda yupqa plyonka (thin liquid films) dinamikasini tavsiflovchi to‘rtinchi tartibli nochiziqli degeneratsiyalangan parabolik tenglamalar nazariyasining rivojlanish bosqichlari hamda avtomodel (self-similar) yechimlar tahliliga bag‘ishlangan ilmiy adabiyotlar sharhi keltirilgan. Maqolada klassik Navye–Stoks tenglamalaridan lubrication approximation (yupqa qatlam yaqinlashuvi) yordamida yupqa plyonka modelining keltirib chiqarilishi, tenglamaning matematik xususiyatlari hamda ushbu sohada xalqaro miqyosda olib borilgan fundamental tadqiqotlar tahlil qilingan. Xususan, F. Bernis, A. Friedman, A. Oron, S.H. Davis, R.V. Bankoff, A.L. Bertozzi, M.P. Pugh, G.I. Barenblatt, J.R. King va V.A. Galaktionov kabi olimlarning nazariy va amaliy natijalari o‘zaro taqqoslanib, sohadagi ochiq ilmiy muammolar va istiqbolli yo‘nalishlar yoritib berilgan.

Kalit so‘zlar: yupqa plyonka tenglamalari, lubrication approximation, degeneratsiyalangan parabolik tenglamalar, avtomodel yechimlar, oraliq asimptotikalar, erkin chegara.

Kirish. So‘nggi o‘n yilliklarda nochiziqli xususiy hosilali differensial tenglamalar nazariyasi amaliy matematika, fizikaviy modellashtirish va muhandislikning eng faol rivojlanayotgan yo‘nalishlaridan biriga aylandi. Ayniqsa, yuqori tartibli degeneratsiyalangan parabolik tenglamalar bilan tavsiflanadigan yupqa suyuqlik qatlamlari (thin liquid films) dinamikasini tadqiq etish nazariy va amaliy jihatdan katta ahamiyat kasb etmoqda. Bunday modellar mikro- va nanoo‘lchamli texnologiyalar, biologik tizimlar, qoplama (coating) jarayonlari, mikrofluidika, optik qurilmalar hamda materialshunoslik kabi ko‘plab sohalarda uchraydi.

Yupqa plyonka nazariyasining shakllanishi klassik gidrodinamika qonunlariga, xususan, Navye-Stoks tenglamalariga borib taqaladi. Suyuqlik qatlamining qalinligi uning uzunligi yoki

kengligiga nisbatan juda kichik bo‘lgan hollarda maxsus asimptotik soddalashtirish – lubrication approximation (yupqa qatlam yaqinlashuvi) qo‘llaniladi. Ushbu yondashuv natijasida uch o‘lchovli Navye – Stoks tizimi yuqori tartibli nochiziqli evolyutsion tenglamaga keltiriladi. Bu usul Reynolds tomonidan moylash nazariyasini ishlab chiqishda asoslangan bo‘lib, keyinchalik Oron, Davis va Bankoff tomonidan yupqa plyonkalar uchun tizimli ravishda rivojlantirilgan. Yupqa plyonka tenglamalari odatda quyidagi umumiy ko‘rinishda yoziladi:

$$\frac{\partial u}{\partial t} + \nabla \cdot (u^n \nabla \Delta u) = 0,$$

bu yerda $u(x,t)-$ plyonka qalinligi, n esa fizik mexanizmni tavsiflovchi parametr hisoblanadi. Mazkur model sirt tarangligi, yopishqoqlik va boshqa fizik omillarning o‘zaro ta’sirini ifodalaydi. Ayrim fizik jarayonlarda tenglamaga gravitatsiya, bug‘lanish, termo-kapillyar kuchlar yoki tashqi manbalarni ifodalovchi qo‘shimcha hadlar ham kiritiladi.

Mazkur tenglamalarning asosiy matematik xususiyati ularning to‘rtinchi tartibli, nochiziqli va degeneratsiyalangan bo‘lishidadir. Shu sababli bunday tenglamalar uchun klassik parabolik tenglamalar nazariyasini bevosita qo‘llash mumkin emas. Degeneratsiya tufayli yechimlarning mavjudligi, yagonaligi, musbatligi, silliqligi va erkin chegara (free boundary) xossalarini tadqiq etish murakkab matematik muammolarni yuzaga keltiradi.

1980–1990-yillardan boshlab yupqa plyonka tenglamalari nazariyasi yangi bosqichga ko‘tarildi. Ayniqsa, Bernis va Friedman tomonidan degeneratsiyalangan to‘rtinchi tartibli parabolik tenglamalar uchun kuchsiz yechimlarning mavjudligi va ayrim sifat xossalari isbotlandi. Mazkur natijalar keyingi tadqiqotlar uchun muhim nazariy asos bo‘ldi. Shundan so‘ng King, Bertozzi, Pugh, Witelski, Galaktionov, Giacomelli va boshqa olimlar tomonidan avtomodel yechimlar, erkin chegaralar, uzilish (rupture), dewetting jarayonlari, sonli modellashtirish va asimptotik tahlil bo‘yicha ko‘plab ilmiy natijalar olindi.

Yupqa plyonka tenglamalari nazariyasining rivojlanishida avtomodel (self-similar) yechimlar alohida o‘rin egallaydi. Bunday yechimlar murakkab xususiy hosilali differensial tenglamalarni oddiy differensial tenglamalarga keltirish imkonini beradi hamda yechimlarning uzoq vaqtli xatti-harakatini, massaning tarqalish qonuniyatlarini, oraliq asimptotikalarni va kritik rejimlarni tahlil qilishda samarali vosita hisoblanadi. Barenblatt tomonidan ishlab chiqilgan oraliq asimptotikalar nazariyasi keyinchalik ko‘plab evolyutsion tenglamalar, jumladan yupqa plyonka tenglamalari uchun ham muvaffaqiyatli qo‘llanildi.

Yupqa plyonka tenglamalarining matematik modeli va uning fizik asoslari. Yupqa suyuqlik qatlamlari harakati gidrodinamikaning klassik qonunlari asosida tavsiflanadi. Umumiy holda suyuqlikning harakati Navye–Stoks tenglamalari va massa saqlanish qonuni orqali ifodalanadi. Biroq ko‘plab amaliy jarayonlarda suyuqlik qatlamining qalinligi uning gorizontal o‘lchamlariga nisbatan juda kichik bo‘ladi. Bunday hollarda uch o‘lchovli modelni bevosita yechish katta hisoblash resurslarini talab qiladi.

Lubrication approximation g‘oyasiga ko‘ra, agar

$$\varepsilon = \frac{H}{L} << 1$$

bo‘lsa ($H-$ plyonkaning xarakterli qalinligi, $L-$ uning uzunlik bo‘yicha xarakterli o‘lchami), vertikal yo‘nalishdagi tezlik komponentlari gorizontal komponentlarga nisbatan ancha kichik bo‘ladi. Natijada Navye–Stoks tenglamalaridagi ayrim hadlarni kichik parametr ε bo‘yicha tashlab yuborish mumkin. Asimptotik soddalashtirishdan so‘ng suyuqlikning uch o‘lchovli harakat tenglamalari bitta evolyutsion tenglamaga keltiriladi. Sirt tarangligi asosiy fizik omil

bo‘lgan hol uchun klassik yupqa plyonka tenglamasi quyidagicha yoziladi:

$$\frac{\partial u}{\partial t} + \nabla \cdot (\frac{u^3}{3\mu} \nabla p) = 0,$$

bu yerda $u\,(x,t)-$ plyonka qalinligi, $\mu-$ dinamik yopishqoqlik, $p-$ bosim. Agar bosim sirt tarangligi orqali aniqlansa ($p = -\sigma \Delta u$, $\sigma-$ sirt taranglik koeffitsienti, $\Delta-$ Laplas operatori), model quyidagi ko‘rinishga keladi:

$$\frac{\partial u}{\partial t} + \nabla \cdot (\frac{\sigma u^3}{3\mu} \nabla \Delta u) = 0,$$

Eng ko‘p uchraydigan umumlashtirilgan model esa: $u_t + \nabla \cdot (u^n \nabla \Delta u) = 0$, ko‘rinishida bo‘lib, bu yerda $n = 3$ Nyuton suyuqliklari uchun klassik modelni, boshqa qiymatlar esa sirpanish (slip), g‘ovak muhit yoki quvvat qonuniga bo‘ysunuvchi (power-law) suyuqliklarni tavsiflaydi.

Modelning asosiy matematik xususiyatlari:

1. To‘rtinchi tartiblilik: Yuqori tartibli spatial hosilalar qatnashishi tufayli ikkinchi tartibli parabolik tenglamalardan farq qiladi.

2. Nochiziqlilik: Diffuziya koeffitsientining yechimning o‘ziga bog‘liqligi sababli superpozitsiya prinsipi bajarilmaydi.

3. Degeneratsiyalanganlik: $u = 0$ bo‘lganda $u^n = 0$ bo‘lib, diffuziya koeffitsienti nolga tenglashadi. Bu esa erkin chegara (free boundary) paydo bo‘lishiga olib keladi.

Avtomodel yechimlar nazariyasining rivojlanishi va zamonaviy tadqiqotlar

Barenblattning oraliq asimptotikalar nazariyasi. G.I. Barenblatt (1950–1960) tomonidan asos solingan oraliq asimptotikalar (intermediate asymptotics) nazariyasi bo‘yicha evolyutsion jarayonlarda boshlang‘ich shartlar ta’siri vaqt o‘tishi bilan yo‘qoladi va yechim universal avtomodel shaklga keladi. Barenblatt avtomodel yechimlarni ikki turga ajratdi:

• Birinchi tur: O‘xshashlik ko‘rsatkichlari o‘lchamlar tahlili va saqlanish qonunlaridan topiladi.

• Ikkinchi tur: Ko‘rsatkichlar chegaraviy masalani yechish jarayonida aniqlanadi.

Kingning avtomodel yechimlar bo‘yicha ishlari. J.R. King to‘rtinchi tartibli nochiziqli tenglamalarni o‘xshashlik almashtirishlari yordamida oddiy differensial tenglamalarga keltirish orqali manba tipidagi (source-type) va kompakt tayanchli (compactly supported) yechimlarni hamda kontakt chizig‘i harakatini tahlil qildi. Galaktionovning yuqori tartibli nochiziqli tenglamalar nazariyasiga qo‘shgan hissasi. V.A. Galaktionov avtomodel yechimlarni funksional-analitik va spektral tahlil, bifurkatsiya hamda homotopiya usullari bilan boyitdi. U avtomodel yechimlarning ko‘pligi hamda tarmoqlanish (branching) hodisalarini ochib berdi.

Qiyosiy tahlil

• Barenblatt yondashuvi: Avtomodel rejimlarning fizik mazmuni va universal asimptotik ko‘rinishini yoritadi.

• King yondashuvi: Xususiy hosilali tenglamalarni oddiy differensial tenglamalarga keltirib, konkret matematik modellarni tahlil qilish imkonini beradi.

• Galaktionov yondashuvi: Funksional tahlil va spektral nazariya orqali sifat xossalarini chuqur tadqiq etadi.

Xulosa. Tahlil qilingan adabiyotlar shuni ko‘rsatadiki, to‘rtinchi tartibli yupqa plyonka tenglamalarini o‘rganishda avtomodel yechimlar muhim va samarali vosita hisoblanadi. Biroq, mavjud ishlarning katta qismi faqat yechimlar mavjudligi yoki sonli hisoblashlarga qaratilgan. Yuqori tartibli nochiziqli modellar uchun yangi analitik avtomodel almashtirishlar qurish hamda hosil bo‘ladigan oddiy differensial tenglamalarning sifat tahlilini olib borish zamonaviy matematik fizikaning dolzarb ochiq muammolaridan bo‘lib qolmoqda.

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УДК 532.546

VAQT BO'YICHA KO'P HADLI KASR HOSILALI DIFFUZIYA TENGLAMASI UCHUN TO'G'RI VA TESKARI MASALALAR

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So'nggi yillarda fraktal tuzilishga ega bo'lgan g'ovak muhitlarda modda ko'chishi jarayonlarining murakkab va anomal xususiyatlarini adekvat ifodalovchi matematik modellarni ishlab chiqish-ga katta e'tibor qaratilmoqda. Bunday jarayonlarda moddaning tarqalish tezligi va dinamikasi klassik diffuziya qonuniyatlaridan farq qilishi sababli kasr tartibli differensial tenglamalardan foydalanish muhim ahamiyat kasb etadi. Shu nuqtayi nazardan, xususiy hosilali differensial tenglamalarda ishtirok etuvchi kasr tartibli hosilalarning α, β_1 va β_2 tartiblarini tajriba yoki o'lchov ma'lumotlari asosida aniqlash muhim teskari masalalardan biridir. Mazkur yo'nalishdagi teskari masalalarni sonli yechish usullari yetarlicha o'rganilmagan bo'lib, bu kasr tartibli modellar parametrlarini ishonchli identifikatsiya qilishning samarali algoritmlarini ishlab chiqishni taqozo etadi [1, 2]. Modda ko'chishi vaqt bo'yicha ko'p hadli kasr hosilali diffuziya tenglamasi asosida tavsiflanadi:

$$\frac{\partial^\alpha c}{\partial t^\alpha} + \sum_{s=1}^{n} r_s \frac{\partial^{\beta_s} c}{\partial t^{\beta_s}} = D\frac{\partial^\gamma c}{\partial x^\gamma} + f(t,x), \tag{1}$$

bu yerda α, β_s $(s = \overline{1,n})$, γ — hosila tartiblari bo'lib, quyidagi oraliqda o'zgaradi: $0 < \beta_s < \cdots < \beta_1 < \alpha < 1$, $1 \le \gamma \le 2$.

Uch hadli holatda $n = 2, f = 0$ ko'rinishida soddaroq shaklga o'tkaziladi va boshlang'ich-chegaraviy masala quyidagicha yoziladi:

$$\frac{\partial^\alpha c}{\partial t^\alpha} + r_1\frac{\partial^{\beta_1} c}{\partial t^{\beta_1}} + r_2\frac{\partial^{\beta_2} c}{\partial t^{\beta_2}} = D\frac{\partial^\gamma c}{\partial x^\gamma}, \tag{2}$$

$$c(0,x) = 0, \quad c(t,0) = c_0, \quad c(t,\infty) = 0.$$

Mazkur masala chekli ayirmalar usuli yordamida sonli yechildi. Hisoblash natijalari shuni ko'rsatdiki, modeldagi hadlar sonining ortishi bilan konsentratsiyaning muhit bo'ylab tarqalish tezligi kamayadi. Xususan, bir hadli modelga nisbatan ko'p hadli modelda modda konsentratsiyasi-ning muhitda tarqalish jarayoni yanada sust kechadi. r_1, r_2 ortishi esa konsentratsiya profillarining yoyilishini yanada sekinlashtiradi.

Endi $\gamma = 2$ deb olib, α, β_1, β_2 tartiblarini aniqlash bo'yicha teskari masala qaraladi:

$$\frac{\partial^\alpha c}{\partial t^\alpha} + r_1 \frac{\partial^{\beta_1} c}{\partial t^{\beta_1}} + r_2 \frac{\partial^{\beta_2} c}{\partial t^{\beta_2}} = D \frac{\partial^2 c}{\partial x^2}. \tag{3}$$

Buning uchun (2) shartlardan tashqari x_l ($l = 1, 2, 3$) nuqtalardagi qo'shimcha (tajribaviy) ma'lumot beriladi:

$$c(t, x_l) = z_l(t), \quad t \in [0, T_{\max}], \quad l = 1, 2, 3. \tag{4}$$

Masala (4) ma'lumotlar asosida α, β_1, β_2 ni topishga, ya'ni quyidagi kvadratik funksionalni minimallashtirishga keltiriladi:

$$I(\alpha, \beta_1, \beta_2) = \sum_{l=1}^{3} \int_0^{T_{\max}} [c(t, x_l) - z_l(t)]^2 dt \approx \sum_{l=1}^{3} \sum_{j=0}^{M} \tau [c(t_j, x_l) - z_l(t_j)]^2, \tag{5}$$

bu yerda $c(t, x_l)$ — berilgan α, β_1, β_2 da (3), (2) masalaning chekli-ayirmali yechimi.

(5) funksionalni minimallashtirish uchun Nelder–Mid simpleks usuli qo'llaniladi [4]. Barcha hisoblashlar MATLAB muhitida bajarilgan: (3), (2) ning chekli-ayirmali sxemasi funksiya sifatida yozilib, Nelder–Mid algoritmini amalga oshiruvchi standart `fminsearch` protsedurasiga argument sifatida uzatiladi; boshlang'ich yaqinlashish $(\alpha^0, \beta_1^0, \beta_2^0)$, aniqlik esa `TolX`, `TolFun` parametrlari bilan beriladi. Usul barqarorligi kvazireal eksperiment orqali tekshiriladi [3, 4]: (3), (2) masala aniq $\alpha_{\text{exact}}, \beta_{1,\text{exact}}, \beta_{2,\text{exact}}$ da yechilib $z_l(t_j)$ topiladi.

$$z_l^\delta(t_j) = z_l(t_j) + 2\delta(\sigma(t_j) - 1/2), \quad \sigma(t_j) \in (0, 1), \quad l = 1, 2, 3, \tag{6}$$

bu yerda δ — xatolik darajasi; qo'zg'atilgan ma'lumotlarga (5) qo'llanilib tiklangan α, β_1, β_2 aniq qiymatlar bilan nisbiy xatolik orqali solishtiriladi.

Xulosa

Xulosa qilib aytganda, uch hadli kasr tartibli diffuziya tenglamasidagi α, β_1, β_2 tartiblarni nuqtaviy o'lchov ma'lumotlari asosida tiklash masalasi tadqiq etildi. Teskari masalani yechish uchun MATLAB muhitida `fminsearch` funksiyasiga asoslangan Nelder–Mid algoritmidan foydala-nildi. Kvazireal eksperimentlar natijasida qo'zg'atilgan ma'lumotlar sharoitida parametrlarni tiklash imkoniyati va hisoblash yondashuvining turg'unligi baholandi. Olingan natijalar mazkur yondashuvni kasr tartibli diffuziya modellarini identifikatsiya qilishda qo'llash mumkinligini ko'rsa-tadi. Kelgusida usulni ikki zonali fraktal g'ovak muhitlarda modda ko'chishi jarayonlarini tavsiflov-chi kasr tartibli modellarga kengaytirish ko'zda tutiladi.

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MORSHO TIPIDAGI KASR TARTIBLI HOSILA ASOSIDA XOTIRALI MASSA–PRUJINA–AMORTIZATOR TIZIMINI MATEMATIK MODELLASHTIRISH VA SONLI TADQIQ ETISH

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Ko'plab fizik, mexanik va texnik jarayonlarda tizimning joriy holati faqat ayni vaqtdagi tashqi ta'sirlarga emas, balki uning avvalgi holatlariga ham bog'liq bo'ladi. Bunday xususiyat xotira effekti deb ataladi [1,2]. Materiallarning viskoelastik xossalari, anomal diffuziya, murakkab tebranish jarayonlari, biologik tizimlar va ayrim boshqaruv jarayonlarida kuzatiladigan uzoq muddatli xotira hodisalarini klassik butun tartibli differensial tenglamalar yordamida yetarlicha aniq ifodalash har doim ham mumkin emas. Shu sababli xotiraga va nochiziqli vaqtga bog'liq jarayonlarni matematik modellashtirishda kasr tartibli differensial operatorlardan foydalanish so'nggi yillarda muhim tadqiqot yo'nalishlaridan biriga aylandi [3].

Kasr tartibli differensial hisoblashning klassik differensial hisoblashdan asosiy farqlaridan biri shundaki, kasr tartibli hosila odatda lokal operator bo'lmay, funksiyaning ma'lum bir vaqt oraliǵidagi qiymatlari bilan bog'liq bo'ladi. Natijada kasr tartibli model tizimning hozirgi holatini uning oldingi holatlaridan ajratib qarash imkonini bermaydi. Aynan shu xususiyat kasr tartibli modellarni xotira effektiga ega fizik tizimlarni tavsiflash uchun tabiiy matematik vositaga aylantiradi.

Mazkur tadqiqot natijalari Morsho kasr tartibli hosilasi yordamida mexanik tizimlarning uzoq muddatli xotira xususiyatlarini modelga kiritish imkoniyatini baholashga xizmat qiladi. Shuningdek, ishlab chiqilgan sonli yondashuv kelgusida viskoelastik materiallar, xotirali mexanik tizimlar va boshqa kasr tartibli dinamik modellar uchun sonli tadqiqotlar olib borishda asos sifatida foydalanilishi mumkin.

Tadqiq qilinayotgan matematik model

$$m\ddot{x}(t) + cD_M^{\alpha}x(t) + kx(t) = F(t), \qquad 0 < \alpha < 1 \tag{1}$$

ko'rinishga ega. Bu yerda m – massa, c – amortizatsiya koeffitsiyenti, k – prujina bikrligi, $x(t)$ – tizimning siljishi va $F(t)$ – tashqi kuch, D_M^{α} – α tartibli Morsho hosilasi.

Tenglama boshlang'ich shartlar

$$x(0) = x_0, \qquad \dot{x}(0) = v_0$$

va tarixiy shart

$$x(t) = \phi(t), \qquad t \le 0$$

bilan beriladi.

Mazkur tadqiqotning bazaviy hisoblash tajribasida

$$x_0 = 0, \qquad v_0 = 0, \qquad \phi(t) = 0$$

deb olinadi.

Hisoblash intervalini (1) tenglama uchun $0 \le t \le T$ deb olamiz va uni bir xil uzunlikdagi N ta vaqt oralig'iga bo'lamiz. Vaqt qadami

$$h = \frac{T}{N}$$

bo'ladi. Diskret vaqt nuqtalari

$$t_n = nh, \qquad n = 0, 1, \ldots, N$$

orqali belgilanadi. Funksiyaning ushbu nuqtalardagi qiymatini

$$x_n = x(t_n)$$

deb belgilaymiz. Shuningdek, $F_n = F(t_n)$ belgilashdan foydalanamiz.

Sonli hisoblash uchun unga mos Grünwald–Letnikov tipidagi diskret operatorni qo'ladillaymiz va quyidagi hisoblash formulasini hosil qilamiz:

$$x_{n+1} = 2x_n - x_{n-1} + \frac{h^2}{m}\left[F_n - \frac{c}{h^\alpha}\sum_{j=0}^{n} w_j^{(\alpha)} x_{n-j} - kx_n\right]. \tag{2}$$

Ushbu (2) formula Python dasturlash tilida dasturning asosiy rekurrent tenglamasi sifatida ishlatiladi.

Standart Grünwald–Letnikov diskretlashtirishda har bir n-vaqt nuqtasi uchun $n+1$ ta tarixiy Morsho kasr hosilasi asosida qurilgan majburiy tebranish tizimining dinamik xususiyatlarini órganish maqsadida sonli hisoblashlar $\alpha = 0.2$, 0.5 va 0.8 kasr tartiblari uchun amalga oshirildi. Hisoblashlarda tizim parametrlari $m = 1$, $c = 2$, $k = 10$, $F_0 = 10$ va $\omega = 1$ ko'rinishida qabul qilindi. Vaqt oralig'i $0 \le t \le 50$ bo'lib, sonli integrallash $h = 0.01$ qadam bilan bajarildi. Kasr tartibli amortizatsiya hadi Grunwald–Letnikov diskretlashtirishiga asoslangan xotirali sonli sxema yordamida hisoblandi. 1-rasmda turli kasr tartiblar uchun tizim siljishining vaqt bo'yicha o'zgarishi keltirilgan. Natijalardan ko'rinadiki, barcha holatlarda tizim tashqi garmonik kuch t'sirida tebranma harakatni amalga oshiradi va vaqt o'tishi bilan barqaror majburiy tebranish rejimiga yaqinlashadi. Biroq kasr tartibining o'zgarishi nafaqat tebranish amplitudasiga, balki o'tkinchi jarayonning davomiyligi va fazaviy xarakteriga ham ta'sir ko'rsatadi.

$\alpha = 0.2$ holatida siljishning dastlabki vaqt oraliǵidagi ózgarishi nisbatan murakkab xarakterga ega. Tizimning birinchi tebranishida siljishning maksimal qiymati taxminan $x_{\max} \approx 1.1$ ga yetadi. Keyingi davrlarda amplituda kamayib, taxminan 0.9–0.95 atrofidagi barqaror tebranish rejimi shakllanadi. Dastlabki bosqichdagi egri chiziqning lokal o'zgarishlari kasr hosilaga xos bo'lgan xotira effektining kuchli namoyon bo'lishi bilan izohlanadi. Xususan, α kichik bo'lganda Grünwald–Letnikov xotira yadrosining sekin so'nishi tizimning oldingi holatlarining joriy dinamikaga uzoqroq vaqt davomida ta'sir qilishiga olib keladi.

$\alpha = 0.5$ uchun siljish grafigi ancha silliq xarakterga ega. o'tkinchi jarayondan so'ng tizimning tebranish amplitudasi taxminan 0.95–1.0 oralig'ida shakllanadi. Ushbu holatda kasr tartibli amor–tizatsiyaning xotira ta'siri $\alpha = 0.2$ holatiga qaraganda kuchsizroq bo'lib, tizimning barqaror tebranish rejimiga o'tishi nisbatan tartibli kechadi.

$\alpha = 0.8$ qiymatida esa barqaror tebranish amplitudasi yanada kattaroq bo'lib, uning maksimum va minimumlari taxminan $+1.0$ va -1.0 atrofida joylashadi. Egri chiziqning davriy xarakteri aniqroq namoyon bóladi. Bu holatda xotira koeffitsiyentlarining tezroq so'nishi sababli

uzoq o'tmishdagi holatlarning joriy dinamikaga ta'siri kamayadi. Natijada tizimning javobi klassik majburiy tebranish modeliga yaqinlashuvchi xarakter kasb etadi.

Olingan natijalar kasr tartibining tizim dinamikasidagi rolini yaqqol ko'rsatadi. Xususan, hisoblash tajribasida kasr tartibi ortishi bilan barqaror tebranish amplitudasining ortishi kuzatildi:

$$A_{\alpha=0.2} < A_{\alpha=0.5} < A_{\alpha=0.8}.$$

Bu bog'lanish kasr tartibli amortizatsiya operatorining xotira xususiyati bilan bevosita bog'liq. Morsho hosilasining Grünwald–Letnikov ko'rinishidagi diskret approksimatsiyasi

$$D_M^{\alpha} x(t_n) \approx \frac{1}{h^{\alpha}} \sum_{j=0}^{n} w_j^{(\alpha)} x_{n-j}$$

bo'lib, bunda joriy vaqtdagi kasr hosila qiymati nafaqat x_n, balki barcha oldingi x_{n-j} qiymatlariga boǵliq. Shuning uchun tizimning joriy holati uning oldingi dinamik holatlari bilan bog'langan bo'lib, α parametrining o'zgarishi ushbu bog'lanishning kuchi va davomiyligini o'zgartiradi.

Shuni ta'kidlash kerakki, ushbu xulosalar berilgan parametrlar to'plami va tanlangan sonli sxema uchun olingan natijalarga tegishlidir. Kasr tartibining amplitudaga ta'sirini umumlashtirish uchun keyingi bosqichda L_2 va L_∞ xatoliklari, turli h qiymatlaridagi konvergentsiya tartibi hamda barqaror holat amplitudasining α ga funksional bog'liqligini miqdoriy tahlil qilish talab etiladi.

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Nochiziqli chegaraviy shartli issiqlik tarqalish tenglamalari sistemasi yechimlarining sifat xossalari.

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Biz ushbu maqolada $Q_+ = \{(x,t) : x \in R_+^N, 0 < t < T\}$ sohada quyidagi masalani qaraymiz.

$$\begin{cases} u_{it} = \nabla\left(|\nabla {u_i}^{m_i}|^{p_i} \nabla {u_i}^{m_i}\right), & x \in R_+^N, 0 < t < T, \\ -|\nabla {u_i}^{m_i}|^{p_i} \dfrac{\partial {u_i}^{m_i}}{\partial x_1} = u_{i+1}^{q_i}, & u_{k+1} = u_1, x_1 = 0, 0 < t < T, \\ u_i(x,0) = u_{i0}(x), & x \in R_+^N, \end{cases} \tag{1}$$

bu yerda m_i, p_i nochiziqli muhit hususiyatlarini ifodalovchi sonli parametrlar va quyidagi shartga ega: $p_i > \frac{1}{m_i} - 1$, $m_i > 1$, $q_i > 0$., $u_{i0}(x)$ uzluksiz, manfiy bo'lmagan, chegaralangan

funksiyalar. Ular quyidagi shartlarini qanoatlantiradi: $-|\nabla u_{i0}^{m_i}|^{p_i}\frac{\partial u^{m_i}{}_{i0}}{\partial x_1} = u_{(i+1)0}^{q_i}$, $x_1 = 0$, va ba'zi $x_0 \in R_+^N$ nuqta uchun $\sup pu_{i0} \subset B_R(x_0) \cap R_+^N, u_{i0} \neq 0$. Boshlang'ich qiymat $u_{i0}(x)$ kichik soha tashqarisida nolga teng bo'lsa ham, yechimlar chekli vaqt ichida blow-up holatga kelishi mumkin. (0.7) tenglamani o'ziga xos xususiyati shundaki, unda diffuziya koeffitsiyenti gradientga bog'liq.

Qaralayotgan (0.7) matematik model ko'p komponentli muhitlarda issiqlik o'tkazuvchanlik jarayonidan tashqari boshqa jarayonlarni ham tasvirlaydi, ya'ni o'zaro ta'sirlashuvchi ko'p kompo-nentli muhitlarda keng doiradagi fizik va biologik hodisalarni ham umumlashtirilgan tarzda tavsiflaydi: moddalarning o'zaro diffuziyasi, turli jismlar orasidagi issiqlik almashinuvi, yonish jarayonlari, suyuqlik va gazning siqiluvchan (politropik) muhitda filtratsiyasi, shuningdek biologik turlarning populyatsion dinamikasi [1]-[3].

Ta'rif 1. $u_i(x,t)$ funksiya Q sohada (0.7) masalaning yuqori (quyi) yechimi deyiladi, agar $u_i(x,t) = 0$ da $Q\backslash D_t$, $D_t = \{x < l(t)\} \times (0; +\infty)$, $u_i(x,t) \in C_{t,x}^{1,2}(D_t) \cap C(Q)$, $-|\nabla u_i{}^{m_i}|^{p_i}\frac{\partial u_i{}^{m_i}}{\partial x_1} \in C(Q)$, va quyidagi shartlarni qanoatlantirsa:

$$u_{it} \geq (\leq)\nabla\left(|\nabla u_i{}^{m_i}|^{p_i}\nabla u_i{}^{m_i}\right)$$

$$-|\nabla u_i{}^{m_i}|^{p_i}\frac{\partial u_i{}^{m_i}}{\partial x_1} \geq (\leq)u_{i+1}^{q_i}$$

$$u_i(x,0) \geq (\leq)u_{i0}(x),\ x \in R.$$

Теорема 1. Agar quyidagi sharti $q_{i0} < q_i < q_{ic}$. bajarilsa, u holda (0.7) masalaning har qanday notrival yechim chekli vaqt ichida chegaralanmagan (blow up) bo'ladi.

Chiziqsiz chegaraviy shartlar bilan bog'langan, buziluvchan parabolik tipdagi issiqlik tarqalish jarayonini tavsiflovchi tenglamalar sistemasining sifat xossalari tadqiq etildi. Radial-simmetrik almashtirish orqali masalaning avtomodel yechimlari qurilib, avtomodel yechim asimptotikasining bosh hadi aniqlandi. Yuqori va quyi yechimlar qurish hamda solishtirish prinsipidan foydalanib, masala yechimining vaqt bo'yicha global mavjudlik shartlari isbotlandi va Fudjita tipidagi kritik eksponentalar qiymati topildi, shuningdek sonli parametrlarning ma'lum sohasida ixtiyoriy no-trivial yechimning chekli vaqtda chegaralanmagan (blow-up) bo'lishi ko'rsatildi.

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On the Extremal Function of the Hermite-Type Optimal Interpolation Formula in Sobolev Space

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A numerical interpolation technique aims to approximate a function based on available data at distinct points. This data may encompass both function values and various derivatives. In the present context, our emphasis is on an interpolation approach that derives information from the specific points where the function is evaluated.

Accordingly, we consider the problem of interpolating a function $\varphi(x)$ by a more computationally straightforward function $P_\varphi(x)$ in the following form:

$$\varphi(x) \cong P_\varphi(x) = \sum_{\beta=0}^{N} C_\beta(x)\varphi(x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(x)\varphi'(x_\beta). \tag{1}$$

Here $C_\beta(x)$ are the coefficients of the optimal interpolation formula (1).

The error takes as

$$R_\varphi(x) = \varphi(x) - P_\varphi(x). \tag{2}$$

It should be noted that in this work, when we consider the approximation of the form (0.7), we impose the condition that the class of functions that transforms this approximate equality into an exact equality in $L_2^{(2)}(0,1)$ space should be the class of all linear functions. If we take $\varphi_1(x) = 1$ and $\varphi_2(x) = x$ as the basis functions for the space of all linear functions, the imposition of

$$\begin{aligned} R_{\varphi_1}(x) &= \varphi_1(x) - P_{\varphi_1}(x) = 0, \quad (R,x) = 0, \\ R_{\varphi_2}(x) &= \varphi_2(x) - P_{\varphi_2}(x) = 0, \quad (R,1) = 0, \end{aligned}$$

conditions on the error functional $R_\varphi(x)$ is enough for the approximation formula (1) to be exact for all linear functions.

Then in the space $L_2^{(2)}(0,1)$ at every fixed point $x = z$ of the interval $[0,1]$ the error (0.7) defines a linear continuous functional

$$R(x,z) = \delta(x-z) - \sum_{\beta=0}^{N} C_\beta(z)\cdot\delta(x-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z)\cdot\delta'(x-x_\beta), \tag{3}$$

and

$$\begin{aligned} (R,\varphi) &= \int_{-\infty}^{\infty} R(x,z)\cdot\varphi(x)\mathrm{d}x \\ &= \int_{-\infty}^{\infty} \left(\delta(x-z) - \sum_{\beta=0}^{N} C_\beta(z)\cdot\delta(x-x_\beta) + \sum_{\beta=0}^{N} C_{\beta,1}(z)\cdot\delta'(x-x_\beta)\right)\cdot\varphi(x)\mathrm{d}x \\ &= \varphi(z) - \sum_{\beta=0}^{N} C_\beta(z)\cdot\varphi(x_\beta) - \sum_{\beta=0}^{N} C_{\beta,1}(z)\cdot\varphi'(x_\beta). \end{aligned}$$

In order to construct an optimal interpolation formula in the form of (1), it is imperative to compute the norm $|R|_{L_2^{(2)}}$ of the error functional (3). This necessity arises from the fact that, according to the Cauchy-Schwarz inequality, the estimation of the error (0.7) is expressed by the norm as follows:

$$\|(R,\varphi)\| \le \|R\|_{L_2^{(2)*}} \cdot \|\varphi\|_{L_2^{(2)}}.$$

It is easy to see that the norm $\|R\|_{L_2^{(2)*}}$ depends on the coefficients $C_{\beta,1}(z)$. Then it should be found the smallest value of the norm $\|R\|_{L_2^{(2)*}}$ by the coefficient $C_{\beta,1}$. That is, it should be calculated the quantity

$$\inf_{C_{\beta,1}} \|R\|_{L_2^{(2)*}}. \tag{4}$$

The coefficients $\mathring{C}_{\beta,1}$ reaching the value (0.7) we call the optimal coefficients.
Thus, consequently

- we calculate the norm $\|R\|_{L_2^{(2)*}}$,
- we find $\mathring{C}_{\beta,1}$ which gives (0.7).

To calculate $\|R\|_{L_2^{(2)*}}$, we use the extremal function $U_R(x)$ [1,2] satisfying the following equality

$$(R,U_R) = \|R\|_{L_2^{(2)*}} \cdot \|U_R\|_{L_2^{(2)}},$$

here,

$$U_R(x) = R(x) * G_2(x) + p_1 x + p_0, \tag{5}$$

where p_0, p_1 are unknown real coefficients and

$$G_2(x) = \frac{|x|^3}{12}.$$

Now we calculate the convolution

$$R(x)*G_2(x) = \int_{-\infty}^{\infty} R(y)\cdot G_2(x-y)\mathrm{d}y = G_2(x-z) - \sum_{\beta=0}^{N} C_\beta(z) G_2(x-x_\beta) - \sum_{\beta=0}^{N} C_{\beta,1}(z) G_2'(x-x_\beta).$$

We calculate the convolution in the above equation separately, then we get following

$$R(x) * G_2(x) = \frac{|x-z|^3}{12} - \sum_{\beta=0}^{N} C_\beta \frac{|x-x_\beta|^3}{12} + \sum_{\beta=0}^{N} C_{\beta,1} \frac{\operatorname{sgn}(x-x_\beta)(x-x_\beta)^2}{4}. \tag{6}$$

If we substitute expression (6) into expression (5), we get the following result:

Theorem 1. The extremal function of interpolation formula (1) has the following form

$$U_R(x) = \frac{|x-z|^3}{12} - \sum_{\beta=0}^{N} C_\beta \frac{|x-x_\beta|^3}{12} + \sum_{\beta=0}^{N} C_{\beta,1} \frac{\operatorname{sgn}(x-x_\beta)(x-x_\beta)^2}{4} + p_1 x + p_0.$$

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АЛГОРИТМ НА ОСНОВЕ ИНТЕГРАЦИИ ICP И АФФИННОЙ АРИФМЕТИКИ ДЛЯ МОДЕЛИРОВАНИЯ КИНЕМАТИКИ РОБОТОТЕХНИЧЕСКИХ СИСТЕМ

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Задача точного определения кинематических параметров роботов (калибровки) является одной из важных задач современной робототехники, поскольку от качества её решения зависят точность позиционирования, повторяемость движений и надежность функционирования робототехнических систем. Математически задача калибровки сводится к решению системы нелинейных уравнений, параметры которой определяются по экспериментальным измерениям.

Одним из наиболее эффективных подходов к гарантированному решению подобных задач являются методы распространения интервальных ограничений (Interval Constraint Propagation, ICP) [1]. Они позволяют последовательно сужать области возможных значений неизвестных, гарантируя сохранение всех допустимых решений. Однако классический алгоритм ICP обладает рядом недостатков: он может существенно переоценивать области неопределённости из-за эффекта зависимости переменных, а эффективность сужения интервалов снижается при высокой нелинейности рассматриваемых моделей. Для уменьшения указанных недостатков в последние годы всё большее внимание уделяется применению аффинной арифметики [2, 3]. В отличие от классической интервальной арифметики, аффинная арифметика учитывает корреляционные зависимости между неопределёнными величинами, что позволяет существенно уменьшить переоценку результатов вычислений и получать более узкие гарантированные оценки.

Аффинной формой называется выражение вида

$$\hat{x} = x_0 + x_1\varepsilon_1 + x_2\varepsilon_2 + \cdots + x_n\varepsilon_n, \quad \varepsilon_i \in [-1, 1],$$

где x_0 — центральное значение, а ε_i — символы неопределенности. Арифметические операции над аффинными формами определяются следующим образом. Для сложения:

$$\hat{x} + \hat{y} = (x_0 + y_0) + \sum_{i=1}^{n}(x_i + y_i)\varepsilon_i.$$

Для вычитания:

$$\hat{x} - \hat{y} = (x_0 - y_0) + \sum_{i=1}^{n}(x_i - y_i)\varepsilon_i.$$

Для умножения вводится новый символ неопределенности ε_k:

$$\hat{x} \cdot \hat{y} = (x_0y_0) + \sum_{i=1}^{n}(x_0y_i + y_0x_i)\varepsilon_i + \left(\sum_{i=1}^{n}|x_iy_i|\right)\varepsilon_k.$$

Деление $\hat{x}/\hat{y}$ выполняется через умножение на аффинную аппроксимацию обратной величины:

$$\hat{x}/\hat{y} = \hat{x} \cdot \left(\frac{1}{y_0} - \sum_{i=1}^{n} \frac{y_i}{y_0^2}\varepsilon_i + \frac{\sum_{i=1}^{n} y_i^2}{y_0^2(y_0 - \sum_{i=1}^{n} |y_i|)}\varepsilon_{n+1} \right),$$

при условии $y_0 > \sum_{i=1}^{n} |y_i|$ (т.е. $0 \notin \hat{y}$). Для вычисления элементарных функций (например, $\sin \hat{x}$, $\exp \hat{x}$, $\sqrt{\hat{x}}$) используются их разложения в ряды Тейлора или аппроксимации Чебышева с последующим аффинным округлением. Эти операции позволяют контролировать распространение ошибок на каждом шаге вычислительного процесса.

Перейдём к математической постановке задачи кинематической калибровки. Пусть в трёхмерном евклидовом пространстве заданы два множества точек: $P = \{p_i\}_{i=1}^{N}$ — множество точек эталонной модели (шаблона), полученное, например, из CAD-системы, и $Q = \{q_i\}_{i=1}^{N}$ — множество точек, измеренных сенсором робота (например, лазерным сканером или стереокамерой). Предполагается, что между этими множествами существует неизвестное жесткое преобразование $T : \mathbb{R}^3 \to \mathbb{R}^3$, сохраняющее расстояния и углы, т.е. $T(p) = Rp + t$, где $R \in SO(3)$ — ортогональная матрица поворота ($R^T R = I$, $\det R = 1$), а $t \in \mathbb{R}^3$ — вектор переноса. Задача калибровки заключается в нахождении параметров R и t, минимизирующих функционал рассогласования

$$E(R, t) = \sum_{i=1}^{N} \|Rp_i + t - q_{\text{closest}}\|^2,$$

где q_{closest} — ближайшая к трансформированной точке $Rp_i + t$ точка из множества Q. При этом точки q_i содержат погрешности измерений, т.е. $q_i = q_i^* + \delta_i$, где q_i^* — истинное значение, а δ_i — шум, для которого известна оценка $\|\delta_i\| \le \Delta$.

В предлагаемом подходе параметры R и t представляются в аффинной форме:

$$\hat{R} = R_0 + \sum_{i=1}^{n} R_i\varepsilon_i, \quad \hat{t} = t_0 + \sum_{i=1}^{n} t_i\varepsilon_i.$$

Это позволяет на каждой итерации сохранять информацию о предыдущих ошибках и корректировать направление поиска, избегая локальных минимумов, а также получать гарантированные оценки погрешности для каждой компоненты искомого преобразования.

Теорема (О сходимости и гарантированной оценке ошибки). Пусть множества P и Q ограничены, начальное смещение T_0 известно, а погрешность измерений ограничена величиной $\Delta > 0$: $\|\delta\| \le \Delta$. Тогда аффин-ICP алгоритм сходится, и для каждого шага k существует эллипсоид погрешности $\mathcal{E}_k$ такой, что

$$\mathcal{E}_{k+1} \subseteq \mathcal{E}_k + \mathcal{W},$$

где $\mathcal{W}$ — константа, зависящая от Δ. Более того, $\lim_{k\to\infty} \mathcal{E}_k \subseteq \mathcal{E}_\infty$, т.е. итоговая ошибка ограничена и гарантированно не превышает некоторого предельного значения.

Доказательство. Рассмотрим оператор итерационного преобразования $\Phi : \mathbb{R}^m \to \mathbb{R}^m$, который ставит в соответствие вектору параметров $\theta_k = (\text{vec}(R_k), t_k)$ новый вектор $\theta_{k+1} = \Phi(\theta_k)$. В классическом ICP этот оператор является сжимающим в некоторой окрестности решения. В предлагаемом аффинном подходе оператор Φ действует на аффинные формы, что эквивалентно рассмотрению якобиана $J_k = \frac{\partial \Phi}{\partial \theta}\big|_{\theta_k}$.

На каждом шаге k расстояние между соответствующими точками $d_k = \|\hat{R}_k p_i + \hat{t}_k - q_i\|$ является аффинной формой. При умножении аффинных форм появляется новый символ

ε_k, который аккумулирует влияние шумов. В силу монотонности аффинных операций, если $\hat{R}_{k+1}$ и $\hat{t}_{k+1}$ являются функциями от $\hat{R}_k$ и $\hat{t}_k$, то погрешность не возрастает.

Для строгой оценки сходимости введём матричную норму $\|\cdot\|_2$ и рассмотрим спектральный радиус $\rho(J_k)$ матрицы Якоби. В предположении, что точки P находятся в общем положении (т.е. матрица системы имеет полный ранг), справедлива оценка

$$\|\theta_{k+1} - \theta^*\|_2 \leq \rho(J_k)\|\theta_k - \theta^*\|_2 + C\Delta,$$

где θ^* — точное решение, C — константа, зависящая от размерности задачи и геометрии множества P. Поскольку $\rho(J_k) < 1$ в окрестности решения (это следует из свойств ICP-алгоритма [1]), то последовательность $\{\theta_k\}$ сходится к θ^* со скоростью геометрической прогрессии.

Что касается оценки погрешности, то из аффинного представления следует, что радиус эллипсоида $\mathcal{E}_k$ удовлетворяет рекуррентному соотношению

$$r_{k+1} \leq \rho(J_k)r_k + \gamma\Delta,$$

где γ — константа, зависящая от структуры аффинных форм. Решая это неравенство, получаем

$$r_k \leq \rho(J_k)^k r_0 + \gamma\Delta\frac{1-\rho(J_k)^k}{1-\rho(J_k)}.$$

Переходя к пределу при $k \to \infty$, имеем

$$r_\infty \leq \frac{\gamma\Delta}{1-\rho_{\max}}, \quad \rho_{\max} = \sup_k \rho(J_k) < 1.$$

Таким образом, итоговая ошибка ограничена сверху величиной, пропорциональной уровню шума Δ и обратно пропорциональной $1-\rho_{\max}$. Ограниченность $\mathcal{E}_k$ следует из ограниченности исходных данных и шума. Следовательно, алгоритм сходится, а итоговая погрешность гарантированно ограничена. Утверждение теоремы полностью доказано. □

Для проверки эффективности предложенного алгоритма было проведено численное моделирование на модели 3D-манипулятора с шестью степенями свободы. В ходе вычислительных экспериментов сравнивались классический ICP и предлагаемый аффин-ICP алгоритм. Результаты моделирования показали, что предложенный подход позволяет существенно повысить точность регистрации: средняя ошибка позиционирования уменьшилась более чем на 35% по сравнению с классическим методом. Кроме того, использование аффинной арифметики обеспечивает гарантированные оценки погрешности, что особенно важно для задач, связанных с безопасностью роботизированных систем. Полученные доверительные интервалы оказались значительно уже, чем при использовании стандартных интервальных методов. Это свидетельствует о том, что аффинная арифметика эффективно учитывает корреляционные связи между параметрами преобразования.

В работе предложен новый подход к решению задачи кинематической калибровки робототехнических систем на основе интеграции ICP и аффинной арифметики. Доказана теорема о сходимости алгоритма и ограниченности погрешности с использованием аппарата матричных норм и спектрального радиуса оператора итерационного преобразования. Численные эксперименты подтверждают преимущества предложенного метода перед классическим ICP. В дальнейшем планируется распространение подхода на динамические модели и системы реального времени, а также исследование возможности адаптивного выбора параметров алгоритма в зависимости от уровня шума.

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Дисперсионное уравнение продольной устойчивости волны фильтрационного горения газов с учетом диффузии компонентов смеси

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Современные математические модели показывают, что устойчивость фронта определяется совокупным влиянием скорости фильтрации, теплофизических свойств пористой среды, геометрии реакционной области и процессов массопереноса [1]. Особую роль при этом играет молекулярная диффузия компонентов газовой смеси, которая изменяет распределение концентраций реагентов, положение зоны химической реакции и условия существования стационарной волны [2]. В настоящей работе на основе одномерной однотемпературной математической модели фильтрационного горения газов, учитывающей диффузионный перенос компонентов газовой смеси, получено дисперсионное уравнение продольной устойчивости волны горения.

Рассматривается одномерная однотемпературная математическая модель фильтрационного горения газов, соответствующая случаю сильного межфазного теплообмена между газовой и инертной фазами и пренебрежения градиентом давления. Предполагается, что плотность газовой смеси до и после волны горения постоянна, но принимает различные значения. Пусть газовая смесь состоит из n компонентов, из которых m являются компонентами исходной смеси, а остальные $(n-m)$ - продуктами химической реакции. Уравнения сохранения массы для компонентов исходной смеси будем записывать с индексом i, а для продуктов реакции — с индексом j. Через индекс k обозначим концентрацию недостающего компонента газовой смеси. В этом случае система уравнений, описывающая процесс фильтрационного горения, имеет следующий вид:

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}+\Big(\frac{\upsilon_1}{1+\varphi}+\phi'\Big)\frac{\partial\theta}{\partial x}+\frac{QAN\eta_k}{1-\sigma}\exp\Big(N\Big(1-\frac{1}{\sigma+(1-\sigma)\theta}\Big)\Big),$$

$$\frac{\partial\eta_i}{\partial t}=Le_i\frac{\partial^2\eta_i}{\partial x^2}+\Big(\upsilon_1+\phi'\Big)\frac{\partial\eta_i}{\partial x}+\frac{\zeta_i\eta_{k0}}{\eta_{i0}}AN\eta_k\exp\Big(N\Big(1-\frac{1}{\sigma+(1-\sigma)\theta}\Big)\Big), \qquad (1)$$

$$\frac{\partial\eta_j}{\partial t}=Le_j\frac{\partial^2\eta_j}{\partial x^2}+\Big(\upsilon_1+\phi'\Big)\frac{\partial\eta_j}{\partial x}+\frac{\zeta_j\eta_{k0}}{\eta_{je}}AN\eta_k\exp\Big(N\Big(1-\frac{1}{\sigma+(1-\sigma)\theta}\Big)\Big),$$

$$i=\overline{1,m},\quad j=\overline{m+1,n},\quad 1\leqslant k\leqq m,$$

с краевыми условиями

$$x = +\infty: \quad \theta = 0, \quad \eta_i = 1, \quad \eta_j = 0,$$
$$x = -\infty: \quad \theta = 1, \quad \eta_i = 0, \quad \eta_j = 1.$$

Система (1) записана в безразмерном виде и в движущейся с нестационарной скоростью ϕ' системе координат: $\theta = \frac{T-\sigma}{1-\sigma}$ — функция безразмерной температуры среды; $\eta_i = \frac{\eta_{ir}}{\eta_{i0}}$ — функции относительной концентрации компонентов исходной смеси; $\eta_j = \frac{\eta_{jr}}{\eta_{je}}$ — функции относительно концентрации компонентов продуктов реакции; $\eta_k = \frac{\eta_{kr}}{\eta_{k0}}$ — функция относительной концентрации недостающего компонента газовой смеси; t — относительное время; x — относительная координата. Другие параметры, входящие в систему (1) — физико-химические параметры пористой среды и смеси газов.

Согласно [3] аппроксимации скоростей в системе (1) примем в виде

$$J = AN \exp(\alpha(\theta(0;t)-1))\delta(x), \quad AN = u_0 + 1,$$
$$\upsilon_1 = u_0 \exp(\alpha(\theta(0;t)-1)),$$
$$\phi' = \exp(\alpha(\theta(0;t)-1)),$$

где $\alpha = \frac{N(1-\sigma)}{2}$, $N = \frac{E}{RT_e}$, $\sigma = \frac{T_0}{T_e}$, $u_0 = \frac{\upsilon_{10r}}{u_n}$, $\delta(x)$ — дельта-функция Дирака. Тогда систему (1) можно записать в виде

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial x^2} + u\exp(\alpha(\theta(0;t)-1))\frac{\partial\theta}{\partial x} + u\exp(\alpha(\theta(x;t)-1))\delta(x),$$

$$\frac{1}{Le_i}\frac{\partial\eta_i}{\partial t} = \frac{\partial^2\eta_i}{\partial x^2} + w_i\exp(\alpha(\theta(0;t)-1))\frac{\partial\eta_i}{\partial x} + \frac{\eta_0\zeta_i w_i}{\eta_{i0}}\exp(\alpha(\theta(x;t)-1))\delta(x), \tag{2}$$

$$\frac{1}{Le_j}\frac{\partial\eta_j}{\partial t} = \frac{\partial^2\eta_j}{\partial x^2} + w_j\exp(\alpha(\theta(0;t)-1))\frac{\partial\eta_j}{\partial x} + \frac{\eta_0\zeta_j w_i}{\eta_{je}}\exp(\alpha(\theta(x;t)-1))\delta(x),$$

$$i = \overline{1,m}, \quad j = \overline{m+1,n}.$$

$$x = +\infty: \quad \theta = 0, \quad \eta_i = 1, \quad \eta_j = 0,$$
$$x = -\infty: \quad \theta = 1, \quad \eta_i = 0, \quad \eta_j = 1,$$

где

$$u = \begin{cases} u_\varphi, & x > 0, \\ \beta u_\varphi, & x < 0, \end{cases} \qquad u = \begin{cases} w_{l(i)}, & x > 0, \\ \beta w_{l(i)}, & x < 0, \end{cases} \qquad u_\varphi = 1 + \frac{1+u_0}{1+\varphi}, \quad w_{l(i)} = \frac{1+u_0}{Le_i},$$

$\beta = \frac{\rho_{10}}{\rho_{1e}}$ — коэффициент теплового расширения газовой смеси, ρ_{1e} — плотность смеси газов за фронтом волны горения.

Используя стационарные решения системы (2) в областях $x > 0$ и $x < 0$, где $\delta(x) = 0$, методом малых возмущений

$$\theta(x;t) = \theta_{st}(x) + \theta'(x;t), \quad \eta_i = \eta_{i(st)}(x) + \eta_i'(x;t), \quad \eta_j = \eta_{j(st)}(x) + \eta_j'(x;t)$$

линеаризуем систему (2). Краевые условия линеаризованной системы имеют вид: $x = \pm\infty : \theta' = 0, \eta_i' = 0, \eta_j' = 0$. Далее, выбирая решение линеаризованной системы в виде

$$\theta' = \psi(x)\exp(\omega t), \quad \eta_i' = \xi_i(x)\exp(\omega t), \quad \eta_j' = \xi_j(x)\exp(\omega t), \tag{3}$$

получим обыкновенные дифференциальные уравнения относительно функций $\psi(x)$, $\xi_i(x)$ и $\xi_j(x)$, граничные условия которых вытекают из (3): $x = \pm\infty$: $\psi = 0$, $\xi_i = 0$, $\xi_j = 0$.

Явные решения функций $\psi(x)$, $\xi_i(x)$ и $\xi_j(x)$ содержат неизвестные константы A_ψ, $\psi(0)$, A_{ξ_i}, $\xi_i(0)$, A_{ξ_j}, $\xi_j(0)$, $i = \overline{1,m}$, $j = \overline{m+1,n}$. Для определения неизвестных констант воспользуемся условиями сшивки решений системы (2) в окрестности точки $x = 0$

$$\begin{gathered}
\Big[\frac{\partial\theta}{\partial x}\Big] + u_\varphi \exp(\alpha(\theta(0;t)-1)) = 0,\\
\Big[\frac{\partial\eta_i}{\partial x}\Big] + \frac{\eta_{k0}\zeta_i w_{l(i)}}{\eta_{i0}}\exp(\alpha(\theta(0;t)-1)) = 0,\\
\Big[\frac{\partial\eta_j}{\partial x}\Big] + \frac{\eta_{k0}\zeta_j w_{l(j)}}{\eta_{je}}\exp(\alpha(\theta(0;t)-1)) = 0,\\
\Big[\theta\Big] = 0, \quad \Big[\eta_i\Big] = 0, \quad \Big[\eta_j\Big] = 0.
\end{gathered} \tag{4}$$

Здесь в квадратных скобках обозначен скачок функции в окрестности точки $x = 0$: $\Big[f(x;t)\Big] = f(x+0;t) - f(x-0;t)$.

Переходя к малым возмущениям в системе (4), учитывая стационарные решения (2) и решения (3), получим систему из $(2+2n)$ алгебраических уравнений, для определения неизвестных констант A_ψ, $\psi(0)$, A_{ξ_i}, $\xi_i(0)$, A_{ξ_j}, $\xi_j(0)$, $i = \overline{1,m}, j = \overline{m+1,n}$. Для того чтобы из системы (4) стало возможным определить неизвестные константы необходимо выполнения условий

$$w_{l(i)} - w_{l(j)} + \frac{\zeta_i\eta_{k0}w_{l(i)}}{\eta_{i0}} + \frac{\zeta_j\eta_{k0}w_{l(j)}}{\eta_{je}}, \tag{5}$$

которые, устанавливают связь между коэффициентами диффузии компонентов смеси и их начальными и конечными концентрациями

$$\frac{D_j}{D_i} = \frac{\eta_{i0}(\eta_{je}/\eta_{k0} - \zeta_j)}{\eta_{je}(\eta_{i0}/\eta_{k0} + \zeta_i)}, \quad i = \overline{1,m}, \quad j = \overline{m+1,n}.$$

Требуя не тривиальное решение системы алгебраических уравнений при выполнении условий (5), получим следующее дисперсионное уравнение

$$\Big(\gamma_1\Big(\frac{u_\varphi^2\alpha}{\omega}+1\Big) + \frac{u_\varphi^3\alpha}{\omega} - \gamma_2^\beta + u_\varphi\alpha\Big)\cdot\prod_{i=1}^{n}\Big(\mu_{2i-1} - \mu_{2i}^\beta\Big) = 0,$$

где

$\gamma_1 = \frac{1}{2}\Big[-u_\varphi - \sqrt{u_\varphi^2+4\omega}\Big]$, $\gamma_2^\beta = \frac{1}{2}\Big[-\beta u_\varphi + \sqrt{\beta^2u_\varphi^2+4\omega}\Big]$,

$\mu_{2i-1} = \frac{1}{2}\Big[-w_{l(i)} - \sqrt{w_{l(i)}^2+(4\omega)/(Le_i)}\Big]$, $\mu_{2i}^\beta = \frac{1}{2}\Big[-\beta w_{l(i)} + \sqrt{\beta^2w_{l(i)}^2+(4\omega)/(Le_i)}\Big]$, $i = \overline{1,m}$,

$\mu_{2j-1} = \frac{1}{2}\Big[-w_{l(j)} - \sqrt{w_{l(j)}^2+(4\omega)/(Le_j)}\Big]$, $\mu_{2j}^\beta = \frac{1}{2}\Big[-\beta w_{l(j)} + \sqrt{\beta^2w_{l(j)}^2+(4\omega)/(Le_j)}\Big]$, $j = \overline{m+1,n}$

корни характеристических уравнений линеаризованной системы.

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СООТВЕТСТВУЮЩИЕ КВАДРАТИЧНЫЕ ФОРМЫ ОБЛАСТИ ВОРОНОГО СОВЕРШЕННОЙ ФОРМЫ $\varphi_1^5(x)$

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В настоящей работы вычисляется все неэквивалентные квадратичные формы, соответствующие области Вороного совершенной формы $\varphi_1^5(x)$.

Здесь уместно сказать, что эти проблемы в многомерных пространствах имеют не только чисто математический интерес, но и важные приложения, например в теории кодирования.

Г.Ф. Вороной доказал, что квадратичные формы, соответствующие $\frac{(n+2)(n-1)}{2}$-мерным граням $\Psi_k = \Psi_k(\varphi_1^n)$ области $V^{\frac{n(n+1)}{2}}$ совершенной формы

$$\varphi_1^n = \varphi_1^n(x_1, \ldots, x_n) = x_1^2 + \ldots + x_n^2 + x_1x_3 + \ldots + x_{n-1}x_n \tag{1}$$

имеют вид:

$$-x_1x_3 \tag{2}$$

$$x_1x_2 - \delta_{34}x_3x_4 - \ldots - \delta_{n-1\,n}x_{n-1}x_n. \tag{3}$$

Следовательно, чтобы найти совершенные формы, смежные с совершенной формой φ_1^n, необходимо выделить среди форм

$$\delta_{34}x_3x_4 + \ldots + \delta_{n-1\,n}x_{n-1}x_n \tag{4}$$

неэквивалентные относительно группы $G(\varphi_1^n)$ целочисленных автоморфизмов формы φ_1^n, где δ_{ij} равно 0 или 1 при $3 \le i < j \le n$.

Число форм достаточно большое, и равно

$$2^{\frac{(n-2)(n-3)}{2}}. \tag{5}$$

А затем с помощью формулы Вороного строятся совершенные формы, смежные с совершенной формой φ_1^n.

В данной работе доказывается следующее предложение.

Теорема. Число 14-мерных граней области Вороного $V^{15}(\varphi_1^5)$ совершенной формы φ_1^5, неэквивалентных относительно группы S_3 перестановок переменных x_3, x_4, x_5, равно 5: 1) x_1x_2, 2) $x_1x_2 - x_3x_4$, 3) $x_1x_2 - x_3x_4 - x_3x_5$, 4) $x_1x_2 - x_3x_4 - x_3x_5 - x_4x_5$.

Каждая из этих форм вместе с $-x_1x_3$, в силу теории Вороного, определяет 14-мерную грань области Вороного $V^{15}(\varphi_1^5)$ совершенной формы φ_1^5.

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Коэффициенты оптимальной интерполяционной формулы в пространстве Л. Хёрмандера $H_2^{\mu}(R)$.

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Задача о построении интерполяционных формул является одной из классических задач вычислительной математики и численного анализа. Теории интерполяционных формул построены многими авторами, например, [1–3].

Требуется построит интерполяционную формулу $P_f(x)$, совпадающую функцией $f(x)$ в узлах интерполирования:

$$f(x_i) = P_f(x_i), \quad i = 0, 1, ...N \tag{1}$$

Важной задачей в теории интерполирование является нахождение максимума ошибки интерполяционной формулы $f(x) \cong P_f(x)$ над данном классом функций. Значение этой функции в некоторой точки z есть функционал, определенный как

$$< \ell_N(x), f(x) >= \int_{-\infty}^{\infty} \ell_N(x) f(x) dx = f(z) - P_f(z) = f(z) - \sum_{\beta=0}^{N} C_\beta(z) f(x_\beta), \tag{2}$$

где ясно, что

$$P_f(z) = \sum_{\beta=0}^{N} C_\beta(z) f(x_\beta), \tag{3}$$

интерполяционная формула и

$$\ell_N(x) = \delta(x - z) - \sum_{\beta=0}^{N} C_\beta(z) \delta(x - x_\beta), \tag{4}$$

функционал погрешности этой интерполяционной формулы, $C_\beta(z)$ - коэффициенты, а x_λ узлы формулы $P_f(z)$, $x_\beta \in [0,1]$, $\delta(x)$ - дельта- функция Дирака и $f(x) \in H_2^\mu(R)$.

Определение. Функция $\psi_\ell(x)$ называется экстремальной функцией функционала $\ell_N(x)$, если для которой выполняется следующее равенство

$$|< \ell_N(x), \psi_\ell(x) >| = \left\|\ell_N|H_2^{\mu*}(R)\right\| * \|\psi_\ell|H_2^\mu(R)\|.$$

Для нахождении оптимальных коэффициентов решетчатых интерполяционных формул в пространстве Л.Хёрмандера $H_2^\mu(R)$ нам необходимо привести результаты следующих работ: [2, 3], т.е.

Теорема 1. Экстремальная функция функционала погрешности $\ell_N(x)$ для интерполяционной формулы $P_f(z)$ является

$$\psi_\ell(x) = \nu_m(x-z) - \sum_{\beta=1}^{N} C_\beta(z)\nu_m(x-h\beta),$$

а норма функционала погрешности данной интерполяционной формулы выражается следующим равенством

$$\left\|\ell_N|H_2^{\mu*}(R)\right\|^2 = \int_R |\overset{-1}{F}\left\{(1+y^2)^{-m}F[\ell_N(x)](y)\right\}(x)|^2dx, \tag{5}$$

где $\nu_m(x) = F^{-1}[(1+y^2)^{-m}](x)$, $C_\beta(z)-$ коэффициенты интерполяционной формулы $P_f(z)$, $h = \frac{1}{N}$ и N число узлов.

Теорема 2. В пространстве $H_2^\mu(R)$ существует единственная оптимальная интерполяционная формула вида (3), коэффициенты которой определяются из системы

$$\sum_{\beta=1}^{N} C_\beta(z)\nu_m(h\alpha - h\beta) = \nu_m(h\alpha - z), \quad \alpha = 0,1,2,...,N, \tag{6}$$

иначе представляется в следующим ввиде

$$\psi_\ell(h\alpha) = 0. \tag{7}$$

Теорема 3. Среди всех интерполяционных формул вида (3), N-заданное натуральное число, решетчатая интерполяционная формула

$$P_f(x) = \sum_{\beta=1}^{N} \overset{0}{C}{}_\beta(z)f(h\beta),$$

определяемая коэффициентами является единственной оптимальной над пространстве $H_2^\mu(R)$:

$C_\beta^0(z) = cth(2\pi h)e^{-2\pi|h\beta - z|} - \frac{1}{2sh(2\pi h)}e^{-2\pi|h(\beta-1)-z|} - \frac{1}{2sh(2\pi h)}e^{-2\pi|h(\beta+1)-z|}$, при $\beta = 1,2,..,N-1$.

$C_0(z) = cth(2\pi h)e^{-2\pi|0-z|} - \frac{1}{2sh(2\pi h)}e^{-2\pi|-h-z|} - -\frac{1}{2sh(2\pi h)}e^{-2\pi|h-z|} =$
$\frac{1}{2sh(2\pi h)}\left[2ch\,(2\pi h)\,e^{-2\pi z} - e^{-2\pi(h+z)} - e^{-2\pi|h-z|}\right]$, при $\beta = 0$

$C_N(z) = \frac{1}{2sh(2\pi h)}\left[2ch(2\pi h)e^{-2\pi|1-z|} - e^{-2\pi|(1-z)-h|} - e^{-2\pi|(1-z)+h|}\right]$, при $\beta = N$

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Алгоритм построения оптимальная квадратурная формула для интегралов типа Фурье в пространстве Л. Хёрмандера $H_2^\mu(R)$.

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Вычисление определенных интегралов с возможно большой точностью является одной из актуальных задач вычислительной математики и численного анализа [1–3].

В данной работе рассматривается вопрос о вычислении интегралов типа Фурье, где подынтегральная функция имеет с ограниченным числом экстремальных точек в интервале интегрирования и в качестве весовой функции берем быстро колеблющаяся функция.

В настоящей работе рассмотрим следующую квадратурную формулу:

$$\int_0^1 \cos(2\pi\sigma x)\, f(x)dx \approx \sum_{\beta=0}^{N} C_\beta f(x_\beta), \tag{1}$$

с функционалом погрешности

$$\ell_N(x) = \varepsilon_{[0,1]}(x)\cos(2\pi\sigma x) - \sum_{\beta=0}^{N} C_\beta \delta(x - x_\beta), \tag{2}$$

где соответственно, C_β и x_β называют коэффициентами и узлами квадратурной формулы (1), $N \in \mathbb{N}, \sigma \in \mathbb{R}$ $f(x)$ является элементом гильбертова пространства Хёрмандера $H_2^\mu(R)$ [1, 3] и назовем ее квадратурную формулу для интегралов типа Фурье.

Теорема 1. Экстремальная функция функционала погрешности (2) квадратурной формулы (1) имеет вид

$$\psi_\ell(x) = [\cos(2\pi\sigma x)\cdot\varepsilon_{[0,1]}(x)] * \nu_m(x) - \sum_{\beta=0}^{N} C_\beta \nu_m(x - h\beta), \tag{3}$$

квадрат нормы функционала погрешности $\ell_N(x)$ в пространстве Хёрмандера $H_2^{\mu}(R)$ имеет следующий вид

$$\left\|\ell_N\left|H_2^{\mu*}(R)\right.\right\|^2=\int\limits_{-\infty}^{\infty}\left|[\cos(2\pi\sigma x)\cdot\varepsilon_{[0,1]}(x)]*\nu_m(x)-\sum_{\beta=0}^{N}C_\beta\nu_m(x-h\beta)\right|^2dx, \tag{4}$$

где

$$\nu_m(x)=\int\limits_{-\infty}^{\infty}\exp 2\pi iyx(1+y^2)^m dy=\frac{\pi\exp(-2\pi|x|)}{2^{2m-2}\cdot(m-1)!}\sum_{k=0}^{m-1}\frac{(2m-k-2)!\cdot(4\pi)^k}{k!\cdot(m-k-1)!}|x|^k, \tag{5}$$

является фундаментального решения дифференциального оператора

$$\left[1-\frac{1}{(2\pi)^2}\frac{d^2}{dx^2}\right]^m. \tag{6}$$

По теоремы Бабушки [2, 3] известно, что экстремальная функция функционала погрешности оптимальной квадратурной формулы обращается в нуль в узлах квадратурной формулы, имея ввиду это из (6) получим

$$\psi_\ell(h\alpha)=0,\quad \alpha=0,1,...,N. \tag{7}$$

Теорема 2. Решетчатая квадратурная формула для интегралов типа Фурье имеющий вид

$$\int\limits_0^1\cos(2\pi\sigma x)f(x)dx\approx\sum_{\beta=0}^{N}\overset{0}{C}[\beta]f[\beta], \tag{8}$$

является единственной оптимальной, среди всех квадратурных формул вида (1) в пространстве $H_2^{\mu}(R)$, если коэффициенты определяется следующими равенствами:

$$\overset{0}{C}[0]=\frac{2}{\pi\mathrm{sh}(2\pi h)\left[1+\sigma^2\right]}\left\{\mathrm{ch}(2\pi h)-\cos(2\pi\sigma h)\right\},\qquad \beta=0$$

$$\overset{0}{C}[\beta]=\frac{2\cos(2\pi\sigma h\beta)}{\pi\mathrm{sh}(2\pi h)\left[1+\sigma^2\right]}\left\{\mathrm{ch}(2\pi h)-\cos(2\pi\sigma h)\right\}\ ,\qquad \beta=\overline{1,N-1}$$

$$\overset{0}{C}[N]=\frac{2\cos(2\pi\sigma)}{\pi\mathrm{sh}(2\pi h)\left[1+\sigma^2\right]}\left\{\mathrm{ch}(2\pi h)-\cos(2\pi\sigma h)\right\},\qquad \beta=N$$

В заключение отметим, что нахождение оптимапьных коэффициентов квадратурных формул для интегралов типа Фурье вида (1) в пространстве непериодических функций Хёрмандера $H_2(R)$ выпольнена в случае $m=1$.

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Исследование проблемы безработицы методами математического моделирования: прогнозирование и управление

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Аннотация: В работе предлагается математическая модель динамики безработицы, основанная на разделении безработного населения на структурную и циклическую составляющие и введении отдельного класса работников, проходящих профессиональную переподготовку. Построенная модель представляет собой нелинейную систему обыкновенных дифференциальных уравнений и позволяет одновременно исследовать прогнозирование и управление уровнем безработицы. Получены условия ограниченности решений и сформулирован критерий локальной устойчивости стационарного состояния. Показано, что интенсивность профессиональной переподготовки и скорость трудоустройства являются ключевыми управляющими параметрами модели. Численные эксперименты демонстрируют возможность снижения долгосрочной безработицы за счет оптимального распределения ресурсов между программами переподготовки и созданием рабочих мест.[1]

Ключевые слова: безработица, математическое моделирование, динамическая система, прогнозирование, управление, устойчивость, переподготовка.

Безработица является одним из основных социально-экономических показателей, характеризующих состояние рынка труда. Ее динамика определяется совокупным воздействием экономического цикла, структурных изменений, демографических процессов, технологического развития и государственной политики занятости. Поэтому методы математического моделирования представляют значительный интерес для прогнозирования уровня безработицы и выбора эффективных механизмов управления.

В классических моделях рынка труда динамика безработицы часто описывается с использованием агрегированной переменной $U(t)$ [2]. Однако такое представление не позволяет учитывать различия между структурной и циклической безработицей, а также влияние профессиональной переподготовки. В настоящей работе предлагается расширенная модель, в которой выделяются четыре класса населения. Пусть $S(t)$, $C(t)$, $R(t)$, $E(t)$ обозначают соответственно численность структурно безработных, циклически безработных, работников, проходящих переподготовку, и занятых работников.

Рассмотрим следующую систему:

$$\begin{cases} \frac{dS}{dt} = A - \alpha_1 S - \rho S - \mu S, \\ \frac{dC}{dt} = \gamma E - \alpha_2 C - \alpha_3 C - \mu C, \\ \frac{dR}{dt} = \rho S + \alpha_3 C - \alpha_4 R - \mu R, \\ \frac{dE}{dt} = \alpha_1 S + \alpha_2 C + \alpha_4 R - \gamma E - \mu E. \end{cases} \tag{1}$$

Здесь A характеризует приток населения в систему безработицы, α_1 и α_2 — интенсивности перехода структурно и циклически безработных в занятость, ρ — скорость направления структурно безработных на переподготовку, α_3 — переход циклических безработных

в состояние переподготовки, α_4 — скорость трудоустройства после переподготовки, γ — интенсивность потери рабочих мест, а μ — коэффициент естественного выбытия.[1][2]

Общее население системы

$$N(t) = S(t) + C(t) + R(t) + E(t)$$

удовлетворяет уравнению $\frac{dN}{dt} = A - \mu N$. Следовательно, при $\mu > 0$ величина $N(t)$ остается ограниченной, а при постоянном притоке A выполняется $N(t) \longrightarrow N^* = \frac{A}{\mu}$. Таким образом, система (1) позволяет моделировать не только изменение общего числа безработных, но и переход части населения между различными состояниями рынка труда. Уровень безработицы определяется выражением $U(t) = \frac{S(t)+C(t)}{N(t)}$.

Таким образом, система (1) позволяет моделировать не только изменение общего числа безработных, но и переход части населения между различными состояниями рынка труда.

Стационарное состояние системы определяется из условий

$$\frac{dS}{dt} = \frac{dC}{dt} = \frac{dR}{dt} = \frac{dE}{dt} = 0.$$

Для структурной безработицы получаем $S^* = \frac{A}{\alpha_1+\rho+\mu}$.

Для циклической безработицы: $C^* = \frac{\gamma E^*}{\alpha_2+\alpha_3+\mu}$. При этом $R^* = \frac{\rho S^*+\alpha_3 C^*}{\alpha_4+\mu}$.

Линеаризация системы около стационарной точки приводит к матрице Якоби J. Локальная асимптотическая устойчивость равновесия определяется отрицательными действительными частями всех собственных значений:

$$\operatorname{Re}\lambda_i(J) < 0, \qquad i = 1, \ldots, 4.$$

Таким образом, изменение экономических параметров может не только изменить установившийся уровень безработицы, но и существенно повлиять на скорость приближения системы к равновесному состоянию.

Зная начальное состояние

$$X(0) = \begin{pmatrix} S_0 & C_0 & R_0 & E_0 \end{pmatrix}^T,$$

можно получить прогноз

$$X(t) = \begin{pmatrix} S(t) & C(t) & R(t) & E(t) \end{pmatrix}^T$$

на заданном временном интервале.

Управление безработицей может быть введено через управляющие функции $u_1(t)$ и $u_2(t)$, характеризующие соответственно интенсивность программ переподготовки и создание новых рабочих мест:

$$\begin{cases} \frac{dS}{dt} = A - (\alpha_1 + u_2)S - (\rho + u_1)S - \mu S, \\ \frac{dC}{dt} = \gamma E - \alpha_2 C - \alpha_3 C - \mu C, \\ \frac{dR}{dt} = (\rho + u_1)S + \alpha_3 C - \alpha_4 R - \mu R, \\ \frac{dE}{dt} = (\alpha_1 + u_2)S + \alpha_2 C + \alpha_4 R - \gamma E - \mu E. \end{cases} \tag{2}$$

В качестве критерия оптимального управления рассмотрим функционал

$$J(u_1, u_2) = \int_0^T \left[w_1(S + C) + w_2 R + \frac{w_3}{2}(u_1^2 + u_2^2) \right] dt, \tag{3}$$

где $w_1, w_2, w_3 > 0$ — весовые коэффициенты.[4]

Минимизация (3) позволяет найти такой сценарий государственной политики, при котором достигается компромисс между снижением безработицы и затратами на программы занятости.

Для иллюстрации рассмотрим условный набор параметров

$$A = 0.50, \quad \alpha_1 = 0.12, \quad \alpha_2 = 0.10, \quad \alpha_3 = 0.04,$$

$$\alpha_4 = 0.20, \quad \gamma = 0.05, \quad \mu = 0.02, \quad \rho = 0.08.$$

При начальных условиях

$$S(0) = 0.30, \quad C(0) = 0.20, \quad R(0) = 0.05, \quad E(0) = 1.00$$

система (1) может быть решена численно.

Численный эксперимент показывает, что увеличение параметра ρ, соответствующего интенсивности профессиональной переподготовки, приводит к ускорению выхода структурно безработных из состояния безработицы. Одновременно увеличение α_4 сокращает время пребывания работников в состоянии переподготовки и способствует росту занятости. Таким образом, математическая модель позволяет сравнивать различные сценарии государственной политики. В частности, эффективным механизмом управления является одновременное увеличение ρ и α_4, то есть расширение программ переподготовки при наличии достаточного количества рабочих мест для квалифицированных работников.

В работе построена математическая модель динамики безработицы с учетом структурной и циклической безработицы, профессиональной переподготовки и занятости. Полученная система дифференциальных уравнений позволяет исследовать основные характеристики рынка труда, включая равновесные состояния, устойчивость и временную динамику уровня безработицы. Предложенная модель объединяет задачи прогнозирования и управления. Численное моделирование показывает, что программы профессиональной переподготовки и увеличение скорости трудоустройства способны существенно уменьшить структурную составляющую безработицы. В дальнейшем модель может быть расширена за счет учета миграции, сезонности, демографических изменений, автоматизации и дробных производных, позволяющих описывать эффекты памяти на рынке труда.

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ОБОБЩЕНИЕ РОМБОВИДНОГО УСЛОВИЯ ДЛЯ ИНТЕРВАЛЬНОЙ ПЯТИТОЧЕЧНОЙ ПРОГОНКИ

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Решение систем с пятидиагональными матрицами возникает при дискретизации дифференциальных уравнений повышенной точности и в строительной механике [1]. При интервальной неопределённости коэффициентов классическая пятиточечная прогонка сталкивается с двумя проблемами: делением на интервал, содержащий ноль, и экспоненциальным ростом ширины решения с ростом размерности n. Для трёхдиагональных систем Остыловским [3] было предложено «ромбовидное условие», гарантирующее корректность и ограниченность ширины. Цель работы — распространение этого подхода на пятидиагональный случай.

Введём стандартные обозначения интервального анализа [2], т.е. интервальные величины обозначаются полужирным шрифтом, например, как $\mathbf{a}, \mathbf{b}, \mathbf{x}$. Для интервала $\mathbf{a} = [\underline{a}, \overline{a}]$ определены: $\langle \mathbf{a} \rangle = \min_{a \in \mathbf{a}} |a|$ — близость к нулю (mignitude); $\mathrm{wid}(\mathbf{a}) = \overline{a} - \underline{a}$ — ширина; $|\mathbf{a}| = \max(|\underline{a}|, |\overline{a}|)$ — модуль. Используется знаковая близость к нулю:

$$\langle \mathbf{a} \rangle_0 = \begin{cases} \langle \mathbf{a} \rangle, & \text{если } \underline{a} \geq 0 \text{ или } 0 \in \mathbf{a}, \\ -\langle \mathbf{a} \rangle, & \text{если } \overline{a} < 0. \end{cases}$$

Геометрически $\langle \mathbf{a} \rangle_0$ представляет собой ближайшую к нулю точку интервала, взятую со знаком.

Рассмотрим систему $Au = f$ с пятидиагональной матрицей $A = \mathrm{pentadiag}(e_i, a_i, b_i, c_i, d_i)$:

$$e_i u_{i-2} + a_i u_{i-1} + b_i u_i + c_i u_{i+1} + d_i u_{i+2} = f_i.$$

Классический метод прогонки ищет решение в виде $u_i = \alpha_i u_{i+1} + \beta_i u_{i+2} + \gamma_i$, где α_i, β_i — прогоночные коэффициенты, связывающие соседние неизвестные, а γ_i — вспомогательный вектор, аккумулирующий правую часть. Знаменатели Δ_i, возникающие в прямом ходе, имеют вид

$$\Delta_i = e_i(\alpha_{i-2}\alpha_{i-1} + \beta_{i-2}) + a_i\alpha_{i-1} + b_i.$$

В интервальном варианте все операции заменяются интервальными. Корректность метода равносильна условию $0 \notin \mathbf{\Delta}_i$ для всех i.

Предположим, что $0 \notin \mathbf{b}_i$, и проведём нормировку: $\mathbf{p}'_i = \mathbf{p}_i / \langle \mathbf{b}_i \rangle_0$, $\mathbf{p} \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}\}$. После нормировки $\langle \mathbf{b}'_i \rangle_0 = 1$ (штрихи далее опускаем).

Определение. Нормированная матрица $\mathbf{A}$ удовлетворяет *обобщённому ромбовидному условию* с параметром $r \in (0, r_0]$, $r_0 \approx 0,7549$ (корень $1 - r^2 - r^3 = 0$), если для всех i:

$$r^2(|\mathbf{a}_i| + |\mathbf{e}_i|) + (|\mathbf{c}_i| + |\mathbf{d}_i|) \leq r. \tag{1}$$

Геометрически это означает, что точка $(|\mathbf{a}_i| + |\mathbf{e}_i|,\ |\mathbf{c}_i| + |\mathbf{d}_i|)$ лежит внутри ромба. При $\mathbf{e}_i = \mathbf{d}_i = 0$ (1) переходит в условие Остыловского [3].

Теорема 1 (Корректность). Пусть выполнено (1) и дополнительно

$$|\mathbf{a}_i|, |\mathbf{e}_i| \le \frac{r}{1+r^2}. \tag{2}$$

Тогда: 1) $0 \notin \boldsymbol{\Delta}_i$ для всех i; 2) все прогоночные коэффициенты $\boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \boldsymbol{\gamma}_i$ ограничены по модулю константами, зависящими только от r; 3) $\Xi_{uni}(\mathbf{A}, \mathbf{f}) \subseteq \mathbf{u}$ (интервальный вектор гарантированно содержит все точечные решения).

Идея доказательства. Для $i = 1$: $\boldsymbol{\Delta}_1 = \mathbf{b}_1$, $\langle \mathbf{b}_1 \rangle_0 = 1$. Для $i = 2$: $\langle \boldsymbol{\Delta}_2 \rangle_0 \ge 1 - |\mathbf{a}_2| r \ge 1/(1+r^2) > 0$. Индукционный переход: из (1) и (2) получаем

$$\langle \boldsymbol{\Delta}_i \rangle_0 \ge 1 - |\mathbf{e}_i|(r^2 + r) - |\mathbf{a}_i| r \ge 1 - \frac{r}{1+r^2} \cdot r(r+2) = \frac{1 - r^2 - r^3}{1+r^2} > 0,$$

так как $r \le r_0$. Ограниченность α_i, β_i следует из тех же неравенств и формулы для α_i. Включение $\Xi_{uni} \subseteq \mathbf{u}$ вытекает из монотонности интервальных операций. $\square$

Получим оценки ширины решения. Пусть

$$\lambda = \max_i \operatorname{wid}(\mathbf{a}_i, \mathbf{b}_i, \mathbf{c}_i, \mathbf{d}_i, \mathbf{e}_i, \mathbf{f}_i), \quad \overline{f} = \max_i |\mathbf{f}_i|.$$

Из формул прогонки и свойств интервальной арифметики ($\operatorname{wid}(x \pm y) = \operatorname{wid} x + \operatorname{wid} y$, $\operatorname{wid}(xy) \le |x| \operatorname{wid} y + |y| \operatorname{wid} x$) выводятся оценки:

$$\operatorname{wid}(\boldsymbol{\Delta}_i) \le C_\Delta \lambda, \quad \operatorname{wid}(\boldsymbol{\alpha}_i) \le C_\alpha \lambda, \quad \operatorname{wid}(\boldsymbol{\beta}_i) \le C_\beta \lambda,$$

где константы зависят только от r. Применяя их к обратному ходу ($u_n = \gamma_n$, $u_i = \alpha_i u_{i+1} + \beta_i u_{i+2} + \gamma_i$), получаем главный результат.

Теорема 2 (Оценка ширины). В условиях теоремы 1 для компонент $\mathbf{u}$ справедливы равномерные по n оценки:

$$|\mathbf{u}_i| \le \frac{C_1 \overline{f}}{\delta^2}, \qquad \operatorname{wid}(\mathbf{u}_i) \le \left(\frac{C_2 \overline{f}}{\delta^4} + \frac{C_3 \overline{f}}{\delta^3} + \frac{C_4}{\delta^2} \right) \lambda,$$

где $\delta = 1 - r$, C_1, C_2, C_3, C_4 — абсолютные константы. Порядок $O(\lambda \overline{f}/\delta^4)$ совпадает с трёхдиагональным случаем [3], что говорит о сохранении асимптотики при добавлении двух диагоналей.

Численные эксперименты подтвердили теоретические выводы. Для матрицы с $\mathbf{b}_i = 6,0$, $\mathbf{a}_i = \mathbf{c}_i = -1,0$, $\mathbf{e}_i = \mathbf{d}_i = -0,5$ при $\varepsilon = 2\%$ все точные решения попали в интервальные, максимальная ширина для $n = 100$ составила около 3,0 (без экспоненциального роста). Более того, метод работает даже при нарушении классического диагонального преобладания: для $\mathbf{b}_i = 2,0$, $\mathbf{a}_i = -1,4$, $\mathbf{c}_i = -0,15$, $\mathbf{e}_i = -0,3$, $\mathbf{d}_i = -0,1$ сумма модулей внедиагональных (0,975) превышает $r = 0,9$, но условие (1) выполнено, и алгоритм корректен. Зависимость ширины от n оказалась сублинейной ($\propto n^{0,45}$), а от ε — линейной, что согласуется с теоремой 2.

Дополнительно была проведена статистическая верификация свойства включения методом Монте-Карло. Для системы с $n = 8$ и $\varepsilon = 10\%$ было сгенерировано 1000 случайных точечных систем, коэффициенты которых выбирались равномерно из соответствующих интервалов. Каждая точечная система решалась классической пятиточечной прогонкой, и проверялось попадание полученного решения в интервальный вектор $\mathbf{u}$. Во всех 1000 случаях (100%) точечные решения полностью содержались в $\mathbf{u}$, что подтверждает гарантирующий характер интервального алгоритма.

Таким образом, в настоящей работе предложено обобщение ромбовидного условия Остыловского на случай пятидиагональных интервальных систем линейных алгебраических уравнений. Сформулированы и доказаны достаточные условия корректности интервального метода пятиточечной прогонки, гарантирующие отсутствие делений на интервалы, содержащие ноль. Получены априорные оценки модуля и ширины компонент интервального решения, не зависящие от порядка системы n и имеющие порядок $O(1/\delta^4)$, что качественно совпадает с трёхдиагональным случаем. Численные эксперименты подтвердили теоретические выводы: метод устойчив, ширина растёт сублинейно по n и линейно по уровню неопределённости. Главный практический результат — продемонстрирована работоспособность метода для матриц без диагонального преобладания. Перспективы дальнейших исследований включают доказательство неулучшаемости обобщённого ромбовидного условия, его возможное ослабление, а также обобщение на системы с большим числом диагоналей (семидиагональные и более) и применение к прикладным задачам строительной механики и гидродинамики с интервальной неопределённостью.

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Об одной математической модели задачи теории фильтрации в двухслойном пласте

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Построением математической модели задачи фильтрации неньютоновских флюидов занимается определенные числа научных исследователей. Эти исследования проводились сначала для однослойных пластов при различных модификациях нелинейных эффектов с учетом законов движения флюидов. В дальнейшем по мере разработки различных математических моделей и их применение в реальных месторождениях полезных ископаемых направления моделирований расширилось и стали появляться математические модели и для многопластовых пористых сред [1-3]. В зависимости от пластовых характеристик эти среды могут быть взаимосообщаюшими или изолированными. Для неизолированных пластов пластов когда хорошо проницаемые пласты чередуются плохо проницаемыми разработаны различные модели перетекание [4,5]. Изолированные пласты, когда движения флюида могут происходит только по горизонтали, а кровли и подошвы пласта непроницаемые, запасы флюида находится как бы ловушке. А если такие пласты соседствует как слоистые и имеют наличные объемы, то возникает задачи разработки многослойных изолированных пластов. В зависимости от мощности пласта и запаса этих пластов их разрабатывает через отдельно пробуренных скважин соединяющие эти пласты. Поэтому и в научном и в промысловым смысле необходимо разработки математических моделей и методов их решения с учетом

законов движения этих флюидов. Так как они изолированные то естественно пластовые и фильтрационные характеристики пласта и флюида а также структурные свойства флюидов могут быть различными. Следует изучить их, когда пласты сообщаются через совместной скважины, а движения флюидов в них подчиняется различным законам фильтрации. Предположим имеется двух пластовая изолированная среда (области D_1 8 D_2), где нижний пласт (D_1) насыщенно флюидом подчиняющему закону с начальным градиентом давления, а верхний (область D_2) насыщенно флюидом подчиняющим криволинейному [5,6] закону фильтраций. Математическая модель этой физической задачи можно сформулировать так: Необходимо найти непрерывную функцию $U(x,t)$ и $V(x,t)$, а также неизвестные подвижные границы $l(x,t)$ и $R(x,t)$ из следующей системы дифференциальных уравнений.

$$\frac{\partial}{\partial x}\left(\frac{k_1H_1}{\mu_1}\left(1-\frac{\gamma_1\beta_1}{|\nabla U|}\right)\frac{\partial U}{\partial x}\right)=M_1(x,t)\frac{\partial U}{\partial t}, t\in T, x\in(x_0,l(x,t)),\ |\nabla U|>\beta, \tag{1}$$

$$\alpha_0\left.\frac{dU}{dx}\right|_{x=l(t)}=\gamma_1\beta_1,\ U(x,t)|_{x=l(t)}=U_0(x),\ x\in(l(x,t);L_1), \tag{2}$$

$$\frac{\partial}{\partial x}\left(\frac{k_2H_2}{\mu_2}\left(1-\frac{\gamma_2\beta_2}{|\nabla V|}\right)\frac{\partial V}{\partial x}\right)=M_2(x,t)\frac{\partial V}{\partial t},\ t\in T,\ |\nabla V|>\beta_2,\ x\in(x_0,R(x,t)), \tag{3}$$

$$\frac{\partial}{\partial x}\left(\frac{k_2H_2}{\nu_2}\frac{\partial V}{\partial x}\right)=M_2(x,t)\frac{\partial V}{\partial t},\ t\in T,\ |\nabla V|<\beta_2,\ x\in(R(x,t);L_2), \tag{4}$$

С начальными

$$U(x,0)=U_0(x)=V(x,0)=V_0(x),\ l(0,0)=R(0,0)=0, \tag{5}$$

И граничными условиями

$$\alpha_1\left.\frac{dV}{dx}\right|_{x=R(t)-0}=\alpha_2\left.\frac{dV}{dx}\right|_{x=R(t)+0}=\beta_2, \tag{6}$$

$$V(x,t)|_{x=R(t)-0}=V(x,t)|_{x=R(T)+0}, \tag{7}$$

$$\frac{k_1H_1}{\beta_1}\left(1-\frac{\gamma_1\beta_1}{|\nabla U|}\right)\left.\frac{\partial U}{\partial x}\right|_{x=x_0}+\frac{k_2H_2}{\nu_2}\left.\frac{\partial V}{\partial x}\right|_{x=x_0}=q_0(t), \tag{8}$$

$$\alpha_3\left.\frac{dV}{dx}\right|_{x=L_2}=0, \tag{9}$$

Задача (1)-(9) является нелинейной начально-краевой задачей. Здесь k_1, μ_1, k_2, μ_2, ν_2, β_1, β_2, γ_1,γ_2, H_1,H_2-параметры пласта и флюидов выражающей проницаемости , динамической и кинематической вязкости флюида а также мощности пластов. Функции $M_1(x,t)$, $M_2(x,t)$ связаны с пористостью среды и типами флюида, $U_0(x)$, $V_0(x)$-начальные значения флюида. β_1, β_2 –значения критических градиентов давления, α_0, α_1, α_2, α_3 -параметры балансирующие размерности уравнений.

Для построения численных алгоритмов решения этой задачи используется методы итерации, челночных итераций, метод прямых по t и потоковый вариант[4,7] метода разностной прогонки. Влияния возмущений флюидов после начала разработки пласта учитывается через условий (8)) с использованием метода смены стационарных состояний.

Численные алгоритмы апробированы с использованием тестовых данных. Анализ результатов позволяет сделать вывод о применимости данного метода моделировании для конкретных месторождений флюидов, имеющие такие же геометрические характеристики как в постановке задачи.

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Алгоритм решения игровой задачи с произвольными ситуациями

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Рассмотрим множества

$$X = \{x \mid f_* \le x \le f^*\}, Y(x) = \{y \mid g_* \le y \le g^*, Ax + By = b\}.$$

Здесь x, f_*, $f^* \in \mathbb{R}^n$, y, g_*, $g^* \in \mathbb{R}^l$, $b \in \mathbb{R}^m$, $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times l}$, $\mathrm{rank}B = m < l$.

Пусть имеются два игрока, которые выбирают векторы x и y соответственно из множеств $X, Y(x)$ поочередно, сначала первый игрок выбирает x, затем, зная x, второй игрок выбирает y.

Цель первого игрока — найти $\hat{x}$, доставляющий максимальное значение функции

$$\varphi(x) = \min_{y\in Y(x)} \Psi(x,y), x \in X, \text{т.е. } \varphi(\hat{x}) = \max_{x\in X} \varphi(x),$$

цель второго игрока — найти $\hat{y}$, минимизирующий функцию

$$\Psi(x,y), y \in Y(x), \text{ т.е. } \Psi(x,\hat{y}) = \min_{y\in Y(x)} \Psi(x,y).$$

$$\text{Здесь } \Psi(x,y) = \begin{cases} c'x + d'y, \text{ если } x \in X, y \in Y(x), c \in \mathbb{R}^n, d \in \mathbb{R}^l; \\ +\infty, \text{ если } x \in X, Y(x) = \varnothing. \end{cases}$$

Тогда имеем максиминную задачу [1]:

$$\varphi(x) = \min_{y\in Y(x)} \Psi(x,y) \to \max_{x\in X}. \tag{1}$$

Векторы $x \in X$ и $y \in Y(x)$ называются соответственно стратегией первого игрока и x-стратегией второго игрока.

В данной работе предлагается модификация алгоритма [1] решения задачи (1), позволяющая значительно уменьшить число математических операций. Она основана на использовании формулы из [2] для подсчета обратной матрицы при смене одного элемента опоры [3] внутренней задачи (1).

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Оценка погрешности, существовании и единственности оптимальной квадратурной формулы типа эрмита в пространстве Соболева $W_2^m(R)$.

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В этой работе рассматривается квадратурные формулы, которые участвует значения функции и значения ее производных некоторого порядка. Если нам известны не только значения функции $f(x)$ в точках $x_0, x_1, ..., x_N$ на отрезке $[0, 1]$, но и значения ее производных того или иного порядка, то естественно, что при правильном использовании всех этих данных мы можем ожидать более точный результат, чем в случае использования только значений функции, т.е. такие формулы называется квадратурные формулы типа Эрмита ([1–3]).

В настоящей работе именно рассматривается весовые квадратурные формулы типа Эрмита и займёмся нахождением экстремальной функции, существование и единственности этих формул в непериодическом пространстве С.Л.Соболева $W_2^m(R)$.

Рассмотрим следующую весовую квадратурную формулу:

$$\int_0^1 p(x) f(x) dx \approx \sum_{\beta=0}^{N} \sum_{\alpha=0}^{m-1} C_\beta^{(\alpha)} f^{(\alpha)}(x_\beta), \tag{1}$$

с функционалом погрешности

$$\ell_N^{(\alpha)}(x) = \varepsilon_{[0,1]}(x) p(x) - \sum_{\beta=0}^{N} \sum_{\alpha=0}^{m-1} (-1)^{(\alpha)} C_\beta^{(\alpha)} \delta^{(\alpha)}(x - x_\beta), \tag{2}$$

где соответственно, $C_\beta^{(\alpha)}$ и x_β называют коэффициентами и узлами квадратурной формулы (1) и $p(x)$ весовая функция, $\varepsilon_{[0,1]}(x)$-индикатор отрезка $[0,1]$, $\delta(x)$- дельта- функция Дирака и назовем ее квадратурную формулу типа Эрмита. Качество квадратурной формулы оценивается при помощи нормы функционала погрешности:

$$\left\| \ell_N^{(\alpha)}(x) \,\middle|\, W_2^{m*}(R) \right\| = \sup_{\|f\|\neq 0} \frac{\left|\ell_N^{(\alpha)}(f)\right|}{\|f|\, W_2^m(R)\|}. \tag{3}$$

Норма функционала погрешности $\ell_N^{(\alpha)}(x)$ зависит от коэффициентов C_β и узлов x_β. Если

$$\left\| \overset{0}{\ell}{}_N^{(\alpha)}(x) \,\middle|\, W_2^{m*}(R) \right\| = \inf_{C_\beta,\, x_\beta} \left\| \ell_N^{(\alpha)}(x) \,\middle|\, W_2^{m*}(R) \right\|, \tag{4}$$

то говорят, что функционал $\overset{0}{\ell}{}_N^{(\alpha)}$ соответствует оптимальной квадратурной формуле в $W_2^m(R)$. В работе [3] вычислена коэффициентов оптимальных решетчатых весовых квадратурных формул в пространстве С.Л.Соболева $W_2^m(R)$. В настоящей работе мы занимаемся вычислением нормы $\left\|\ell_N^{(\alpha)}(x)|W_2^{m*}(R)\right\|$ функционала погрешности $\ell_N^{(\alpha)}(x)$ и нахождения экстремальная функция функционала погрешности весовой квадратурной формулы типа Эрмита и существованием и единственности.

Таким образом следует

Теорема 1. Экстремальная функция функционала погрешности (2) квадратурной формулы (1) является

$$\psi_\ell^{(\alpha)}(x) = \int_{-\infty}^{\infty} \varepsilon_{[0,1]}(y)\nu_m^{(\alpha)}(x-y)dy - \sum_{\beta=0}^{N}\sum_{\alpha=0}^{m-1} C_\beta^{(\alpha)}\nu_m^{(\alpha)}(x-h\beta) \tag{5}$$

квадрат нормы функционала погрешности в пространстве $W_2^\mu(R)$ имеет вид

$$\left\|\ell_N^{(\alpha)}(x)|W_2^{m*}(R)\right\| = \int_{-\infty}^{\infty} \left| \varepsilon_{[0,1]}(y)\nu_\ell^{(\alpha)}(x-y)dy - \sum_{\alpha=0}^{m-1}\sum_{\beta=0}^{N} C_\beta \nu_m^{(\alpha)}(x-h\beta) \right|^2 dx, \tag{6}$$

где

$$\nu_m(x) == \frac{\pi\cdot e^{-2\pi|x|}}{2^{2m-2}(m-1)!}\sum_{k=0}^{m-1}\frac{(2m-k-2)!(2\pi)^k}{k!(m-k-1)!}|x|^k. \tag{7}$$

Справедливо следующая.

Лемма. Система $\left\{\nu_{m/2}^{(\alpha)}(x-\beta h)\right\}_{\beta=0}^{N}$ является линейно независимой системой в пространстве $L_2(R)$ и линейная оболочка этой системы является $(N+1)$ мерным подпространством в $L_2(R)$.

Решение системы

$$\sum_{\beta=0}^{N}\sum_{\alpha=0}^{m-1} C_\beta^{(\alpha)}\nu_m^{(\alpha)}(x-h\beta) = \int_{-\infty}^{\infty} p(y)\nu_m^{(\alpha)}(h\theta-y)dy, \quad \alpha = 0,1,...,N. \tag{8}$$

есть оптимальные коэффициенты квадратурной формулы (1).

Из леммы и существование и единственности наилучшего приближения подпространством следует следующая

Теорема 2. Оптимальная квадратурная формула, коэффициенты который является решением системы линейных уравненный (8) в пространстве С.Л.Соболева $W_2^m(R)$, существует и она является единственная.

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Оптимальные неявные разностные формулы типа Адамса – Башфорта в фактор пространстве Соболева $L_2^{(2)}(0,1)$

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В общем случае неявная разностная формула имеет вид

$$\sum_{\beta=0}^{k} C[\beta]\varphi[\beta] - h\sum_{\beta=0}^{k} C^{(1)}[\beta]\varphi'[\beta] \cong 0. \tag{1}$$

Функционал погрешности этой формулы выражается формулой

$$\ell^{\text{н.яв}}(x) = \sum_{\beta=0}^{k} C[\beta]\delta(x-h\beta) + h\sum_{\beta=0}^{k} C^{(1)}[\beta]\delta'(x-h\beta)$$

в пространстве $L_2^{(2)}(0,1)$.

В данной работе мы изучаем случай когда коэффициенты имеют вид $C[k]=1$, $C[k-1]=-1$, $C[k-i]=0$, $i=\overline{2,n}$, т.е. мы рассмотрим формулу типа Адамса-Башфорта.

Оптимальной среди таких формул вида (1) называется та которая соответствующая функционал погрешности имел наименьшую норму в сопряженном пространстве.

В данной работе для оптимальной разностной формулы получен следующий рещультат.

Теорема 1. В фактор пространстве Соболева $L_2^{(2)}(0,1)$, существует единственная оптимальная неявная разностная формула, типа Адамса-Башфорда $\varphi[k] - \varphi[k-1] \cong h\sum\limits_{\beta=0}^{k} \overset{\circ}{C}{}^{(1)}[\beta]\,\varphi'[\beta]$ коэффициенты которой определяются формулами

$$\overset{\circ}{C}{}^{(1)}[\beta] = \begin{cases} \frac{1}{2}, & \beta = k, \\ \frac{1}{2}, & \beta = k-1, \\ 0, & \beta = 1,2,...,k-2. \end{cases}$$

Доказательство. Используя метод Лагранжа неопределенных множителей для коэффициентов оптимальной разностной формулы получена система

$$h\sum_{\beta=0}^{k} C^{(1)}[\gamma]\frac{|h\beta - h\gamma|}{2} + a = f_1[\beta], \qquad \beta = \overline{0,k},$$

$$h\sum_{\beta=0}^{k} C^{(1)}[\beta] = 1.$$

Решая данную систему методом предложенным Соболевым получены выражения для оптимальных коэффициентов [1].

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Смешанное уравнение теплопроводности с законом Джоуля-Ленца для электрической дуги

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Объектом исследования в данной работе, является математическая модель электрической дуги, возникающая в высоковольтных сетях при коротких замыканиях. Как известно, короткое замыкание - это аварийное соединение двух точек электрической цепи с разными потенциалами, при которой электрический ток I течет по кратчайшему пути, минуя полезную нагрузку, в следствии чего его сила резко и многократно возрастает, что согласно закону Джоуля — Ленца ($\sim I^2$), приводит к значительному тепловыделению, и, как следствие, возможно плавление токоведущих элементов с последующим возникновением возгорания и распространением пожара.

Короткое замыкание в одном из элементов энергетической системы способно нарушить её функционирование в целом — у других потребителей может снизиться питающее напряжение, что может привести к повреждению устройства; в трёхфазных сетях при коротких замыканиях возникает асимметрия напряжений, нарушающая нормальное электроснабжение. В больших энергосетях короткое замыкание может вызывать тяжёлые системные аварии, поэтому, для обеспечения защиты от подобных явлений, используются силовые выключатели, которые являются ключевыми коммутационными аппаратами и предназначены для безопасного включения и отключения цепей высокого напряжения (свыше 1000 В) в нормальных и аварийных режимах.

Однако, при размыкании контактов силовых выключателей, по которым течет ток короткого замыкания, между ними, как правило, возникает электрическая дуга, которая представляет из себя мощный, самоподдерживающийся электрический разряд в газах, т.е. плазму, с колоссальным выделением тепла, температура которой составляет от $5000°C$ до $20000°C$ и является одним из самых разрушительных последствий аварий в электросетях.

Для данного высокоинтенсивного теплового процесса, в зазоре между разомкнутыми контактами силового выключателя и последующего аварийного гашения дуги в дугогасительной камере построена математическая модель в контексте термодинамики, которая представляет из себя третью краевую задачу для смешанного уравнения теплопроводности по закону Джоуля – Ленца и с учетом светового излучение тепла [1].

В указанных переходных процессах классические параболические модели теплопроводности, основанные на традиционной теории Фурье для теплового потока q:

$$q = -k\nabla u, \tag{1}$$

где k — коэффициент теплообмена, создают значительные искажения температурных полей u. Следовательно, для высокоинтенсивных кратковременных переключающих процессов классическая теория переноса становится некорректной, и для описания таких систем необходимо использовать локально-неравновесные методы.

Рассмотрим механизм гашения электрический дуги в дугогасительной камере, при токе короткого замыкания в высоковольтной сети (Рис.1).

На Рис.1а, контакты соединены и по ним протекает ток короткого замыкания. До момента времени T_1, релейной защитой фиксируется короткое замыкание и подается сигнал выключателю на размыкание контактов 1 на Рис.1. За период времени $T_1 \leq t \leq 0$, контакты выключателя разъединяются, однако из – за пробоя воздушной изоляции возникает электрическая дуга (2 на Рис.1б), в которой по инерции течет значительный ток, выделяя при этом тепло по закону Джоуля – Ленца: $dT_{JL}(t) = I_m^2(t)Rdt$, и вызывая свечение ($\sim u^4(x,t)$). Т.к. данный ток значительно превышает номинальный $I_m >> I_n$, то примем его значение равным току короткого замыкания.

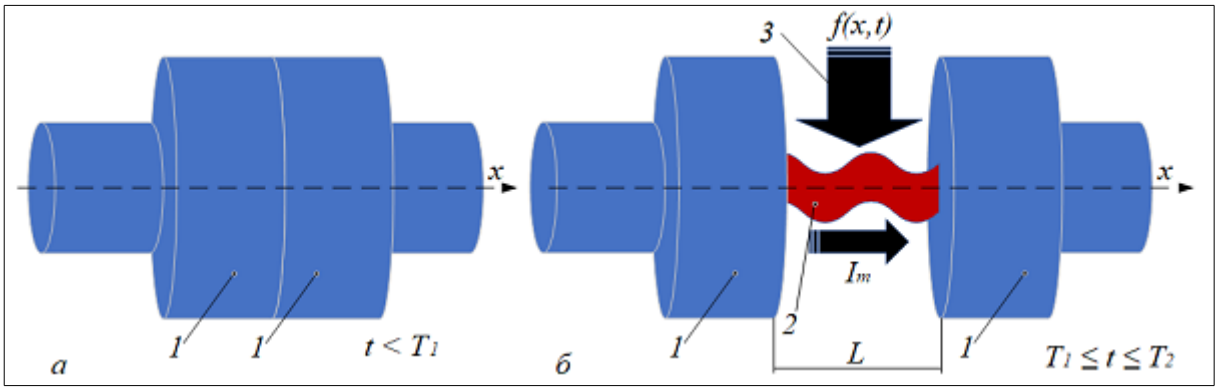


Рис. 7: Схема размыкания воздушного выключателя: 1 – контакт выключателя; 2 – электрическая дуга; 3 - поток воздуха.

В следующем периоде времени при $0 < t \leq T_2$ (Рис.1б) на электрическую дугу воздействует мощный поток воздуха (3 на Рис.1) из резервуара и в момент T_2 она гаснет, в результате цепь обрывается $I_m = 0$ и свечение дуги исчезает. Для учета процесса гашения дуги потоком воздуха в модель вводится управляющая функция $f(x,t)$.

Очевидно, что весь процесс размыкания цепи и подавления дуги протекает мгновенно и крайне интенсивен, поэтому в уравнение теплопроводности (1), включается вторая производная температуры по времени, которую предложили для подобных явлений Вернотт и Каттанео и позже было проанализировано Лыковым [3]. Результатом стало уравнение Максвелла-Каттанео, обобщающее закон Фурье (1) в одномерном случае:

$$q = -k\frac{\partial u}{\partial x} - \tau_r \frac{\partial q}{\partial t}, \tag{2}$$

где τ_r — время релаксации (инерции).

Исходя из вышеизложенного и, применяя уравнение (2) и уравнение теплового баланса к рассматриваемой задаче, получаем следующее гиперболо – параболическое уравнение теплопроводности:

$$k(x,t)\frac{\partial^2 u(x,t)}{\partial t^2} + a(x,t)\frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\lambda(u)\frac{\partial u(x,t)}{\partial x}\right] + \int_{T_1}^{T_2} I_m^2(t)Rdt + cu^4(x,t) + f(x,t), \tag{3}$$

в области $G = [0, L] \times [T_1, T_2]$.

Здесь, при $t \leq 0, k(x,t) = 0, f(x,t) = 0$ а для $t > 0, k(x,t) > 0, f(x,t) \neq 0, k(x,t)$ — коэффициент тепловой релаксации, $a(x,t) = c_v\rho$ – произведение коэффициента удельной теплоемкости при постоянном объеме на коэффициент удельной плотности, $\lambda(u)$ - коэффициент теплопроводности, c - коэффициент бокового теплового оттока за счет свечения, I_m - ток дуги, R - сопротивление дуги.

Начально-краевая задача: найти решение уравнения (3) при выполнении следующих краевых условий третьего рода и начального профиля температуры $u_0(x)$:

$$\left[\mp\lambda(x,t)\frac{\partial u(x,t)}{\partial x} + \alpha_{0,L}u(x,t)\right]_{x=0,L} = q_{0,L}, t \in [T_1, T_2], \tag{4}$$

$$u(x, T_1) = u_0(x), \tag{5}$$

где $\alpha_{0,L}$ и $q_{0,L}$ — коэффициенты теплоотдачи и плотности теплового потока поверхностных источников на левом и правом концах стержня.

Далее, для задачи (3-5) строится консервативная разностная схема (интегро - интерполяционный метод) и численно решается методом прогонки в математическом пакете MathCAD-15 [2]. При этом, т.к. в смешанном уравнении теплопроводности (3) при $t > 0$ присутствует вторая производная по времени, то в конечно-разностной схеме применяется второе начальное условие, которое выводится из уравнения (3) с применением разложения в ряд Тейлора в окрестности точки $t = 0$. Полученные результаты хорошо согласуются с экспериментальными данными.

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ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ ДИНАМИКИ РЕШЕНИЙ УРАВНЕНИЙ НАВЬЕ–СТОКСА В ПОГРАНИЧНОМ СЛОЕ БЛАЗИУСА

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Исследование динамики решений уравнений Навье–Стокса в пограничном слое является актуальной задачей теории гидродинамической устойчивости. В работе [1] рассмотрены методы преобразования неограниченных областей, применяемые при численном исследовании спектральных задач. В работе [2] предложен спектрально-сеточный метод на основе полиномов Чебышёва первого рода с разбиением расчётной области на отдельные элементы. В работе [3] спектральный подход применён для численного решения классической задачи Блазиуса. В настоящей работе спектрально-сеточный метод используется для исследования динамики продольной и поперечной компонент скорости течения Блазиуса.

Безразмерное движение вязкой несжимаемой жидкости описывается системой уравнений Навье–Стокса

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \vartheta\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \\ \frac{\partial \vartheta}{\partial t} + u\frac{\partial \vartheta}{\partial x} + \vartheta\frac{\partial \vartheta}{\partial y} &= -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial^2 \vartheta}{\partial y^2}\right), \\ \frac{\partial u}{\partial x} + \frac{\partial \vartheta}{\partial y} &= 0. \end{aligned} \tag{1}$$

Здесь u и ϑ — продольная и поперечная компоненты скорости, p — давление, Re — число Рейнольдса.

Течение представляется как сумма основного ламинарного решения и малого возмущения:

$$u = U(y) + \widetilde{u}, \qquad \vartheta = \widetilde{\vartheta}, \qquad p = P + \widetilde{p}. \tag{2}$$

Для возмущённого течения вводится функция тока

$$\Psi(x, y, t) = \psi(y)e^{i(kx-\omega t)}, \qquad \lambda = \frac{\omega}{k} = \lambda_r + i\lambda_i, \tag{3}$$

причём

$$\widetilde{u} = \psi'(y)e^{i(kx-\omega t)}, \qquad \widetilde{\vartheta} = -ik\psi(y)e^{i(kx-\omega t)}.$$

После линеаризации системы (1) и исключения давления получаем уравнение Орра–Зоммерфельда

$$\frac{1}{ikRe}\left(\psi^{(4)} - 2k^2\psi'' + k^4\psi\right) - (U-\lambda)\left(\psi'' - k^2\psi\right) + U''\psi = 0, \tag{4}$$

с граничными условиями

$$\psi(\eta_0) = \psi'(\eta_0) = 0, \qquad \psi(\eta_l) = \psi'(\eta_l) = 0.$$

При $\lambda_i > 0$ возмущения возрастают, а при $\lambda_i < 0$ затухают.

Профиль основного течения определяется соотношением $U(\eta) = f'(\eta)$, где $f(\eta)$ удовлетворяет уравнению Фолкнера–Скэна

$$f''' + ff'' + \beta\left[1 - (f')^2\right] = 0. \tag{5}$$

Уравнение (5) решается методом Рунге–Кутты четвёртого порядка, после чего полученный профиль скорости интерполируется в коллокационные узлы полиномов Чебышёва.

В спектрально-сеточном методе [2] область интегрирования разбивается на N элементов, а решение на каждом элементе представляется в виде

$$\psi_j(y) = \sum_{n=0}^{p_j} a_n^{(j)} T_n(y), \qquad j = 1, \ldots, N, \tag{6}$$

где $T_n(y)$ — полиномы Чебышёва первого рода. На границах соседних элементов обеспечивается непрерывность функции ψ и её производных до третьего порядка.

Спектрально-сеточная аппроксимация приводит к обобщённой задаче на собственные значения

$$(A - \lambda B)x = 0. \tag{7}$$

По найденному собственному значению и собственному вектору восстанавливаются функция тока и компоненты скорости возмущённого движения.

При $Re = 580$, $k = 0.179$, $\eta_l = 20$ для одного спектрального элемента при $\bar{m} = 46$ получено

$$\lambda = 0.36419492 + 0.00792907i.$$

В [1] приведено значение

$$\lambda_{\text{ref}} = 0.36412286 + 0.00795972i,$$

при этом абсолютные погрешности составляют

$$|\Delta\lambda_r| = 7.20 \times 10^{-5}, \qquad |\Delta\lambda_i| = 3.06 \times 10^{-5}.$$

Для оценки влияния распределения полиномов расчётная область разбивалась на два элемента. Результаты приведены в табл. 1.

Таблица 1. Результаты вычисления собственных значений

p_1+1	p_2+1	λ_r	λ_i	$\lvert\Delta\lambda_r\rvert$	$\lvert\Delta\lambda_i\rvert$
23	23	0.37184542	0.01451747	$7.7 \cdot 10^{-3}$	$6.5 \cdot 10^{-3}$
27	19	0.36295784	0.00853757	$1.1 \cdot 10^{-3}$	$5.7 \cdot 10^{-4}$
33	13	0.36418391	0.00793507	$6.1 \cdot 10^{-5}$	$2.4 \cdot 10^{-5}$

При распределении 33+13 полиномов абсолютные погрешности достигают порядка 10^{-5}, а порядок комплексных матриц уменьшается с 46×46 до 38×38. Это показывает, что перераспределение полиномов между элементами позволяет снизить размерность алгебраической задачи без существенной потери точности.

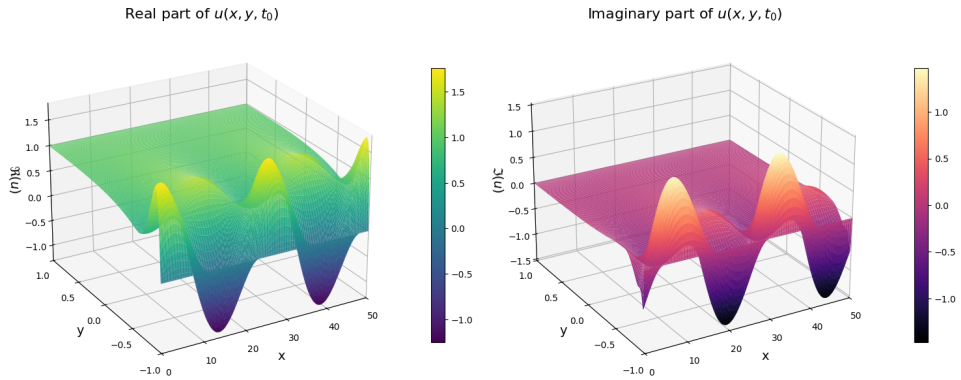


Рис. 1. Действительная и мнимая части продольной компоненты скорости $u(x, y, t_0)$ при $t_0 = 25$.

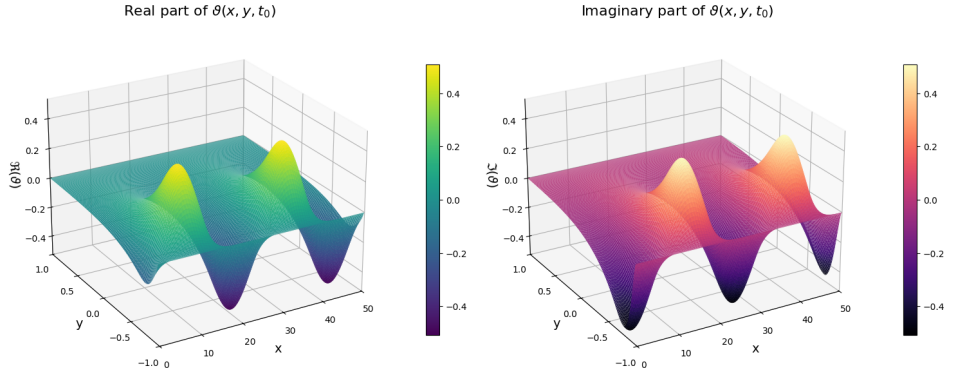


Рис. 2. Действительная и мнимая части поперечной компоненты скорости $\vartheta(x, y, t_0)$ при $t_0 = 25$.

Рис. 1 и 2 характеризуют пространственную структуру продольной и поперечной компонент скорости при $t_0 = 25$. С увеличением времени амплитуды возмущений возрастают по экспоненциальному закону. Поскольку $\lambda_i > 0$, рассматриваемый режим течения Блазиуса является неустойчивым.

Таким образом, спектрально-сеточный метод позволяет с высокой точностью определять собственные значения задачи Орра–Зоммерфельда, восстанавливать компоненты скорости и исследовать развитие неустойчивых возмущений в пограничном слое. Разбиение области на элементы позволяет уменьшить размерность алгебраической задачи при сохранении точности порядка 10^{-5}.

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Численное моделирование краевой задачи для сингулярно возмущенного уравнения второго порядка методом предварительного интегрирования

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В данной статье для численного моделирования краевой задачи для сингулярно возмущённого дифференциального уравнения четвёртого порядка предлагается высокоточный метод предварительного интегрирования. Численное решение сингулярно возмущенной задачи четвертого порядка реакционно-диффузионного типа с применением локального разрывного метода Галеркина изложено в [1]. В работе [2] предложен спектрально-сеточный метод на основе по линомов Чебышёва первого рода с разбиением расчётной области на отдельные элементы. Цель работы [3] заключается в разработке высокоточного и эффективного численного метода для исследования динамики производных различных порядков сингулярно возмущённого дифференциального уравнения второго порядка. Требуется найти приближённое решение сингулярно возмущенного дифференциального уравнения второго порядка:

$$4\varepsilon\frac{d^2y}{dx^2}+2\left(\varepsilon+1\right)\frac{dy}{dx}+y\left(x\right)=f(x),\qquad y\in[-1,1], \tag{1}$$

при следующих краевых условиях:

$$y\left(-1\right)=0,\quad y\left(1\right)=1 \tag{2}$$

где $\varepsilon-$малый параметр $(0<\varepsilon<1)$. Точное решение задачи (1)–(2) имеет следующий вид:

$$y_e(x)=\frac{e^{-(x+1)/2\varepsilon}-e^{-(x+1)/2}}{e^{-1/\varepsilon}-e^{-1}},\quad y_e=y_{exact}. \tag{3}$$

Эта функция удовлетворяет условиям, заданным уравнениями (1)– (2). В методе предварительного интегрирования старшей производной и правая часть уравнения (1) представляются в виде следующих конечных рядов по полиномам Чебышева первого рода

$$y^{(2)}(x)=\sum_{j=0}^{N}a_jT_j(x),f(x)=\sum_{i=0}^{N}b_iT_i(x) \tag{4}$$

где $T_i(y)$-полиномы Чебышева, они определяются по следующей рекуррентной формуле:

$$T_{n+1}(y)=2yT_n(y)-T_{n-1}(y),$$

здесь $c_0=2\,,\ c_n=1\ \ (n\geq1)\ ,\ T_0(y)=1\,,\ T_1(y)=y$. Ряд для $y^{(2)}(x)$ в формуле (4) последовательно дважды интегрируем:

$$\begin{aligned}
y^{(1)}(x)&=\sum_{i=0}^{N+1}\sum_{j=0}^{N}f_{ij}^{(3)}a_jT_i+C_1T_0(x)\,,\\
y^{(0)}(x)=y(x)&=\sum_{i=0}^{N+2}\sum_{j=0}^{N}f_{ij}^{(2)}a_jT(x)_i+C_1T_1(x)+C_2T_0(x)\,,
\end{aligned} \tag{5}$$

где $C_1,\ C_2$ - константы интегрирования, которые определяются из краевых условий (2). Используя краевые условия (2) формулу для y(x) из (5) и следующие свойство полиномов Чебышева $T_n(\pm 1) = (\pm 1)^n$ имеем:

Решая систему (5) определяются константы интегрирования $C_1,\ C_2$:

$$\begin{aligned} C_1 &= \sum_{j=0}^{N} \left[\frac{1}{2} + \frac{1}{2} \left(\bar{s}_j^{(0)} - s_j^{(0)} \right) \right] a_j \\ C_2 &= \sum_{j=0}^{N} \left[\frac{1}{2} - \frac{1}{2} \left(s_j^{(0)} - \bar{s}_j^{(0)} \right) \right] a_j \end{aligned} \tag{6}$$

где $s_j^{(\beta)} = \sum\limits_{i=0}^{N+2-\beta} f_{ij}^{(\beta)}, \qquad \bar{s}_j^{(\beta)} = \sum\limits_{i=0}^{N+2-\beta} (-1)^i f_{ij}^{(\beta)}, \qquad \beta = 0, 1, 2,$ параметр β указывает порядок производной.Функции $f_{ij}^{(\beta)}$при $\beta = 0, 1$, в формула (6) вычисляются по формулам

$$\begin{aligned} f_{ij}^{(1)} &= \delta_{i,j+1} \beta_j^{(1)} + \delta_{i,j-1} \gamma_j^{(1)}, \\ f_{ji}^{(0)} &= \delta_{i,j+2} \beta_j^{(0)} + \delta_{i,j} \gamma_i^{(0)} + \delta_{i,j-2} \delta_i^{(0)}, \end{aligned}$$

где δ_{ij}-символ Кронекера,а ненулевые коэффициенты $\beta_j, \gamma_j, \delta_j$ вычисляются по следующим формулам:.

$$\beta_j^{(1)} = \frac{c_j}{2(j+1)},\ j \geq 0, \qquad \beta_j^{(0)} = \frac{\beta_j^{(1)}}{2(j+2)},\ j \geq 0,$$

$$\gamma_j^{(1)} = \frac{-1}{2(j-1)},\ j \geq 2, \qquad \gamma_j^{(0)} = \frac{-\beta_j^{(1)} + \gamma_j^{(1)}}{2j},\ j \geq 0,$$

$$\delta_j^{(0)} = \frac{-\gamma_j^{(1)}}{2(j-2)},\ j \geq 3.$$

Учитывая значения этих констант в коэффициентах интегрирования составляются суммы:

$$u^{(\beta)}(y) = \sum_{i=0}^{N+2-\beta} \sum_{j=0}^{N} g_{ij}^{(\beta)} a_j T_i, \quad \beta = 0, 1\,, \tag{7}$$

Точки коллокации для полиномов Чебышева имеют вид:

$$y_l = \cos\left(\frac{\pi l}{N} \right), \qquad l = 0, 1, \ldots, N$$

Коэффициенты разложения b_i для правой части $f(x)$ в (4) вычисляются по формуле:

$$b_i = \frac{2}{N c_i} \sum_{l=0}^{N} \frac{1}{c_l} f(y_l) T_i(y_l), \qquad i = 0, 1, \ldots, N \tag{8}$$

Подставляя ряды (5) и (8) в дифференциальное уравнение (1) и приравнивая коэффициенты при одинаковых степенях полиномов Чебышева имеем следующую систему

$$\sum_{k=0}^{N} \left[4\varepsilon \delta_{jk} + 2(\varepsilon + 1) g_{jk}^{(1)} + g_{jk}^{(0)} \right] a_k = b_j \tag{9}$$

где j = 0, 1, . . . , N. Систему (9) удобно записать в матрично-векторной форме

$$A \cdot a = \text{b}.$$

Приводятся результаты расчётов по исследованию динамики приближенного решения и ее производных до второго порядка включительно при различных значениях характерных параметров: малого параметра ε и число полиномов N.

Таблица 5: Сравнение точных и приближенных значений решения и его производных

№	$S\{y_j\}_1^N$	Решение	Производная	
			Первая	Вторая
50	10	2.646e−12	3.025347e−11	2.314424e−08
	20	5.865530e−12	1.750248e−10	1.709739e−08
	30	3.369971e−12	3.259345e−10	8.221678e−09
	40	1.024070e−12	2.386380e−10	9.430262e−09
	Максимальная погрешность	7.337686e−12	3.719909e−10	2.725321e−08

Результаты табл.1 графически изображены на рис.1-3

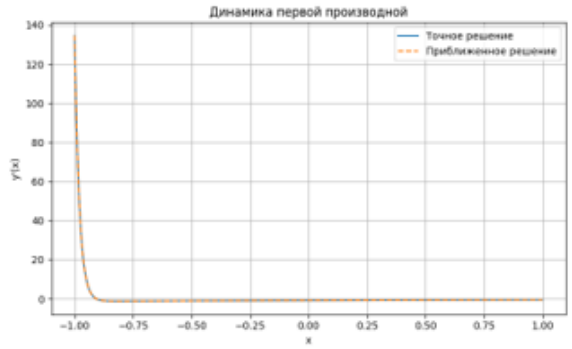


Рис. 1. Динамика первой производной

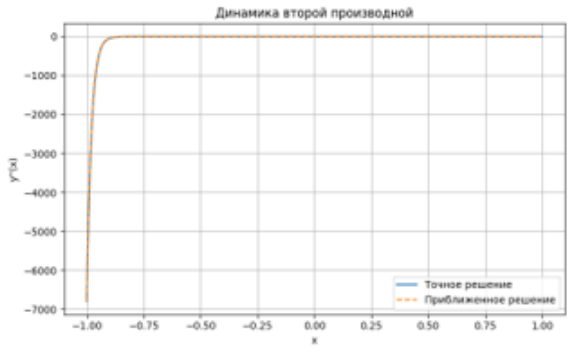


Рис. 2. Динамика второй производной

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О применении полиномов Чебышёва к решению сингулярно возмущённых краевых задач

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Требуется найти приближённое решение сингулярно возмущенного дифференциального уравнения второго порядка:

$$4\varepsilon\frac{d^2y}{dx^2}+2\left(\varepsilon+1\right)\frac{dy}{dx}+y\left(x\right)=f(x),\qquad y\in[-1,1], \tag{1}$$

при следующих краевых условиях:

$$y\left(-1\right)=0,\quad y\left(1\right)=1 \tag{2}$$

где $\varepsilon-$малый параметр $(0<\varepsilon<1)$. Точное решение задачи (1)–(2) имеет следующий вид:

$$y_e(x)=\frac{e^{-(x+1)/2\varepsilon}-e^{-(x+1)/2}}{e^{-1/\varepsilon}-e^{-1}},\quad y_e=y_{exact}. \tag{3}$$

Эта функция удовлетворяет условиям, заданным уравнениями (1)– (2). Для численного моделирования дифференциальной задачи (1)–(2) используется дискретный вариант метода предварительного интегрирования. Приближённое решение y_aпредставляется в виде разложения по полиномам Чебышёва первого рода следующим образом:

$$\frac{dy}{dx}=\sum_{i=0}^{N}{}'a_iT_i\left(x\right) \tag{4}$$

В данной постановке a_i обозначает неизвестный коэффициент, а штрих у знака суммирования указывает, что $i=0$ соответствующий коэффициент учитывается вместе с $\frac{1}{2}$соответствующим множителем. Для вычисления второй производной используется следующее соотношение [2-5]:

$$\frac{d^2y}{dx^2}=\sum_{i=0}^{N}{}'a_i{}^{(2)}T_i\left(x\right). \tag{5}$$

Путём подстановки соотношений (4)–(5) в уравнение (1) получаем следующее выражение:

$$4\varepsilon\sum_{i=0}^{N}{}'a_i{}^{(2)}T_i\left(x\right)+2\left(1+\varepsilon\right)\sum_{i=0}^{N}{}'a_i{}^{(1)}T_i\left(x\right)+\sum_{i=0}^{N}{}'a_iT_i\left(x\right)=0,$$

Из полученного выражения выводим следующее уравнение:

$$4\varepsilon a_i^{(2)}+2\left(1+\varepsilon\right)a_i^{(1)}+a_i=0.$$

Применяя к обеим частям данного уравнения дискретную формулу метода предварительного интегрирования:

$$a_i{}^{(k-1)}=\frac{a_{i-1}{}^{(k)}-a_{i+1}{}^{(k)}}{2i}, \tag{6}$$

выполняем двукратное последовательное интегрирование. Путём сложения и вычитания уравнений, полученных из граничных условий (2), получаем следующую систему уравнений.

$$\begin{cases} (i+1)a_{i-2} + 4(1+\varepsilon)\left(i^2-1\right)a_{i-1} + \left(16\varepsilon i\left(i^2-1\right) - 2i\right)a_i - \\ -4(1+\varepsilon)\left(i^2-1\right)a_{i+1} + (i-1)a_{i+2} = 0, \quad (i=2,3,...,N) \\ \dfrac{1}{2}a_0 + a_2 + a_4 + ... + a_N = \dfrac{1}{2}, \\ a_1 + a_3 + a_5 + ... + a_{N-1} = \dfrac{1}{2}. \end{cases} \tag{7}$$

Система алгебраических уравнений (7) была численно решена, и получены следующие результаты. Решая систему алгебраических уравнений (7) относительно и подставляя найденные значения в формулу приближённого решения (4), вычисляем значения приближённого решения в узловых точках $x_l = \cos\frac{\pi l}{N}$, $l = 0, 1, 2, ..., N$. Для демонстрации высокой точности предложенного метода максимальные абсолютные погрешности приближённого решения в коллокационных узлах полиномов $x_l\,(\,l = 0, 1, 2, ..., N)$ вычисляются по следующей формуле:

$$\Delta = \max_l |y_e(x_l) - y_a(x_l)|$$

.

Таблица 6: Максимальная погрешность Δ точного и приближённого решения.

ε	N	Погрешность	Производная	
			Первая	Вторая
0.1	24	2.401×10^{-15}	1.682×10^{-14}	3.567×10^{-12}
	48	2.387×10^{-15}	5.329×10^{-15}	4.263×10^{-14}
0.01	24	4.372×10^{-3}	0.821	217.238
	48	2.089×10^{-10}	1.508×10^{-7}	1.292×10^{-4}

Графические представления результатов, приведённых в Таблице 1 и полученных с использованием предложенного метода, представлены на рисунках 1–2.

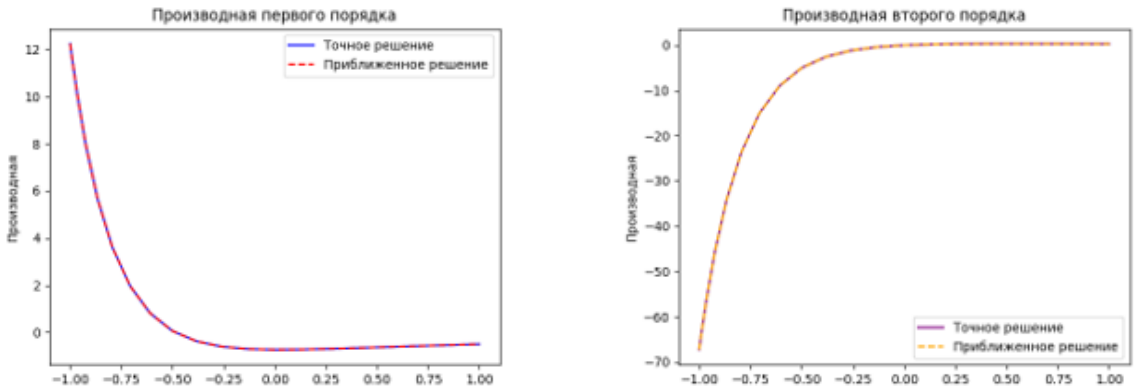


Рис. 8: Динамика производных при $\varepsilon = 0.1$ и N=24.

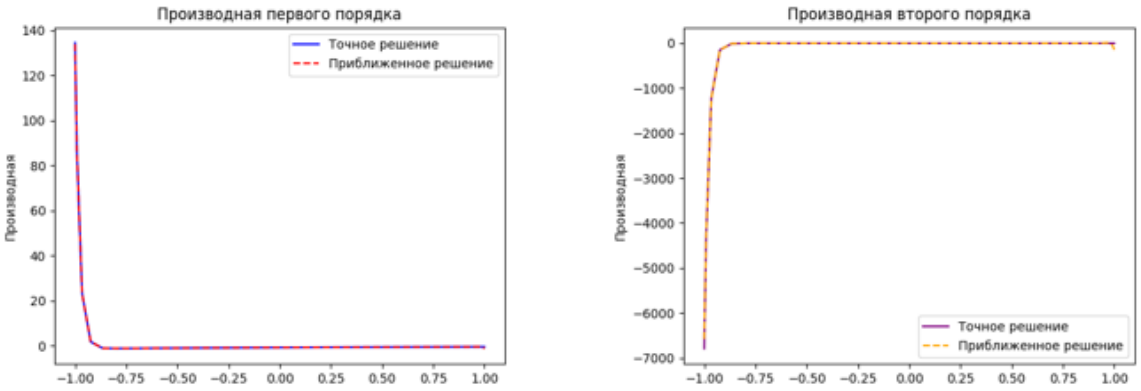


Рис. 9: Динамика производных при $\varepsilon = 0.01$ и N=24.

Стабильный и точный метод обеспечивает, что не только само решение, но и его производные плавно сходятся к точному поведению по мере увеличения числа базисных функций (например, полиномов Чебышёва). Таким образом, анализ динамики производных позволяет проверить сходимость, устойчивость и надёжность вычислительного подхода.

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АЛГОРИТМ РЕШЕНИЯ КРАЕВЫХ ЗАДАЧ ДЛЯ СИСТЕМЫ ОБЫКНОВЕННЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ЧЕТВЕРТОГО ПОРЯДКА С МЕТОДОМ КОНЕЧНЫХ РАЗНОСТЕЙ

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Аннотация: Для численной решения краевых задач систем обыкновенных дифференциальных уравнений четвертого порядка применяется абсолютно устойчивые векторно-разностные схемы с точностью $O(h^2)$, предложенные И.В. Фрязиновым.

Ключевые слова: квадратные матрицы дифференциальных уравнений, граничных условиях, матричной прогонки, вектор функции, абсолютно устойчивые, векторно-разностные схемы.

В данной работе расоматривается вопрос о построении приближенного решения линейных обыкновенных дифференциальных уравнений четвертого порядка с переменными коэффициентами и сравнительно общими краевыми условиями.

Требуется определить в области $[a,b]$ неизвестный вектор функции

$$U(x)=\{U_1(x),U_2(x),\dots,U_n(x)\}$$

удовлетворяющей системе дифференциальных уравнений:

$$\big[K(x)U''(x)\big]''+a_5(x)\big[a_7(x)U''(x)\big]'+a_4(x_1)\big[a_6(x)U'(x)\big]'+$$

$$+a_3(x)U''(x)+a_2(x)U'(x)+a_1(x)U(x)=f(x). \tag{1}$$

записанной в матричной форме при граничных условиях

$$\Big\{\alpha_iU(x)+\beta_iU'(x)+\gamma_ik(x)U''(x)+\theta_i\big[K(x)U''(x)\big]'\Big\}\Big|_{x=a}=d_i. \tag{2}$$

$$\Big\{\alpha_{i+2}U(x)+\beta_{i+2}U'(x)+\gamma_{i+2}K(x)U''(x)+\theta_{i+2}\big[K(x)U''(x)\big]'\Big\}\Big|_{x=b}=d_{i+2}, \tag{3}$$

где $K(x),a_j(x)$ $(j=1,7)$, $d_\nu,\beta_\nu,\gamma_\nu,\theta_\nu$ $(\nu=1,4)$ - заданные квадратные матрицы в порядке n;

$F(x)$ и d_ν - n-мерные векторы функции, причем существует матрица $K^{-1}(x)$ для всех $x\in[a,b]$, а также рассмаривается система дифференциальных уравнений вида

$$\big[K(x)U_1''(x)\big]''+\sum_{j=1}^{2}a_{ij}(x)U_j''(x)+\sum_{j=1}^{2}b_{ij}(x)U_j'(x)+$$

$$+\sum_{j=1}^{2}C_{ij}(x)U_j(x)+\rho_1(x)\big[K_1(x)U_1''(x)\big]'=f_1(x); \tag{4}$$

$$K_2(x)U_2''(x)+\sum_{j=1}^{2}b_{2j}(x)U_j'(x)+\sum_{j=1}^{2}C_{2j}(x)U_j(x)+a_{21}(x)U_1''(x)+$$

$$+\rho_2(x)\big[K_1(x)U_1''(x)\big]'=f_2(x). \tag{5}$$

с соответствующими граничными условиями

$$\left.\begin{aligned}\Big\{\sum_{j=1}^{2} d_{ij}U_j(x)+\sum_{j=1}^{2}\beta_{ij}U_j'(x)+\delta_i K_1(x)U_1''(x)+\rho_i\big[K_1(x)U_1''(x)\big]'\Big\}\Big|_{x=a}&=\varphi_i\\ \Big\{\sum_{j=1}^{2} \eta_{ij}U_j(x)+\sum_{j=1}^{2}\mu_{ij}U_j'(x)+\theta_i K_1(x)U_1''(x)+\gamma_i\big[K_1(x)U_1''(x)\big]'\Big\}\Big|_{x=b}&=\Psi_i\end{aligned}\right\},\quad i=1,2,3; \tag{6}$$

где

$$U_1(x)=\{U_{11}(x),U_{12}(x),\dots,U_{1n(x)}\};\quad U_2(x)=\{U_{21}(x),U_{22}(x),\dots,U_{2n(x)}\}$$

- искомые векторы.

Будем предполагать, что на отрезка $[a,b]$ решение задач (1)-(3), а также (4)-(6) существует и единственно.

Гладкость входных данных и решения задач предположим такими, какие нам будут нужны в каждом случае.

Прейдем к построению численного решения дифференциальной краевой задачи (4)-(6).

действительно сделав в уравнениях (4)-(6) замену перемменой $K_1U_1''=U_0$, перепишем систему (4)-(6):

$$\left.\begin{aligned}U_0''+\sum_{j=0}^{2}\bar b_{1j}U_j'+\sum_{j=0}^{2}c_{1j}U_j&=f_1\\ K_1U_1''-U_0&=0\\ K_2U_2''+\sum_{j=0}^{2}b_{2j}U_j'+\sum_{j=0}^{2}c_{2j}U_j&=f_2\end{aligned}\right\} \tag{7}$$

с граничными условиями

$$\left.\begin{aligned}\Big(\sum_{j=0}^{2}\alpha_{ij}U_j+\sum_{j=0}^{2}\beta_{ij}U_j'\Big)\Big|_{x=a}&=\varphi_i\\ \Big(\sum_{j=0}^{2}\eta_{ij}U_j+\sum_{j=0}^{2}\mu_{ij}U_j'\Big)\Big|_{x=b}&=\Psi_i\end{aligned}\right\} \tag{8}$$

Далее в задаче (7)-(8) введя обозначение

$$\begin{gathered}\bar b_{10}=d_1-a_{12}K_2^{-1},\quad \bar c_{10}=a_{11}K_1^{-1}-a_{12}K_2^{-1}a_{21}K_1^{-1},\quad \bar b_{1K}=b_{1K}-a_{12}K_2^{-1}b_{2K},\\ \bar c_{1K}=-a_{12}K_2^{-1}c_{2K}+c_{1K},\quad K=1,2,\qquad \bar f_1=f_1-a_{12}K_2^{-1}f_2;\qquad \alpha_{i0}=\delta_iK_1^{-1},\\ \beta_{i0}=d_iK_1^{-1},\quad n_{i0}=\theta_iK_1^{-1},\quad \mu_{i0}=\gamma_iK_1^{-1},\quad i=1,2,3.\end{gathered} \tag{9}$$

получаем

$$\begin{pmatrix}U_0\\U_1\\U_2\end{pmatrix}=V$$

здесь

$$KV''+BV'+CV=D; \tag{10}$$

$$\left.\begin{aligned}(\alpha V+\beta V')\big|_{x=a}&=\varphi\\ (\eta V+\mu V')\big|_{x=b}&=\Psi\end{aligned}\right\} \tag{11}$$

$$K=\begin{pmatrix}E & 0 & 0\\ 0 & K_1 & 0\\ 0 & 0 & K_2\end{pmatrix};\qquad B=\begin{pmatrix}\bar{b}_{10} & \bar{b}_{11} & \bar{b}_{12}\\ 0 & 0 & 0\\ \bar{b}_{20} & \bar{b}_{21} & \bar{b}_{22}\end{pmatrix};$$

$$C=\begin{pmatrix}\bar{C}_{10} & \bar{C}_{11} & \bar{C}_{12}\\ -1 & 0 & 0\\ \bar{C}_{20} & \bar{C}_{21} & \bar{C}_{22}\end{pmatrix};\qquad D=\begin{pmatrix}\bar{f}_1\\ 0\\ \bar{f}_2\end{pmatrix}$$

Здесь вводя обозначения

$$\alpha=\{\alpha_{ij}\},\ \beta=\{\beta_{ij}\};\ \varphi=\{\varphi_{ij}\},\ \eta=\{\eta_{ij}\};\ \mu=\{\mu_{ij}\},\ \phi=\{\phi_{ij}\};\quad i=1,2,3;\ j=0,1,2.$$

Теперь для заддачи (10)-(11) можно применить абсолютно устойчивые векторно-разностные схемы с точностью $O(h^2)$, предложенные И.В. Фрязиновым [2];

$$K_i\frac{V_{i+1}^{(1)}-2V_{i-1}^{(1)}+V_{i-1}^{(1)}}{h^2}+\beta_i\frac{V_{i+1}^{(2)}-V_i^{(2)}}{h}+C_iV_i^{(1)}=D_i;$$

$$K_i\frac{V_{i+1}^{(2)}-2V_i^{(2)}+V_{i-1}^{(2)}}{h^2}+\beta_i\frac{V_i^{(1)}-V_{i-1}^{(1)}}{h}+C_iV_i^{(2)}=D_i;$$

Здесь вводя обозначения

$$V_i=\begin{pmatrix}V_i^{(1)}\\ V_i^{(2)}\\ i\end{pmatrix},$$

Приходим к разностным задачам

$$\left.\begin{array}{l}A_iV_{i-1}-C_iV_i+B_iV_{i+1}=F_i\\ A_0V_0-C_0V_1+B_0V_2=-F_0\\ A_NV_{N-2}-C_NV_{V_{n-1}}B_nV_n=-F_N,\quad i=1,2,\ldots,N-1\end{array}\right\}\tag{12}$$

где

$$A_i=\begin{pmatrix}K_i & hB_i\\ 0 & K_i\end{pmatrix};\qquad C_i=\begin{pmatrix}2K_i-h^2C_i & hB_i\\ -hB_i & 2K_i-h^2C_i\end{pmatrix};$$

$$B_i=\begin{pmatrix}K_i & 0\\ -hB_i & K_i\end{pmatrix};\qquad F_i=\begin{pmatrix}h^2D_i\\ h^2D_i\end{pmatrix};\quad A_0=2h\alpha-3\beta;$$

$$C_0=-4\beta;\quad \beta_0=-\beta;\quad F_0=-2h\varphi;\quad A_N=\mu;\quad C_N=4\mu;$$

$$B_N=2h\eta+3\mu;\quad F_N=2h\varphi.$$

задача (12), решается с методом матричной прогонки [3].

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НОВЫЕ РАСЩЕПЛЕННЫЕ СИСТЕМЫ УРАВНЕНИЙ СЕН-ВЕНАНА

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Нахождения решений системы уравнений Сен-Венана актуально следующими ключевыми аспектами. Во-первых, эти уравнения лежат в основе математического описания неустановившихся течений жидкости в открытых руслах. Они применяются при моделировании рек, каналов, водохранилищ и ирригационных систем, что напрямую связано с задачами водного хозяйства. Во-вторых, решения системы необходимы для прогнозирования паводков и наводнений. Точные расчеты позволяют заранее оценивать распространение волны, глубину и скорость потока, что критически важно для защиты населения и инфраструктуры. В-третьих, уравнения Сен-Венана широко используются при проектировании гидротехнических сооружений: плотин, шлюзов, каналов. Надежные решения помогают оптимизировать конструкции и обеспечить их безопасность. Кроме того, развитие численных методов (в частности, метода характеристик) делает задачу нахождения решений особенно актуальной с точки зрения повышения точности и устойчивости вычислений. Наконец, в условиях изменения климата и увеличения экстремальных гидрологических явлений возрастает потребность в более точных и универсальных моделях течений, что усиливает значимость исследований, связанных с системой уравнений Сен-Венана.

В классической работе С.А. Христиановича [1] показано, что неустановившееся движение воды в открытых руслах описывается системой уравнений Сен-Венана, выведенным из законов сохранения массы и импульса, представляющей собой гиперболическую систему уравнений мелкой воды [2-3].

Среди решений системы уравнений Сен-Венана особое место занимает метод характеристик. Однако в существующей теории гидравлики рек в уравнения характеристик не входит параметр, характеризующий число Фруда ($Fr = v_{cp}/\sqrt{gh}$) на выбранном участке реки, и, соответственно, полученные системы уравнений относительно локальных производных по времени от неизвестных функций глубины потока и средней скорости течения не содержат указанного параметра и требуют пересмотра.

В связи с этим целью настоящей работы является нахождение параметризованного вида характеристик и условий вдоль них, и с учётом этого параметра получить расщепленные уравнения для численного расчета гидравлических параметров по времени и координат.

Математическая модель. Уравнения Сен-Венана в консервативной форме имеет вид

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q,$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + gI_1\right) = gA(i_0 - i_f). \tag{1}$$

Здесь A - площадь живого сечения, Q - расход воды, q - приток или отток воды, I_1 - гидростатический член (в призматическом русле $I_1 = By^2/2$, B - ширина реки по поверхности воды, y - глубина потока), i_0- уклон дна, i_f - уклон трения, t - время, x - продольная координата.

Переходим от формы с расходом Q (1) к "классической"системе Сен-Венана в переменных y и v (средняя скорость течения по живому сечению) без бокового притока ($q = 0$). В итоге имеем следующую систему уравнений для дальнейшего исследования

$$\frac{\partial y}{\partial t} + v\frac{\partial y}{\partial x} + y\frac{\partial v}{\partial x} = 0,$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x} - k = 0. \tag{2}$$

Далее, из системы (2) посредством преобразования, то есть умножением второго уравнения на $\pm\sqrt{y/g}$ и сложением с первым уравнением, получим следующую уравнению

$$\frac{\partial y}{\partial t} + v\frac{\partial y}{\partial x} + y\frac{\partial v}{\partial x} \pm \sqrt{\frac{y}{g}}\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x} - k\right) = 0. \tag{3}$$

Нетрудно убедиться, что следующая система дифференциальных уравнений в частных производных, разрешенная относительно производных по времени от неизвестных функций y и v, и содержащий отличный от нуля произвольную константу α удовлетворяет тождественно уравнению (3)

$$\begin{aligned}\frac{\partial y}{\partial t} &= -\frac{1}{\alpha}\left[v\frac{\partial y}{\partial x} + y\frac{\partial v}{\partial x} \pm \sqrt{\frac{y}{g}}\left(v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x}\right)\right] \pm \frac{1}{\alpha}\sqrt{\frac{y}{g}}\cdot k,\\ \frac{\partial v}{\partial t} &= \mp\frac{\alpha-1}{\alpha}\sqrt{\frac{g}{y}}\left[v\frac{\partial y}{\partial x} + y\frac{\partial v}{\partial x} \pm \sqrt{\frac{y}{g}}\left(v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x}\right)\right] + \frac{\alpha-1}{\alpha}\cdot k.\end{aligned} \tag{4}$$

Заметим, что система (4) допускает первый интеграл

$$v \mp 2(\alpha-1)\sqrt{gy} = const. \tag{5}$$

Далее, можно убедиться, что вдоль характеристик

$$\frac{dx}{dt} = v \mp 2(\alpha-1)\sqrt{gy}, \tag{6}$$

система уравнений (2) выполняется тождественно.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ФИЛЬТРОВАНИЯ СУСПЕНЗИЙ С УЧЁТОМ РЕЛАКСАЦИОННЫХ СВОЙСТВ КЕЙК-СЛОЯ

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Следуя [1,2], дадим вывод уравнений фильтрования с образованием релаксирующего кейк-слоя. При этом воспользуемся моделями релаксационной фильтрации, приведенными в [3,4].

В кейк-слое скорость движения частиц гораздо ниже, чем скорость движения жидкой фазы. Скорость движения частиц на границе суспензии и кейк-слоя может приниматься равной скорости движения суспензии, а на границе кейк-слоя с поверхностью фильтра эта скорость равна нулю. Так что скорость фильтрации твердых частиц уменьшается вдоль кейк-слоя от скорости движения суспензии до нуля. Скорость же движения жидкой фазы увеличивается вдоль кейк-слоя от границы с суспензией до границы с фильтром, так как общая скорость фильтрации в соответствии с [1, 2] остается постоянной. В силу этого, темп изменения скорости фильтрации твердой фазы по сравнению с жидкой фазой можно считать замедленным. Это обстоятельство позволяет пренебречь релаксационными эффектами скорости фильтрации твердой фазы по сравнению с жидкой фазой. Закон Дарси для относительной скорости фильтрования запишем

$$\left(1+\lambda_{q\ell}\frac{\partial}{\partial t}\right)\frac{q_\ell}{\varepsilon}-\frac{q_s}{\varepsilon_s}=-\frac{1}{\varepsilon}\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_\ell}{\partial x}, \tag{1}$$

где индекс l - для жидкой фазы, s - для твердых частиц, ε_i - пористость i-ой фазы (здесь и далее вместо ε_ℓ используется просто ε), $\varepsilon+\varepsilon_s=1$, q_ℓ и q_s соответственно скорость жидкой и твердой фазы, p_ℓ - давление в жидкой фазе, k - проницаемость, μ - вязкость, $\lambda_{q\ell}$ - времена релаксации скоростей фильтрации q_ℓ, $\lambda_{p\ell}$ - время релаксации градиента давления, t - время.

Для простоты рассмотрим сначала случай с релаксацией градиента давления, т.е. примем $\lambda_{q\ell}=0$ и с использованием уравнения неразрывности

$$\frac{\partial\varepsilon}{\partial t}=-\frac{\partial q_\ell}{\partial x}$$

получим

$$\frac{\partial\varepsilon_s}{\partial t}=-\frac{\partial}{\partial x}\left\{\varepsilon_s\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\left(f'\frac{\partial p_s}{\partial x}\right)\right\}-q_{\ell m}\frac{\partial\varepsilon_s}{\partial x}. \tag{2}$$

Это основное уравнение фильтрования суспензии с образованием кейк-слоя, соответствующее релаксационному закону фильтрования.

Параметры ε_s и k зависят от давления p_s. Эти зависимости примем в виде [1]

$$\varepsilon_s=\varepsilon_s^0\left(1+\frac{p_s}{p_A}\right)^{\beta},\qquad k=k^0\left(1+\frac{p_s}{p_A}\right)^{-\delta}, \tag{3}$$

где ε_s^0, k^0 - значения ε_s, k при $p_s=0$, соответственно, p_s - компрессионное давление, p_A - характерное давление, β, δ - показатели – постоянные величины.

Используя (3) и соотношение

$$\frac{dp_\ell}{dp_s}=f'=-1$$

из (2) можно получить уравнение относительно p_s

$$\frac{\varepsilon_s^0\beta}{p_A}\left(1+\frac{p_s}{p_A}\right)^{\beta-1}\frac{\partial p_s}{\partial t}=\frac{k^0\varepsilon_s^0}{\mu}\frac{\partial}{\partial x}\left[\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\left(\frac{\partial p_s}{\partial x}\right)\right]-\frac{\varepsilon_s^0\beta}{p_A}q_{\ell m}\left(1+\frac{p_s}{p_A}\right)^{\beta-1}\frac{\partial p_s}{\partial x}. \tag{4}$$

Начальные и граничные условия для решения (4) имеют вид

$$p_s(0,x)=0, \qquad -k\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\bigg|_{x=0}=\frac{p_0-p_s}{R_m}\bigg|_{x=0}, \qquad p_s(t,L(t))=0. \tag{5}$$

где R_m приведенное сопротивление фильтра, p_0 - давление подачи суспензии.

Для (4) используя закон сохранения массы выведено следующее уравнение роста толщины слоя осадка $L(t)$:

$$\frac{dL}{dt}=-\frac{\varepsilon_s^0}{\varepsilon_s^0-\varepsilon_{s_0}}\left[\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\right]_{L^-}+\left[\frac{k}{\mu}\left(1+\lambda_{p\ell}\frac{\partial}{\partial t}\right)\frac{\partial p_s}{\partial x}\right]_{x=0}. \tag{6}$$

Из уравнения (6) определяется подвижный фронт $L(t)$ - граница между суспензией и кейк-слоем. Это уравнение решается совместно с основным уравнением фильтрования (4) при условиях (6) и $L(0)=0$.

Для уравнений (4), (6) поставленная задачи является задачей Стефана. Для ее решения применим метод конечных разностей с ловлей подвижной границы [5]. Результаты расчетов показывают, что увеличение времени релаксации при прочих равных условиях приводит к ускоренному росту толщины осадка. Другими словами, релаксационные эффекты в зависимости фильтрационного потока от градиента давления приводят к интенсификации осаждения частиц суспензии на поверхности фильтра в виде осадка.

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ОБОБЩЁННАЯ ИНТЕРВАЛЬНАЯ ЗАДАЧА НА СОБСТВЕННЫЕ ЗНАЧЕНИЯ В ЗАДАЧАХ МОДАЛЬНОГО АНАЛИЗА И РОБАСТНОЙ УСТОЙЧИВОСТИ

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Обобщённая задача на собственные значения

$$K\phi=\lambda M\phi$$

возникает при моделировании свободных колебаний механических систем, анализе устойчивости динамических систем, в задачах сплайн-интерполяции и многих других областях.

В реальных приложениях матрицы K и M часто известны лишь с интервальной неопределённостью из-за погрешностей измерений, технологических допусков, старения материалов. Классические детерминированные методы дают единственный набор собственных значений, тогда как для оценки надёжности и работоспособности конструкции необходимо знать гарантированные границы возможных значений. Это приводит к обобщённой интервальной задаче на собственные значения.

В дальнейшем изложении используются стандартные интервальные обозначения [1], т.е. интервальные величины записываются полужирным шрифтом, точечные (неинтервальные) величины — без специального выделения.

Модальный анализ конструкций с неопределёнными параметрами. Уравнение свободных колебаний дискретизированной упругой конструкции имеет вид

$$K(p)\,\phi = \lambda\, M(p)\,\phi,$$

где $K(p)$ — матрица жёсткости, $M(p)$ — матрица масс, $\lambda = \omega^2$ — квадрат собственной круговой частоты, ϕ — форма колебаний, а $p \in \mathbb{R}^m$ — вектор физических параметров (модули упругости, плотности, геометрические размеры).

Предположим, что каждый параметр p_i известен с интервальной неопределённостью, т.е. $p_i \in [\underline{p}_i, \overline{p}_i]$. Тогда матрицы $[\mathbf{K}]$ и $[\mathbf{M}]$ становятся интервальными. Требуется найти интервалы для собственных частот $\omega_i = \sqrt{\lambda_i}$, определяющих резонансные режимы. Частным случаем этой постановки является анализ вибраций балок, пластин и оболочек при интервальных материальных свойствах.

Робастная устойчивость систем управления. Также рассмотрим линейную динамическую систему

$$\dot{x} = A(p)\,x, \quad A(p) \in \mathbb{R}^{n\times n},$$

где матрица состояния $A(p)$ зависит от вектора параметров $p \in [\underline{p}, \overline{p}]$. Система асимптотически устойчива тогда и только тогда, когда все собственные значения $A(p)$ лежат в левой полуплоскости, т.е. $\operatorname{Re}\lambda_i(A(p)) < 0$. Если матрица A симметрична и положительно определена, задача о границах спектра сводится к обобщённой задаче на собственные значения $A\phi = \lambda\phi$, а при наличии массовой матрицы то рассматривается задача $K\phi = \lambda M\phi$. Таким образом, оценка запаса устойчивости в условиях интервальной неопределённости естественно формулируется как обобщённая интервальная задача на собственные значения.

Постановка задачи. Пусть заданы интервальные матрицы $[\mathbf{K}] = [\underline{K}, \overline{K}]$ и $[\mathbf{M}] = [\underline{M}, \overline{M}]$ размера $n \times n$. Рассмотрим обобщённую интервальную задачу на собственные значения:

$$[\mathbf{K}]\,\phi = \lambda\,[\mathbf{M}]\,\phi.$$

Требуется для каждого $i = 1, \ldots, n$ найти интервал $[\underline{\lambda}_i, \overline{\lambda}_i]$ такой, что для любых $K \in [\mathbf{K}]$, $M \in [\mathbf{M}]$ соответствующее обобщённое собственное значение $\lambda_i(K, M)$ принадлежит этому интервалу:

$$\underline{\lambda}_i = \min_{K\in[\mathbf{K}],\, M\in[\mathbf{M}]} \lambda_i(K,M), \quad \overline{\lambda}_i = \max_{K\in[\mathbf{K}],\, M\in[\mathbf{M}]} \lambda_i(K,M).$$

Аналогично ставится задача для собственных векторов и для границ $\operatorname{Re}\lambda_i$ в несимметричном случае.

Для симметричных положительно определённых матриц K и M собственные значения вещественны и положительны и выражаются через минимум и максимум отношения Рэлея:

$$\lambda_{\min}(K,M) = \min_{x\neq 0} \frac{x^T K x}{x^T M x}, \quad \lambda_{\max}(K,M) = \max_{x\neq 0} \frac{x^T K x}{x^T M x}.$$

В интервальном случае, если элементы $[\mathbf{K}]$ и $[\mathbf{M}]$ независимы, в силу монотонности отношения Рэлея по K и M границы достигаются на вершинных матрицах [2].

Ниже мы опираемся на центральную и радиусную матрицы, ассоциированные с интервальной матрицей. Для интервальной матрицы $[\mathbf{K}] = [\underline{K}, \overline{K}]$, где $\underline{K} = (\underline{k}_{ij})$ и $\overline{K} = (\overline{k}_{ij})$, центральная матрица K_c и радиусная матрица ΔK определяются покомпонентно:

$$(K_c)_{ij} = \frac{\underline{k}_{ij} + \overline{k}_{ij}}{2}, \qquad (\Delta K)_{ij} = \frac{\overline{k}_{ij} - \underline{k}_{ij}}{2}, \quad i, j = 1, \ldots, n. \tag{1}$$

Тогда интервальная матрица допускает представление

$$[\mathbf{K}] = [K_c - \Delta K,\ K_c + \Delta K] = K_c + [-\Delta K, \Delta K]. \tag{2}$$

Аналогично для интервальной матрицы $\mathbf{M}$ используются обозначения

$$[\mathbf{M}] = [M_c - \Delta M,\ M_c + \Delta M]. \tag{3}$$

Теорема. *Пусть* $[\mathbf{K}]$ *и* $[\mathbf{M}]$ *— симметричные интервальные матрицы порядка* n*, причём* $[\mathbf{M}]$ *положительно определена, а* S^i *— знаковая матрица, полученная из* $x^i(K_c, M_c)$*. Если* S^i *сохраняет свой знак для всех пар* $(K, M) \in ([\mathbf{K}], [\mathbf{M}])$*, то каждое собственное значение* $\lambda_i([\mathbf{K}], [\mathbf{M}])$ *принадлежит интервалу*

$$\boldsymbol{\lambda}_i = \left[\underline{\lambda}_i(K_c - S^i \Delta K S^i, M_c + S^i \Delta M S^i),\ \overline{\lambda}_i(K_c + S^i \Delta K S^i, M_c - S^i \Delta M S^i)\right], \tag{4}$$

где $i = 1, 2, \ldots, n$.

Из (4) следует, что для нахождения границ каждого $\lambda_i([\mathbf{K}], [\mathbf{M}])$ и соответствующих собственных векторов $x^i([\mathbf{K}], [\mathbf{M}])$ требуется построить всего две пары специальных вершинных матриц $(K_c \mp S^i \Delta K S^i,\ M_c \pm S^i \Delta M S^i)$. Однако для произвольной квадратной вещественной интервальной матрицы размерности n общее число вершинных матриц составляет 2^{n^2}, что делает задачу вычислительно неразрешимой уже при $n \geq 5$ (например, при $n = 5$ число вершинных матриц превышает 33 миллиона). Предлагаемый метод сокращает число рассматриваемых вершинных матриц до $4n$, то есть до двух пар для каждого λ_i.

Для несимметричных матриц задача существенно сложнее. Здесь можно использовать интервальные итерационные методы, например, метод окаймления, который позволяет получать гарантированные границы собственных значений с контролем переоценки. Идея метода состоит в последовательном уточнении интервалов, содержащих собственные значения, с использованием интервальных расширений функций и теорем о неподвижной точке.

На основе полученных теоретических результатов разработан вычислительный алгоритм, включающий: (i) решение детерминированной обобщённой задачи на собственные значения для центральной пары (K_c, M_c) и построение знаковых матриц S^i; (ii) проверку условия сохранения знака; (iii) формирование вершинных пар и решение двух детерминированных обобщённых задач на собственные значения для вычисления нижней и верхней границ каждого λ_i. Алгоритм имеет полиномиальную сложность по n и не требует перебора вершин интервальных матриц.

Для верификации теоретических утверждений проведён вычислительный эксперимент на модельных и прикладных интервальных системах. Численные результаты подтвердили следующие: корректность локализации собственных значений в границах (4), работоспособность метода при уровнях неопределённости до $\varepsilon = 5\%$, отсутствие эффекта «разбухания» ширины при росте размерности задачи, а также применимость метода к матрицам без классического диагонального преобладания. Сравнение с прямым перебором вершинных матриц

для малых n показало совпадение границ, что подтверждает неулучшаемость полученных оценок в рамках сделанных предположений. Перспективы дальнейших исследований связаны с ослаблением условия сохранения знака матрицы S^i, распространением метода на несимметричные интервальные матрицы и на матрицы с блочной и ленточной структурой.

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ДИНАМИКА ВЯЗКОУПРУГОЙ ТОРОИДАЛЬНОЙ ОБОЛОЧКИ С ПРОТЕКАЮЩЕЙ ЖИДКОСТЬЮ

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Исследуется задача о собственных изгибных колебаниях криволинейного участка трубопровода большого диаметра, моделируемого как тонкая вязкоупругая тороидальная оболочка, с учётом динамического влияния протекающей внутри идеальной несжимаемой жидкости. Получено разрешающее уравнение относительно амплитудных значений радиального перемещения срединной поверхности оболочки; исследовано влияние скорости потока жидкости и относительной толщины оболочки на частоты собственных изгибных колебаний.

Ключевые слова: вязкоупругая тороидальная оболочка, протекающая жидкость, изгибные колебания, оператор упругости, ядро релаксации, частоты собственных колебаний.

Тонкостенные криволинейные трубы широко используются во многих областях народного хозяйства; в процессе эксплуатации они, как правило, находятся в контакте с жидкой или газообразной средой и подвергаются динамическим воздействиям [1,2].

Постановка задачи. Рассматривается криволинейный участок трубопровода в виде тонкостенной трубы большого диаметра, по которому протекает идеальная несжимаемая жидкость с постоянной скоростью $U = const$ и постоянным гидростатическим давлением $p_0 = const$. Ставится задача об исследовании частот и форм собственных изгибных колебаний в плоскости кривизны данного участка трубопровода как тонкой тороидальной оболочки с учётом динамического влияния протекающей жидкости. Срединная поверхность оболочки задаётся в тороидальных криволинейных координатах β, θ, где β – центральный угол тора ($0 \le \beta \le \pi$), θ – угол в поперечном сечении оболочки ($0 \le \theta \le 2\pi$), продольная ось оболочки составляет половину окружности радиуса R. Дифференциальное уравнение движения криволинейного участка трубопровода со стационарным потоком жидкости, записанное в перемещениях u, v, w, W_y, ϑ в координатах β, θ с учётом составляющих сил инерции X_i^*, имеет вид:

$$\widetilde{E}\Big(\frac{r^2}{R^2}\frac{\partial^3 u}{\partial\beta\partial\theta^2}\cos\theta + \frac{r^3}{R^3}\frac{\partial^3 u}{\partial\beta^3} - \frac{r^2}{R^2}\frac{\partial}{\partial\theta}\Big(\frac{\partial u}{\partial\beta}\sin\theta\Big) + \frac{r^3}{R^3}\frac{\partial^2 W_y}{\partial\beta^2} +$$

$$+\frac{r^2}{R^2}\frac{\partial}{\partial\theta}\Big[\frac{\partial}{\partial\theta}(W_y\cos\theta)-W_y\sin\theta\Big]+\frac{h^2}{r^2 12(1-\nu^2)}\frac{\partial^3}{\partial\theta^3}\Big(\frac{\partial^2\vartheta}{\partial\theta^2}+\vartheta\Big)\Big)=$$
$$=\frac{r^3}{R}p\frac{\partial}{\partial\beta}\Big(\frac{\partial^2 u}{\partial t^2}\Big)-r^2p\frac{\partial}{\partial\theta}\Big(\frac{\partial^2 v}{\partial t^2}\Big)+\frac{r}{h}p_0\frac{\partial^3\vartheta}{\partial\theta^3}-r^2p\frac{\partial^4 w}{\partial\theta^2\partial t^2}+\frac{r}{h}\frac{\partial^2}{\partial\theta^2}(p_{\text{ж}}), \tag{1}$$

где $\widetilde{E}$ – операторный модуль упругости вязкоупругого материала,

$$\widetilde{E}\varphi(t)=E_{01}\Big[\varphi(t)-\int_0^t R_E(t-\tau)\varphi(\tau)\,d\tau\Big], \tag{2}$$

$\varphi(t)$ – произвольная функция времени, $R_E(t-\tau)$ – ядро релаксации, E_{01} – мгновенный модуль упругости. Полагая интегральные члены в (1) малыми и применяя к (2) процедуру замораживания [3] с трёхпараметрическим ядром релаксации $R(t)=Ae^{-\beta t}/t^{1-\alpha}$, приходим к комплексному оператору $\overline{E}\varphi=E\big[1-\Gamma^C(\omega_R)-i\Gamma^S(\omega_R)\big]\varphi$, где Γ^C,Γ^S – косинус- и синус-образы Фурье ядра релаксации.

Нормальную составляющую перемещения $w(\beta,\theta,t)$, удовлетворяющую граничным условиям $w|_{\beta=0,\pi}=0,\ \partial^2 w/\partial\beta^2|_{\beta=0,\pi}=0$ и условию цикличности по координате θ, ищем в виде

$$w(\beta,\theta,t)=f(t)\,b_m\cos m\theta\sin n\beta, \tag{3}$$

где $b_m=const$, m,n – волновые числа, определяющие формы колебаний оболочки в окружном и продольном направлениях. Подставив соответствующие данному виду выражения для остальных составляющих перемещения u,v,ϑ,W_y в уравнение движения (1) и вычислив частные производные по β и θ, получаем разрешающее уравнение относительно неизвестных амплитудных значений b_m, линейное однородное относительно $f(t)$ и $f''(t)$. Поскольку решение этой однородной системы отлично от нуля ($b_m\neq 0$), определитель коэффициентов однородной системы должен равняться нулю, что и даёт частотное уравнение для ω_{mn}.

Численный анализ частотного уравнения показывает, что частоты собственных изгибных колебаний ω_{mn} снижаются при увеличении скорости потока жидкости от 0 до 25 м/с не более чем на 10%; при увеличении относительной толщины оболочки h/r (при постоянной кривизне трубы) собственные частоты возрастают. Таким образом, чем больше кривизна трубы и чем толще её стенки, тем более жёсткой является конструкция.

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МОДЕЛИРОВАНИЕ И ИССЛЕДОВАНИЕ ВЛИЯНИИ СТЕПЕНИ РАСШИРЕНИИ КАНАЛА НА ПАРАМЕТРЫ СТРУИ ПРИ СМЕШЕНИИ И РАСПРОСТРАНЕНИИ В ПЛОСКОМ КАНАЛЕ

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Численное моделирование турбулентных течений (CFD-Computational Fluid Dynamics) позволяет существенно ускорить разработку различных современных технических устройств, начиная от создания авиационных газотурбинных двигателей и заканчивая различной бытовой техникой, а так же таких как моделирование горение газов, моделирование выброса вредных газовых веществ в атмосфере, биохимических процессов и т.д. С развитием вычислительной техники и совершенствованием методов вычислительной математики и математическое моделирования, вполне возможно CFD займет основное место, сравнением по использованию экспериментальные и аналитические методы.

Широкий круг проблем, стоящих перед современной наукой и техникой, связан с изучением внутренние турбулентные течения жидкостей или газов, которые происходят внутри замкнутых пространств и твердых границ, таких как трубы, каналы, сопла или элементы проточных частей машин. Турбулентные течения в трубах и каналах отличаются сложной, хаотичной структурой движения жидкости, которая сопровождается интенсивным перемешиванием и значительным переносом импульса, тепла и массы.

Целью данной работы является изучение основных закономерностей и особенностей турбулентного течения при смешении и распространении струи, истекающие из коаксиальной щели в комбинированном плоском расширяющемся канале при помощи математического моделирования. Для моделирования данного процесса используются двумерные полные нестационарные уравнения Навье – Стокса, в векторно –консервативном виде для течения сжимаемого газа и для замыкания системы дифференциальных уравнений привлечением уравнения состояния, связь полной удельной энергии с внутренней и кинетической энергиями, и выражение для коэффициента динамической вязкости [1-2]:

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}=\frac{\partial V_1(U,U_x)}{\partial x}+\frac{\partial V_2(U,U_y)}{\partial x}+\frac{\partial W_1(U,U_x)}{\partial y}+\frac{\partial W_2(U,U_y)}{\partial y}, \quad (1)$$

$$p=\rho T R_m, E=\rho C_\vartheta T+\frac{1}{2}\rho(u^2+\vartheta^2), \mu=\mu_l+\mu_{\text{т}}. \quad (2)$$

Здесь, μ – эффективная вязкость, μ_l - ламинарная вязкость, $\mu_{\text{т}}$ - турбулентная вязкость, U-вектор консервативных переменных; F, G, V_1, V_2, W_1, W_2-векторы потоков.

В расчетах предполагается, что течения является сжимаемым воздухом, двумерным, плоским, отсутствует сила тяжести и симметричным относительно оси канала, и это позволяет ограничиться исследованием течения в области между одной из стенок канала и осью.

Граничные условия к уравнению (1) задаются следующего вида:
1) на стенке ставится условие обращения в ноль вязкости, поперечной, продольной скорости и по температуры первого или второго рода, а также условие по P вида $\frac{\partial p}{\partial \bar{n}}=0$, а только поперек прилегающего к стенке подслоя толщиной Δy.
2) на входной границе устанавливается условие первого рода по всем искомым неизвестным;
3) на выходной границе значение всех неизвестных экстраполируется из внутренних точек области.
4) на оси канала ставится условия симметрии. В качестве начальных условия задавались некоторые значения искомых неизвестных.

Для численного решения системы уравнений (1) с краевыми и начальными условиями, использована эффективная неявная конечно-разностная схема Бима-Уорминга с трехточечной аппроксимацией назад.

Особый интерес представляет малоизученная область, где рассматривается влияние степени расширения канала на процесс смешения и распространения газовых смесей.

В настоящей работе приводятся результаты исследования струи, истекающие из плоских щелей и распространяющиеся в спутном потоке воздуха в расширяющемся канале, где угол расширения изменяется до 11°. Форма расширяющего канала задается в виде $f(x) = ax+b$. В частности, исследовано влияние соотношения исходных скоростей, соотношения размеров входных щелей и давления на параметры потока. Выявлено, что при в равных давлениях струи зоны рециркуляции не прослеживаются, параметры потока плавно изменяются по поперечному направлению вдоль канала, а при меньших значениях соотношения щелей наблюдается образования меньшего ядра струи и быстрое убывание осевого значения продольной скорости при постоянных исходных значениях других параметров. При смешении двух сверхзвуковых потоков со скоростью основного потока 500 м/с, давлением 2 атм, а спутного потока?со скоростью 340 м/с и давлением 1 атм., поперечное распределения температуры и давления с приведенными данными, они быстро выравниваются и переходят к одинаковым и постоянным значениям. Осевые значения этих параметров в начальном сечении канала и в зоне смешения увеличиваются, а увеличение угла расширения приводит к быстрому падению продольной скорости. Отметить, что с регулированием угла расширения канала можно получить необходимые параметры технических устройств.

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МОДЕЛИРОВАНИЕ И АЛГОРИТМ РАСЧЕТА ТРЕХМЕРНЫХ ТУРБУЛЕНТНЫХ РЕГИРУЮЩИХ СТРУЙ

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Трехмерные реагирующие струи - это сложные газодинамические течения, в которых поток газа или жидкости не просто перемешивается с окружающей средой в трех измерениях, но и вступает с ней в химические реакции. Моделирование и изучение и таких струй особенно необходимо и важно для проектирования камер сгорания ракетных, прямоточных и авиационных двигателей, проектирование промышленные горелки и химической технологии.

В общем случае, моделирование турбулентных струйных течений реагирующих газовых смесей основано на общепринятой системе связанных уравнений в частных производных, выражающих законы сохранения массы, импульса, энергии и вещества [1, 2]. Эти связанные уравнения описывают динамику движения жидкости, химические реакции между

основными компонентами и процессы диффузионного переноса, такие, как теплопроводность, вязкость и диффузию.

Среди различных задач прикладной газовой динамики струйных течений значительный интерес представляют исследования распространения струй, вытекающих из отверстия прямоугольной формы. Подобные течения, часто встречающиеся в вентиляционной и топочной технике (щелевые горелки и т.п.), и его можно моделировать с помощью системы полных уравнений Навье-Стокса для многокомпонентных химически реагирующих газовых смесей [1]. Моделирование струйные течения, в частности, в случае истечения из прямоугольного отверстия с конечным отношением длин сторон использовании упрощенных или параболизованных уравнений Навье-Стокса, вообще говоря, оправданным лишь в тех случаях, когда в течение имеется некоторое преобладающее направления движения.

Вероятно, наиболее общая форма параболизованных уравнений Навье-Стокса получена в предположении, что вязкие члены с производными в продольном направлении полагают малыми по сравнению с вязкими членами с производными в нормальном и поперечном направлениях [3] и для трехмерных турбулентных струй реагирующих газов приведен в работе [2]. Главная особенность трехмерной параболической модели закючается в разделении членов с градиентами давления по продольному и поперечным напралениям. При расчете трехмерных течений, градиентом давления в продольном направлении можно пренебречь или он известен заранее. Такая ситуация возникает при истечении дозвуковой свободной струи через сопло прямоугольного сечения в среду, которая либо покоится, либо движется в направлении оси сопла. При расчетах нами тоже получено, что форма такой струи в поперечном сечении постепенно меняется в продольном направлении и, наконец, становится круглой. Для расчета таких течений разумным является предположение о пренебрежении продольным градиентом давления. В расчетах диффузионного горения пропана-бутановой смеси в воздухе, в зоне горение, т.е. на фронте пламени рост давления наблюдается. Численные исследования показали, что влияние учета давления на параметры диффузионного факела, заметно влияет на профили скоростей и температур в начальных участках струи, а при удалении от среза сопла влияние давления можно считать незначительным.

В данной работе излагается модификации алгоритма SIMPLE (SIMPLER) для расчета трехмерных турбулентных струй реагирующих газов (пропана-бутановой смеси) истекающих из прямоугольного сопла и распространяющихся в спутном (затопленном) потоке окислителя-воздуха диффузионном горение.

Уравнение неразрывности используется для расчета дисбаланса массы в каждой расчетной точке сетки (i,j,k). Алгоритм изложен в двух вариантах:

1) Найденные решения u, v, w в итерации $(S+1)$ выражаются как расчетные (u_p, v_p, w_p) и плюс поправочные (u_c, v_c, w_c) в виде

$$u = u_p + u_c, \qquad v = v_p + v_c, \qquad w = w_p + w_c. \tag{1}$$

2) Истинные решения u, v, w и P будем выражать соответственно, как найденные расчетные (или промежуточные) плюс поправочные (u_c, v_c, w_c, P_c) в виде:

$$u = u_p + u_c, \qquad v = v_p + v_c, \qquad w = w_p + w_c, \qquad P = P_p + \alpha_1 P_c, \tag{2}$$

где α_1 – параметр релаксации.

Предполагается, что поправки к скорости определяются поправками к давлению в соответствии с приближенными уравнениями движения, в которых продольные конвективные члены уравновешены членами с давлением:

$$\rho u\frac{\partial u_c}{\partial x}=-\frac{\partial P_c}{\partial x},\qquad \rho u\frac{\partial v_c}{\partial x}=-\frac{1}{L}\frac{\partial P_c}{\partial y},\qquad \rho u\frac{\partial w_c}{\partial x}=-\frac{\partial P_c}{\partial z}. \tag{3}$$

В обоих случаях приходится решать уравнение Пуассона.

С помощью предложенных вариантов исследовано влияние исходных значений скорости и температуры горючего газа, в данном случае смеси пропана-бутана на параметры факела при диффузионном горении и результаты сопоставлены.

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НИЖНИЕ ГРАНИЦЫ ЭФФЕКТИВНОСТИ ОЦЕНОК В ЧАСТИЧНО-ИНФОРМАТИВНОЙ МОДЕЛИ СЛУЧАЙНОГО ЦЕНЗУРИРОВАНИЯ

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Задачи оценивания параметров распределения по цензурированным наблюдениям возникают в анализе выживаемости, теории надёжности, актуарной математике и других прикладных областях, где проводится численное моделирование времени жизни объекта в условиях конкурирующих рисков. Классические непараметрические методы (в частности, оценка Каплана–Мейера) не используют структурную информацию о связи между цензурирующим и целевым распределениями, что снижает точность получаемых численных оценок. В настоящей работе рассматривается модель, в которой такая связь задаётся в явном виде – условием пропорциональности интенсивностей, что позволяет построить более эффективную оценку и обосновать её преимущество аналитически, посредством нижних границ дисперсии.

Пусть $\{(X_i, Y_{1i}, Y_{2i}),\, i \geq 1\}$ – независимые, одинаково распределённые случайные векторы с независимыми компонентами и абсолютно непрерывными функциями распределения $F(x;\theta)$, $G_1(x)$, $G_2(x)$, где $\theta \in \Theta \subset \mathbb{R}$ – скалярный параметр. Наблюдается выборка $\varepsilon^n = \{(Z_i, \delta_i),\, i = 1, \ldots, n\}$, где $Z_i = \min(X_i, Y_{1i}, Y_{2i})$, а индикатор δ_i принимает значение 1, если $X_i \leq \min(Y_{1i}, Y_{2i})$; значение 0, если $Y_{1i} \leq \min(X_i, Y_{2i})$; и значение -1, если $Y_{2i} \leq \min(X_i, Y_{1i})$. Величина X_i цензурируется справа величиной $\min(Y_{1i}, Y_{2i})$ и наблюдается лишь при $\delta_i = 1$. Предполагается, что пара (X_i, Y_{1i}) удовлетворяет условию пропорциональности интенсивностей: существует $\alpha = \alpha(\theta) > 0$ такое, что для всех x:

$$1 - G_1(x) = \big(1 - F(x;\theta)\big)^{\alpha},\qquad \alpha = \alpha(\theta). \tag{1}$$

Модель называется *частично-информативной*, поскольку зависимость $\alpha(\theta)$ от θ переносит часть информации о параметре в цензурирующее распределение G_1. При этом функция распределения минимума $K(x;\theta) = P\big(\min(X_i, Y_{1i}) \le x\big)$ принимает вид

$$K(x;\theta) = 1 - \big(1 - F(x;\theta)\big)^{1+\alpha}. \tag{2}$$

Величины X_i информативно цензурируются через Y_{1i} (условие (1)) и неинформативно через Y_{2i}.

Вводятся субраспределения $\widetilde{H}(x;\theta)$ и $\widetilde{\widetilde{H}}(x;\theta)$, соответствующая плотность выборки $\omega_n(\varepsilon^{(n)};\theta)$, а также стандартные для теории Крамера–Рао условия регулярности (I)–(IV) (существование и интегрируемость производных субраспределений по θ, конечность информации Фишера, дифференцируемость оцениваемой функции). При этих условиях доказывается, что

$$E_\theta\left[\frac{\partial}{\partial\theta}\log\omega_n(\varepsilon^{(n)};\theta)\right] = 0, \qquad E_\theta\left[\frac{\partial}{\partial\theta}\log\omega_n(\varepsilon^{(n)};\theta)\right]^2 = nI(\theta),$$

где информация Фишера модели

$$I(\theta) = \int\limits_{-\infty}^{\infty}\left(\frac{\partial}{\partial\theta}\log\widetilde{h}(x;\theta)\right)^2 d\widetilde{H}(x;\theta) + \int\limits_{-\infty}^{\infty}\left(\frac{\partial}{\partial\theta}\log\widetilde{\widetilde{h}}(x;\theta)\right)^2 d\widetilde{\widetilde{H}}(x;\theta).$$

Отсюда следует неравенство типа Крамера–Рао: для любой регулярной оценки $\hat{g}_n(\varepsilon^{(n)})$ гладкой функции $g(\theta)$ параметра выполняется

$$D_\theta\big[\hat{g}_n(\varepsilon^{(n)})\big] \;\ge\; \frac{\big(g'(\theta)\big)^2}{n\cdot I(\theta)}, \tag{3}$$

а для смещённой оценки со смещением $B_n(\theta)$ – обобщённая форма

$$E_\theta\big(\hat{g}_n(\varepsilon^{(n)}) - \theta\big)^2 \;\ge\; B_n^2(\theta) + \frac{\big(1 + B_n'(\theta)\big)^2}{n\cdot I(\theta)}. \tag{4}$$

Далее сравниваются две оценки функции распределения $F(x;\theta)$: параметрически – непараметрическая оценка $F_n(x)$, учитывающая условие (1), и классическая оценка типа Каплана–Мейера $F_n^{KM}(x)$, построенная без учёта информативности цензурирования. Для линейного функционала

$$\mu(x;\theta) = \int_{-\infty}^{x}\Psi(u)\,dF(u;\theta)$$

через $\sigma_\Psi^2(x)$ и $\sigma_\Psi^{*2}(x)$ обозначены асимптотические дисперсии оценок, построенных на базе $F_n^{KM}(x)$ и $F_n(x)$ соответственно. Доказано, что при всех $x < T_H = \inf\big\{x : \widetilde{H}(x;\theta) + \widetilde{\widetilde{H}}(x;\theta) = 1\big\}$:

$$\frac{\gamma}{p_0+p_1}\cdot\big(1-F(x;\theta)\big)^2\big(\log(1-F(x;\theta))\big)^2 \;<\; \sigma_\Psi^{*2}(x;\theta) \;<\; \sigma_\Psi^2(x;\theta), \tag{5}$$

где

$$\gamma = \frac{p_1}{p_0+p_1}, \qquad p_k = P(\delta_i = k).$$

В частности, для $\Psi(u;\theta) = (1 - F(x;\theta))\,I(u \le x)$ неравенство (5) даёт

$$\frac{\gamma}{n\cdot(p_0+p_1)}\big(1-F(x;\theta)\big)^2\log^2\big(1-F(x;\theta)\big) \;<\; \sigma^2(x;\theta) \;<\; \sigma_{KM}^2(x;\theta). \tag{6}$$

Неравенства (5)–(6) показывают, что дисперсия оценки, построенной с учётом частичной информативности цензурирования, строго меньше дисперсии классического аналога Каплана–Мейера при любом объёме выборки n.

Проведено численное (Монте-Карло) моделирование модели с

$$F(x,\theta) = 1 - e^{-\theta x},$$

$$G_1(x) = 1 - e^{-\lambda_1 x}\ (\lambda_1 = \alpha\theta,\ \alpha = 1{,}5),$$

$$G_2(x) = 1 - e^{-\lambda_2 x}\ (\lambda_2 = 0{,}8)$$

при $\theta = 1$. Для каждого объёма выборки n генерировалось 3000 повторов, в каждом из которых по выборке (Z_i, δ_i) вычислялись: оценка $\hat{\Lambda}_n(x)$, $\Lambda_n(x)$, отношение

$$R_n(x) = \frac{\hat{\Lambda}_n(x)}{\Lambda_n(x)}$$

и доля

$$\gamma_n = \frac{p_{1n}}{p_{1n} + p_{0n}},$$

после чего по формуле

$$F_n(x) = \begin{cases} 0, & x < Z_{(1)}, \\ 1 - \left(\dfrac{n-j}{n}\right)^{\gamma_n R_n(x)}, & Z_{(j)} \le x < Z_{(j+1)},\ 1 \le j \le n-1, \\ 1, & x > Z_{(n)}, \end{cases}$$

находилась оценка $F_n(x_0)$ и, для сравнения, стандартная оценка Каплана–Мейера $F_n^{KM}(x_0)$ (событие – $\delta_i = 1$), в точке $x_0 = 0{,}7$ (истинное значение $F(x_0;\theta) = 0{,}5034$).

Таблица 1. Эмпирическая дисперсия оценок $Var(F_n)$ и $Var(F_n^{KM})$ по 3000 повторам

n	$Var(F_n)$	$Var(F_n^{KM})$	$Var(F_n)/Var(F_n^{KM})$
50	0.01149	0.01458	1.27
100	0.00494	0.00659	1.33
200	0.00265	0.00356	1.34
400	0.00129	0.00168	1.31

Из таблицы видно, что при всех рассмотренных n дисперсия оценки F_n, учитывающей условие пропорциональности интенсивностей (1), устойчиво меньше дисперсии классической оценки Каплана–Мейера: выигрыш составляет 27–34% (отношение $Var(F_n)/Var(F_n^{KM}) \approx 1.27$–$1.34$), что численно подтверждает теоретическое неравенство (5)–(6) и иллюстрирует практическую ценность учёта информативности цензурирования при обработке данных типа конкурирующих рисков.

Полученные неравенства (3)–(6), подтверждённые численным экспериментом раздела 4, показывают, что оценка, построенная с учётом частичной-информативности цензурирования, асимптотически эффективнее классического непараметрического аналога Каплана–Мейера. Результаты могут быть использованы при численном моделировании и статистической обработке цензурированных данных в задачах анализа выживаемости и надёжности, где выбор вычислительно более точного метода оценивания параметра имеет практическое значение.

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СЕКЦИЯ 6. ПЕРСПЕКТИВНЫЕ ТЕХНОЛОГИИ ИСКУСТВЕННОГО ИНТЕЛЛЕКТА

DIABETDA O‘Z-O‘ZINI KUZATISH MA’LUMOTLARI ASOSIDA YASHIRIN REJIMLARNI ANIQLASH UCHUN ALMASHINUVCHI AVTOREGRESSIV HOLAT-FAZO MODELI

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Kirish

Qandli diabetni boshqarish jarayonida generatsiya qilinadigan glyukoza va insulin dozasi bo‘icha o‘z-o‘zini kuzatish yozuvlari notekis, bemorga individual vaqt qatorlarini tashkil etadi, bunda bevosita kuzatilmaydigan latent dinamik rejim hech qanday tashqi klinik jihatdan tasdiqlangan yorliq bilan ta’minlanmaydi. Bunday yashirin rejimni tashqi klinik jihatdan tasdiqlangan yorliqlarsiz ma’lumotlardan aniqlash klassifikatsiya emas, balki latent o‘zgaruvchini baholash masalasidir. Latent o’zgaruvchilar dastlabki holatda bevosita kuzatish imkoni bo’lmagan o’zgaruvchilar hisoblanadi. Ushbu ishda diskret yashirin rejimni statik klasterlashdan (vaqt bog‘liqligini hisobga olmaydi) va standart emissiyali yashirin Markov modelidan (HMM, kuzatiluvchi vektorning oldingi vaqt nuqtasiga bevosita rejimga bog‘liq avtoregressiv bog‘lanishini modellashtirmaydi) farqli o‘laroq, kunlik dinamikaning o‘zi rejimga bog‘liq bo‘lgan almashinuvchi avtoregressiv holat-fazo modeli yordamida aniqlash taklif etiladi.

Matematik model

Kuzatiluvchi kattalik — kunlik olti o‘lchamli xususiyat vektori

$$y_t = [G_t, I_t, \bar{G}_t, \bar{I}_t, \Delta G_t, \Delta I_t]^T$$

(o‘rtacha glyukoza, jami insulin dozasi, ularning 7 kunlik siljuvchi o‘rtachalari va birinchi ayirmalari), yashirin kattalik esa diskret rejim $s_t \in \{1, \ldots, M\}$ hisoblanadi. Yetti kunlik siljuvchi o‘rtachalar va ayirmalar ketma-ket kuzatuvlar asosida hisoblandi; dastlabki oyna preprocessing bosqichida bir xil qoida bo‘yicha to‘ldirildi. Model quyidagi ko‘rinishga ega:

$$y_t = A_{s_t} \cdot y_{t-1} + w_t, \qquad w_t \sim N(0, Q_{s_t}),$$

$$P(s_t = j \mid s_{t-1} = i) = a_{ij},$$

bu yerda A_j – rejimga xos avtoregressiya matritsasi, Q_j – rejimga xos kovariatsiya, a_{ij} — o‘tish ehtimolligi. Parametrlar to‘plami Θ kuzatiluvchi ma’lumotning to‘liq ehtimollik funksiyasini maksimallashtirish yo‘li bilan, rejim ketma-ketligi kuzatilmaganligi sababli, forward–backward filtrlash-silliqlash rekursiyasiga asoslangan EM-algoritmi orqali baholanadi; regressiya matritsalari kichik ridge-regulyarizatsiya ($\lambda = 10^{-2}$) bilan vaznli kichik kvadratlar usulida qayta baholanadi.

Ma’lumotlar bazasi va taklif etilgan usul.

Tadqiqot UCI “Diabetes” (AIM’94) ochiq ma’lumotlar bazasidan (Kahn, 1994; DOI: 10.24432/C5T59G) foydalanadi 1988–1991-yillarda yig‘ilgan, 70 bemorning insulin dozasi va

glyukoza o‘lchovlari bo‘yicha xom hodisa jurnallari. Xom fayllar to‘g‘ridan-to‘g‘ri qayta ishlandi: sana formati, hodisa kodlari va qiymatlarning raqamliligi tekshirildi, nostandart yozuvlar (masalan, o‘lchov diapazonidan tashqari “Hi”/“Lo” ko‘rsatkichlari, mavjud bo‘lmagan sanalar, buzilgan qatorlar) rad etildi va son jihatdan qayd etildi. Qamrov muddati ≥ 60 kun va ≥ 40 ta alohida glyukoza qayd etilgan kun mezoni asosida 70 bemordan 20 tasi tanlab olindi (jami 2189 bemor-kuni). Rejim uchun tashqi/klinik tasdiqlangan yorliq mavjud emasligi sababli, model ichki klasterlash sifati ko‘rsatkichlari (silhouette, Calinski–Harabasz, Davies–Bouldin), vaqt bo‘yicha barqarorlik (o‘rtacha turg‘unlik davomiyligi, o‘tishlar soni) va so‘nggi $\sim 20\%$ kuzatuv bo‘yicha bir qadamli oldindan aytish log-ehtimolligi asosida, K-means, PCA+K-means, Gauss HMM, bir bosqichli almashinuvchi baza modeli va sayoz avtokoduvchi+K-means bilan taqqoslandi; barcha usullar bir xil boshlang‘ich shart (K-means) va bir xil $M = 3$ rejim soni ostida moslashtirildi.

Hisoblash tajribasi va natijalar

Taklif etilgan model so‘nggi $\sim$ 20% kuzatuv bo‘yicha bir qadamli oldindan aytish log-ehtimolligi bo‘yicha Gauss HMM’ga nisbatan (median $-28,1$ ga qarshi $-171,8$; juftlashtirilgan Wilcoxon testi, $N = 20$, $p = 0,00059$, Holm tuzatilgan $p = 0,0023$) va bir bosqichli baza modeliga nisbatan (median $-37,9$; $p = 0,0266$) statistik jihatdan yuqori natija ko‘rsatdi, biroq silhouette ko‘rsatkichi bo‘yicha sof geometrik klasterlash usullaridan past chiqdi ($0,132 \pm 0,150$ ga qarshi K-means uchun $0,278 \pm 0,036$). Ko‘p martalik tekshiruv uchun barcha besh juft taqqoslash Holm tuzatishidan keyin ham bir xil xulosani qo‘llab-quvvatladi.

Bu – modelning geometrik jihatdan ixcham emas, balki vaqt bo‘yicha silliqlangan rejimlarni aniqlashining tabiiy natijasidir: model tomonidan aniqlangan yashirin rejimlarning vaqt bo‘yicha o‘rtacha turg‘unlik davomiyligi (5,96 kun) sof geometrik usullarnikidan ($\leq 2,6$ kun) sezilarli yuqori bo‘ldi – bu klinik davomiylik emas, balki model natijasi sifatida talqin etiladi. Glyukoza o‘rtacha qiymati bo‘yicha tartiblangan uchta rejim mos ravishda 143,6, 164,5 va 192,7 mg/dL o‘rtacha glyukoza hamda gipoglikemik va giperglikemik kunlar ulushi bo‘yicha monoton, tavsifiy gradient namoyon etdi; bu gradient klinik jihatdan tasdiqlangan kasallik bosqichlari sifatida emas, balki faqat tavsifiy assotsiatsiya sifatida talqin etiladi.

Qo‘shimcha additiv Gauss shovqini tajribasi shuni ko‘rsatdiki, kunlik rejim yorliqlari shovqinga sezgir: noise-free reference run tomonidan hosil qilingan rejim yorliqlariga nisbatan moslashtirilgan Rand indeksi (adjusted Rand index, ARI) 1,00 dan $\sigma = 0,05$ da 0,53 gacha va $\sigma = 0,30$ da 0,15 gacha pasaydi. Model esa 10 xil tasodifiy boshlang‘ich qiymat (seed) bo‘yicha o‘qitilganda takrorlanuvchanlikning qoniqarli darajasini ko‘rsatdi (o‘qitish log-ehtimolligining variatsiya koeffitsiyenti 7,3%).

Xulosa

Olingan natijalar shuni ko‘rsatadiki, diabetda o‘z-o‘zini kuzatish ma’lumotlarida rejim-almashinuvchi dinamikani bevosita modellashtirish sof statik klasterlashga yoki standart emissiyali yashirin Markov modeliga nisbatan o‘lchanadigan oldindan aytish afzalligini beradi, biroq bu geometrik ajratilganlik hisobiga keladi – shu sababli baholash bir nechta mustaqil o‘lchov (geometrik, vaqtinchalik, prognozli) bo‘yicha olib borilishi zarur. Tadqiqot yagona ochiq ma’lumotlar bazasidagi 20 bemorga asoslangan bo‘lib, rejimlar soni ($M = 3$) oldindan belgilangan va rasmiy model tanlash mezoni bilan aniqlanmagan, tashqi klinik tasdiqlash esa mavjud emas.

Model klinik diagnostika, kasallik bosqichini aniqlash yoki β-hujayra faoliyati/insulin sezuvchanligini baholash vositasi emas – u faqat tavsifiy, gipoteza generatsiya qiluvchi tahlil vositasi sifatida taklif etiladi. Kelgusi tadqiqotlar zichroq uzluksiz glyukoza monitoringi ma’lumotlarida rejimlar sonini rasmiy model tanlash mezonlari asosida aniqlashga hamda tavsifiy glyukoza gradientini klinik jihatdan tasdiqlangan yorliqlar bilan solishtirishga qaratilishi lozim.

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REGISTON ANSAMBLI MADRASA DEVORLARINING YEMIRILISH DARAJASINI FRAKTAL O‘LCHOV VA SUN’IY INTELLEKT YORDAMIDA BAHOLASH

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Samarqandning tarixiy qismi “Samarkand – Crossroad of Cultures” nomi bilan UNESCO Butunjahon merosi ro‘yxatiga kiritilgan. Registon ansamblidagi Ulug’bek, Sherdor va Tillakori madrasalarining dekorativ sirtlari harorat, namlik, yog’in, shamol eroziyasi, tuz kristallanishi, seysmik va antropogen omillar ta’sirida yemiriladi. An’anaviy vizual nazorat subyektiv va mehnat talabchan bo‘lgani uchun kompyuter ko‘rishi orqali yoriq, qoplama yo‘qolishi va sirt deformatsiyalarini avtomatik aniqlash muhimdir [1]. Fraktal tahlil esa yoriqlarning tarmoqlanishi va geometrik murakkabligini miqdoriy ifodalaydi.



1-rasm. Registon devor bezagida yoriqlar va lokal qoplama yo‘qolishi.

Tadqiqotda frontal va lokal fragment tasvirlari olinadi; kalibrlangan marker yoki fotogrammetrik tayanch nuqtalar piksel o‘lchovlarini millimetrga o‘tkazishga xizmat qiladi. Rangli tasvir

$$I(x,y) = \{R(x,y),\, G(x,y),\, B(x,y)\}. \tag{1}$$

Bu yerda x, y – piksel koordinatalari, R, G, B – rang kanallari intensivligi. Tasvirlar perspektiv buzilishlardan tozalanib, yoritish va kontrast normallashtiriladi hamda 512×512 yoki 1024×1024 o‘lchamli ROI fragmentlariga ajratiladi.

Shikastlanishlarni aniqlash semantik segmentatsiya masalasi sifatida qaraladi:

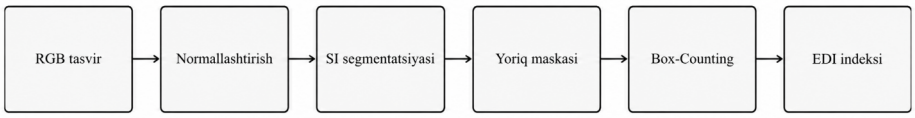
$$f_\theta(I) \to M. \tag{2}$$

Bu yerda f_θ – parametrlari θ bo‘lgan neyron tarmoq, M – piksel darajasidagi segmentatsiya xaritasi. Piksellar sog’lom sirt, yoriq, koshin yo‘qolishi, eroziya, rang-tekstura buzilishi va namlik belgilariga ajratilishi mumkin. U-Net, Attention U-Net, DeepLabV3+ va SegFormer turidagi modellar mos keladi [2]. Model sifati ekspert maskasi M_{GT} va bashorat M_{PR} orqali baholanadi:

$$IoU = \frac{|M_{GT} \cap M_{PR}|}{|M_{GT} \cup M_{PR}|}. \tag{3}$$

$$Dice = \frac{2|M_{GT} \cap M_{PR}|}{|M_{GT}| + |M_{PR}|}. \tag{4}$$

IoU va $Dice$ ning 1 ga yaqin qiymatlari yuqori segmentatsiya aniqligini bildiradi. Ornament chiziqlari yoriqqa o‘xshashi mumkinligi sababli fraktal o‘lchov avvalo SI ajratgan yoriq maskasiga hisoblanadi.



2-rasm. Sun’iy intellekt va fraktal tahlil asosidagi diagnostika ketma-ketligi.

Box-Counting usulida binar yoriq tasviri ε o‘lchamli kvadratlarga ajratiladi, yoriqni o‘z ichiga olgan kataklar soni $N(\varepsilon)$ bilan belgilanadi. Fraktal o‘lchov [3]:

$$D_F = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)}. \tag{5}$$

Raqamli tasvirda turli ε_i qiymatlar uchun

$$x_i = \log(1/\varepsilon_i), \qquad y_i = \log N(\varepsilon_i). \tag{6}$$

nuqtalari qurilib, chiziqli regressiya bajariladi:

$$y = a + D_F x. \tag{7}$$

Regressiya chizig’ining burchak koeffitsiyenti D_F yoriq tarmog’ining fraktal o‘lchovi bo‘lib, ikki o‘lchamli ingichka yoriq tarmog’i uchun

$$1 \le D_F < 2. \tag{8}$$

D_F ning 1 ga yaqinligi sodda, uning ortishi murakkabroq va zichroq tarmoqlanishni bildiradi.

SI aniqlagan shikastlangan piksellar soni N_d, jami piksellar soni N bo‘lsa, yemirilgan maydon koeffitsiyenti

$$A_d = \frac{N_d}{N}. \tag{9}$$

Foiz ko‘rinishida esa

$$P_d = \frac{N_d}{N} \times 100\%. \tag{10}$$

Masalan, 512×512 tasvirda $N = 262144$ va $N_d = 52429$ bo‘lsa,

$$P_d = \frac{52429}{262144} \times 100\% \approx 20.0\%. \tag{11}$$

Demak, fragment sirtining taxminan 20 foizi vizual shikastlanish belgisiga ega.

Koshin elementining yo‘qolishi naqshning lokal simmetriyasi, periodikligi va teksturasini buzadi. Etalon sog’lom fragmentning tekstura-fraktal ko‘rsatkichi D_R, tekshirilayotgan fragmentniki D_T bo‘lsa, nisbiy o‘zgarish

$$\Delta D = \frac{|D_T - D_R|}{D_R}. \tag{12}$$

Masalan, $D_R = 1.78$ va $D_T = 1.49$ bo‘lganda

$$\Delta D = \frac{|1.49 - 1.78|}{1.78} = 0.163. \tag{13}$$

Bu qiymat dekorativ strukturaning etalon fragmentga nisbatan sezilarli o‘zgarganini ko‘rsatadi. Ko‘rsatkich sog’lom geometrik ornamentning tabiiy murakkabligini hisobga olgan holda kalibrlanishi kerak.

Taklif etilgan yondashuv chuqur o‘qitish va fraktal geometriyani birlashtiradi: neyron tarmoq shikastlanishning joylashuvini, P_d zararlangan maydon ulushini, D_F esa yoriq tarmog’ining geometrik murakkabligini baholaydi. P_d kichik, ammo D_F yuqori bo‘lsa, lokal lekin kuchli tarmoqlangan yoriqlar mavjud bo‘lishi mumkin; P_d yuqori va D_F nisbatan past holat esa katta maydondagi qoplama yo‘qolishiga mos kelishi mumkin. Registonning murakkab ornamentlari sabab o‘quv bazasiga sog’lom va shikastlangan fragmentlarni turli yoritish sharoitlarida kiritish, annotatsiyalarni restavratorlar bilan tekshirish va modelni alohida madrasa hamda ornament turlarida sinash zarur. Periodik kuzatuvda bir xil rakurs va masshtabdagi tasvirlar yemirilish dinamikasini miqdoriy baholashga xizmat qiladi.

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RAQAMLI TA’LIM MUHITIDA O‘QUVCHILARNI SUN’IY INTELLEKT VOSITASIDA O‘QITISHDA INTEGRATIV YONDASHUV METODIKASINI TAKOMILLASHTIRISH

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O‘zbekistonda raqamli transformatsiya va sun’iy intellektni rivojlantirish bo‘yicha davlat siyosatining ta’lim tizimiga ta’siri tahlil qilinib, Prezident Shavkat Mirziyoyevning ta’lim, innovatsiya, yoshlar va raqamli texnologiyalar haqidagi nutq va asarlarida ilgari surilgan g‘oyalar bilan bog‘liqligi ochib berilgan. Yaqinda O‘zbekiston maktablarida sun’iy intellekt asosidagi ta’limni rivojlantirish bo‘yicha amaliy taklif va istiqbolli yo‘nalishlar ishlab chiqilgan. XXI asrda ta’lim tizimining rivojlanishi raqamli texnologiyalar, katta ma’lumotlar, bulutli

xizmatlar, virtual muhitlar va sun'iy intellekt texnologiyalarining jadal taraqqiyoti bilan chambarchas bog‘liq. Bugungi kunda sun'iy intellekt oddiy axborot texnologiyasi sifatida emas, balki inson faoliyatining turli sohalarini, jumladan, ta'lim jarayonini tubdano‘zgartiruvchi strategik omil sifatida namoyon bo‘lmoqda. O‘zbekiston ham mazkur global jarayondan chetda qolmay, raqamli iqtisodiyot, zamonaviy ta'lim va sun'iy intellektni rivojlantirishga ustuvor yo‘nalish sifatida qaramoqda. “O‘zbekiston - 2030” strategiyasida ta'lim tizimini rivojlantirish ustuvor vazifalardan biri sifatida belgilangan. Maktab ta'limi sifatini oshirishga qaratilgan davlat siyosatida zamonaviy pedagogik texnologiyalar, muhandislik, axborot texnologiyalari va STEM yo‘nalishlarini rivojlantirishga katta e'tibor berilmoqda. Umumta'lim maktablarini rivojlantirish,o‘qitish metodlarini yangilash, integrativ yondashuv vositasida zamonaviy bilim va ko‘nikmalarni shakllantirish hamda ilg‘or xorijiy tajribalardan foydalanish zarurligi qayd etilgan. Bugungi bosqichda ushbu vazifalarni amalga oshirishda sun'iy intellekt texnologiyalaridan samarali foydalanish yangi metodikasini yuzaga keltirmoqda.

O‘zbekiston Respublikasi Prezidenti Sh. Mirziyoyevning 2026-yil 7-avgustdagi taqdimotida mamlakatning sun'iy intellektga tayyorlik bo‘yicha xalqaro reytingdagio‘rnini yaxshilash, 2030-yilgacha ilg‘or 50 davlat qatoriga kirish hamda sun'iy intellekt asosidagi mahsulot va xizmatlar hajmini sezilarli darajada oshirish maqsadlari belgilangan. Shu nutq sun'iy intellekt nazardan, maktabo‘quvchilarini sun'iy intellekt vositasidao‘qitish masalasi nafaqat texnologik, balki pedagogik, metodik va ijtimoiy muammo sifatida ham dolzarbdir. Raqamli ta'lim muhitida sun'iy intellekt texnologiyalaridan foydalanish asosidao‘quvchilarning bilim olish samaradorligini oshirish, ta'limni individuallashtirish va zamonaviy kompetensiyalarni shakllantirishning istiqbolli mexanizmlarini aniqlashdan iborat. An'anaviy ta'lim modelidao‘quvchi ko‘pincha tayyor bilimlarni qabul qiluvchi subyekt sifatida namoyon bo‘ladi. Raqamli ta'lim esao‘quvchini axborotni mustaqil izlash, tahlil qilish, taqqoslash, qayta ishlash va yangi bilim yaratishga yo‘naltiradi. Raqamli ta'lim muhitining muhim xususiyati uning interaktivligi va individuallashtirilganligidir. Bunda elektron darsliklar, ta'lim platformalari, videodarslar, virtual laboratoriyalar, test tizimlari, simulyatorlar va sun'iy intellekt asosidagi yordamchi dasturlar yagona ta'lim ekotizimini shakllantiradi.

O‘zbekistonda mazkur jarayon uchun zarur infratuzilma bosqichma-bosqich kengayib bormoqda. Umumta'lim maktablarda zamonaviy kompyuter xonalari, STEM-laboratoriyalar, muhandislik va texnologiya yo‘nalishlarini rivojlantirishga e'tibor qaratilgani buning amaliy ifodasidir. Raqamli muhitning keyingi bosqichi esa oddiy elektronlashtirishdan "aqlli ta'lim"modeligao‘tishdir. Bunda sun'iy intellekto‘quvchi faoliyati haqidagi ma'lumotlarni tahlil qilib, uning bilim darajasi, xatolari, qiziqishlari va o‘zlashtirish sur'atiga mos ta'limiy tavsiyalar bera oladi. An'anaviy darsda o‘qituvchi barchao‘quvchilarga bir xil topshiriq berishga majbur bo‘lishi mumkin. Sun'iy intellekt esa o‘quvchining avvalgi natijalarini tahlil qilib, unga murakkablik darajasi moslashtirilgan topshiriqlarni tavsiya qilishi mumkin.

O‘zbekistonda ta'limni sun'iy intellekt asosida rivojlantirish davlat siyosatining umumiy strategik maqsadlari bilan uzviy bog‘liq. Sun'iy intellekt o‘qituvchining o‘rnini egallashi emas, balki uning pedagogik imkoniyatlarini kengaytirishi kerak. Bunda o‘qituvchining kasbiy roli kamaymaydi, aksincha, mazmunan murakkablashadi. O‘zbekistonda sun'iy intellektni rivojlantirishga doir davlat siyosati ta'lim tizimining kelajakdagi transformatsiyasi uchun muhim zamin yaratmoqda. 2025-yilda sun'iy intellekt texnologiyalarini 2030-yilgacha rivojlantirish bo‘yicha strategik hujjatlar qabul qilinib, zarur infratuzilma va kadrlar tayyorlash masalalariga e'tibor qaratildi. O‘zbekiston Respublikasi umumta'lim maktablarida sun'iy intellektdan samarali foydalanish uchun milliy sun'iy intellektni – ta'lim platformasini yaratish, o‘zbek tilidagi ta'limiy sun'iy intellektni modellarini rivojlantirish, pedagoglarning sun'iy intellektni kompetensiyasini oshirish, o‘quvchilarda sun'iy intellekt va axborot savodxonligini shakllantirish, akademik halollik

va ma'lumotlar xavfsizligi standartlarini joriy etish maqsadga muvofiq. Raqamli ta'lim muhitida sun'iy intellekt texnologiyalaridan foydalanish masalasini ilmiy jihatdan tadqiq etishda, avvalo, "raqamli ta'lim muhiti", "sun'iy intellekt asosidagi ta'lim", "adaptiv ta'lim", "intellektual tutorlik" va "shaxsga yo'naltirilgan ta'lim" tushunchalariningo'zaro nisbatini aniqlash muhimdir.

Raqamli ta'lim muhiti – bu ta'lim subyektlari, raqamli kontent, dasturiy vositalar, kommunikatsiya texnologiyalari, ta'lim platformalari va pedagogik metodlarning o'zaro integratsiyasiga asoslangan yaxlit pedagogik tizimdir. Bunday muhitda o'quvchi faqat bilimni qabul qiluvchi emas, balki axborotni izlovchi, tahlil qiluvchi, qayta ishlovchi va yangi bilimni mustaqil shakllantiruvchi faol subyektga aylanadi. Sun'iy intellekt esa ushbu jarayonda "bilim beruvchi" emas, balkio'quv faoliyatini qo'llab-quvvatlovchi intellektual vosita sifatida ishlatilishi lozim. Sun'iy intellektni tizimi ma'lumot asosida o'quvchi uchun individual mashqlar ketma-ketligini shakllantiradi. Sun'iy intellekt texnologiyalaridan foydalanish natijasida o'quvchilarning o'zlashtirish ko'rsatkichlaridan tashqari, ularning mustaqilligi, kreativligi, tanqidiy fikrlash ko'nikmalarini baholash zarurligi asoslanadi.

Raqamli ta'lim muhiti – bu ta'lim subyektlari, raqamli kontent, dasturiy vositalar, kommunikatsiya texnologiyalari, ta'lim platformalari va pedagogik metodlarning o'zaro integratsiyasiga asoslangan yaxlit pedagogik tizimdir. Sun'iy intellekt esa ushbu muhitning "aqlli" komponenti sifatida namoyon bo'lib, o'quvchining ta'limiy faoliyati to'g'risidagi ma'lumotlarni qayta ishlash, o'quv materiallarini moslashtirish, tavsiyalar ishlab chiqish, o'zlashtirish dinamikasini aniqlash va ta'lim jarayonini individuallashtirish imkonini beradi.

Umuman olganda, raqamli ta'lim muhitida o'quvchilarni sun'iy intellekt texnologiyalari vositasida o'qitish zamonaviy ta'lim tizimining strategik yo'nalishlaridan biri hisoblanadi. Sun'iy intellekt ta'limni individuallashtirish, o'quvchining ta'limiy ehtiyojlarini aniqlash, o'quv materiallarini moslashtirish, ta'lim natijalarini tahlil qilish va o'qituvchi faoliyatini optimallashtirish uchun yangi imkoniyatlarni yaratmoqda. Sun'iy intellektning ta'lim tizimiga joriy etilishi oddiy texnologik yangilanish emas, balki ta'lim paradigmasining transformatsiyasidir.

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SIYAM NEYRON TARMOQLARI YORDAMIDA QO'LYOZMA IMZOLARNI STATIK VERIFIKATSIYA QILISH USULI

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Annotatsiya. *Ushbu maqolada Siyam neyron tarmog‘idan foydalangan holda qo‘lyozma imzolarni verifikatsiya qilish yondashuvi taqdim etilgan. Tadqiqot imzolarning haqiqiyligini aniqlash uchun ularni samarali tahlil qilish va solishtirishga qodir bo‘lgan modelni ishlab chiqish hamda o‘qitishga qaratilgan. Metodologiya boshlang‘ich ma’lumotlarni tayyorlash va modelni keyingi bosqichda sinovdan o‘tkazishni o‘z ichiga oladi. Taklif etilayotgan usul yuridik va moliyaviy muassasalar kabi imzolarni aniq va tezkor verifikatsiya qilish muhim bo‘lgan tizimlarda qo‘llanilish salohiyatiga ega.*

Kalit so‘zlar: *Siyam tarmoqlari, imzo verifikatsiyasi, konvolyutsion neyron tarmoqlari (CNN), biometrik autentifikatsiya, kontrastiv yo‘qotish, chuqur o‘rganish, shakllarni tanish.*

Kirish.

Zamonaviy dunyoda qo‘lyozma imzolarni verifikatsiya qilish yuridik, bank sohalar va shaxsiy hujjatlar xavfsizligi va haqiqiyligini ta’minlashning muhim jihati bo‘lib qolmoqda. Ekspertlarning vizual baholashiga asoslangan an’anaviy verifikatsiya usullari xatoliklarga va subyektiv talqinlarga moyildir. Bu esa o‘z navbatida avtomatlashtirilgan tizimlarni ishlab chiqish zaruriyatini keltirib chiqaradi.

Machine learning and computer vision texnologiyalarining rivojlanishi bilan imzolarni verifikatsiya qilish jarayonlarini avtomatlashtirish va takomillashtirish uchun yangi imkoniyatlar paydo bo‘ldi. Bunday tizimlardan foydalanish ma’lumotlarga ishlov berish tezligi va aniqligini sezilarli darajada oshirishga, shu bilan birga inson omilini minimallashtirishga imkon beradi. Avtomatik verifikatsiyaning asosiy jihati tizimning haqiqiy va qalbaki imzolarni aniq farqlash qobiliyatidir.

Hozirgi vaqtda imzolarni verifikatsiya qilishning ikkita asosiy yondashuvi mavjud:

1) Dinamik verifikatsiya,

2) Statik verifikatsiya. Dinamik verifikatsiya imzo yaratish jarayonini real vaqt rejimida tahlil qilsa, statik verifikatsiya tayyor skanerlangan imzolar bilan ishlaydi. Ushbu tadqiqot aynan statik verifikatsiya usulini takomillashtirishga qaratilgan bo‘lib, buning uchun Siyam neyron tarmoqlari imkoniyatlaridan foydalaniladi.

Metodologiya va model arxitekturasi.

Siyam neyron tarmoqlari (Siamese Neural Networks) chuqur o‘qitish arxitekturalaridan foydalangan holda imzo tasvirlarini solishtirish vazifasiga o‘ziga xos yondashuvni taqdim etadi, chunki ular tasvirlar juftligidan belgilarni ajratib olish va solishtirishni o‘rganadi. Metodologiya bir necha asosiy bosqichlarni o‘z ichiga oladi: ma’lumotlarni tayyorlash, modelni ishlab chiqish, o‘qitish va sinovdan o‘tkazish.

1. Ma’lumotlarni tayyorlash. Modelni samarali o‘qitish va testdan o‘tkazish uchun ma’lumotlar to‘plami quyidagi tartibda tayyorlandi:

- **Skanerlash va formatlash** — qo‘lyozma imzolar skanerlandi va barcha tasvirlar yagona o‘lchamga keltiriladi.
- **Ranglarni o‘zgartirish** — hisoblash murakkabligini kamaytirish maqsadida tasvirlar kulrang (grayscale) tuslariga o‘tkaziladi.
- **Juftliklarni shakllantirish** — ma’lumotlar to‘plami imzo juftliklaridan tashkil topgan bo‘lib, har bir juftlikka “haqiqiy” yoki “qalbaki” ekanligini ko‘rsatuvchi yorliq (label) biriktiriladi.
- **Balanslash** — o‘qitish jarayonida biror sinfga (haqiqiy yoki qalbaki) ustunlik berib yubormaslik uchun sinflar balansi ta’minlanadi.

2. Modelning umumiy arxitekturasi. Ushbu tadqiqotda imzolarni verifikatsiya qilish uchun chuqur o‘rganishga asoslangan Konvolyutsion Neyron Tarmog‘i (CNN) arxitekturasi tanlab olindi. Taklif etilayotgan model kirish ma’lumotlaridan iyerarxik xususiyatlarni ajratib olish va ularni ixcham vektor ko‘rinishiga keltirishga qaratilgan. Model ketma-ket (Sequential) arxitekturada qurilgan bo‘lib, u konvolyutsion qatlamlar (Convolutional layers), pooling qatlamlari (Max-Pooling layers) va to‘liq bog‘langan qatlamlardan (Dense layers) iborat.

Model quyidagi bosqichlardan tashkil topgan:

- **Konvolyutsion bloklar:** Har bir konvolyutsion qatlam kirish tasviridan fazoviy xususiyatlarni (qirralar, chiziqlar va murakkab geometrik shakllar) ajratib oladi. Dastlabki qatlamlarda umumiy xususiyatlar aniqlansa, chuqurroq qatlamlarda imzoga xos bo‘lgan nozik detallar tahlil qilinadi.

- **Aktivatsiya funksiyasi:** Har bir konvolyutsion qatlamdan so‘ng ReLU (Rectified Linear Unit) aktivatsiya funksiyasi qo‘llanilgan. ReLU funksiyasi tarmoqqa nochiziqli (non-linear) xususiyatlarni kiritadi, bu esa modelga murakkab qonuniyatlarni o‘rganish imkonini beradi. ReLU funksiyasi manfiy qiymatlarni nolga tenglashtirib, hisoblash tezligini va modelning o‘rganish samaradorligini oshiradi.

- **Pooling (Hajmni qisqartirish):** Max-Pooling qatlamlari xususiyatlar xaritasining (feature map) o‘lchamini kichraytiradi, bu esa ortiqcha ma’lumotlarni (noise) kamaytirishga va hisoblash yuklamasini optimallashtirishga xizmat qiladi.

- **Flatten va Klassifikatsiya:** Yakuniy qatlamlarda konvolyutsion qatlamlardan olingan ikki o‘lchovli xususiyatlar “Flatten” qatlami orqali bir o‘lchovli vektorga aylantiriladi va klassifikatsiya (yoki o‘xshashlikni aniqlash) qismiga uzatiladi.

Quyi tarmoqlarning chiqishlari Evklid masofasi yordamida belgilar vektorlari orasidagi masofani hisoblaydigan qatlam bilan birlashtiriladi. Ushbu qatlam natijasi ikkala imzo bir kishiga tegishli yoki har xil shaxslarga tegishli ekanligini klassifikatsiya qilish uchun ishlatiladi.

O‘qitish jarayoni va eksperimental natijalar.

Modelning o‘qitilish jarayoni Contrastive Loss (Kontrastiv yo‘qotish) funksiyasiga asoslangan bo‘lib, u juftlik asosidagi taqqoslash (pair-based learning) uchun eng samarali yechim hisoblanadi. Kontrastiv yo‘qotish funksiyasi bir shaxsga tegishli haqiqiy imzo juftliklari orasidagi evklid masofasini (Euclidean distance) minimallashtirishga, turli shaxslarga tegishli yoki qalbaki imzolar juftligi orasidagi masofani esa maksimallashtirishga xizmat qiladi.

Model parametrlari Adam (Adaptive Moment Estimation) optimizatori yordamida yangilanib borildi. O‘qitish jarayonida “Backpropagation” (teskari xatolikni tarqatish) algoritmi orqali model og‘irliklari (weights) ketma-ket optimallashtirildi, bu esa modelning imzo xususiyatlarini aniqlash aniqligini sezilarli darajada oshirdi. Eksperimentlar natijasida ishlab chiqilgan Siyam modeli imzo verifikatsiyasi jarayonida barqaror va yuqori samaradorlik ko‘rsatdi. Sinov (test) bosqichida model 88% umumiy aniqlik (accuracy) ko‘rsatkichiga erishdi.

Xulosa va cheklovlar.

Yuqori unumdorlikka qaramay, ba’zi cheklovlar mavjud. Insonning jismoniy holatidagi o‘zgarishlar yoki imzo qo‘yish sharoitlari imzo xususiyatlariga ta’sir qilishi mumkin. Kelgusi tadqiqotlar yanada takomillashtirilgan arxitekturalarni ishlab chiqishni va modelni real hujjatlarni verifikatsiya qilish tizimlariga integratsiya qilishni o‘z ichiga olishi mumkin.

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LOGISTIC REGRESSION FOR BINARY CLASSIFICATION IN MACHINE LEARNING

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Problem statement. Suppose that the following training data describe the relationship between study time x and examination result y:

x	1	2	3	4	5
y	0	0	0	1	1

(1)

where $y = 1$ means that the student passed the examination and $y = 0$ means that the student failed.

We consider the logistic regression model

$$P(Y = 1 \mid x) = p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}}. \tag{2}$$

The main problem is to determine the parameters β_0 and β_1 and then predict the examination result of a student who studies for $x = 3.5$ hours.

Maximum likelihood formulation. For a binary observation $y_i \in \{0, 1\}$, the probability can be written as

$$P(Y_i = y_i) = p_i^{y_i}(1 - p_i)^{1-y_i}, \tag{3}$$

where

$$p_i = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_i)}}.$$

Therefore, the likelihood function for all observations is

$$L(\beta_0, \beta_1) = \prod_{i=1}^{5} p_i^{y_i}(1 - p_i)^{1-y_i}. \tag{4}$$

Taking logarithms gives

$$\ell(\beta_0, \beta_1) = \sum_{i=1}^{5} \left[y_i \ln p_i + (1 - y_i) \ln(1 - p_i) \right]. \tag{5}$$

Thus, maximizing the likelihood is equivalent to minimizing the cross-entropy loss

$$J(\beta_0, \beta_1) = -\sum_{i=1}^{5} \left[y_i \ln p_i + (1 - y_i) \ln(1 - p_i) \right]. \tag{6}$$

The first derivatives are

$$\frac{\partial J}{\partial \beta_0} = \sum_{i=1}^{5} (p_i - y_i), \tag{7}$$

and

$$\frac{\partial J}{\partial \beta_1} = \sum_{i=1}^{5} (p_i - y_i) x_i. \tag{8}$$

Consequently, the parameters can be updated numerically by

$$\beta_0^{(k+1)} = \beta_0^{(k)} - \eta \sum_{i=1}^{5} (p_i - y_i), \tag{9}$$

$$\beta_1^{(k+1)} = \beta_1^{(k)} - \eta \sum_{i=1}^{5} (p_i - y_i) x_i. \tag{10}$$

Numerical solution. To avoid perfect separation in this small illustrative dataset, we use a regularized logistic model. The objective function is

$$J_\lambda(\beta_0, \beta_1) = J(\beta_0, \beta_1) + \frac{\lambda}{2} \beta_1^2, \qquad \lambda > 0. \tag{11}$$

Taking $\lambda = 1$ and solving the corresponding optimization problem numerically gives approximately

$$\beta_0 = -3.64, \qquad \beta_1 = 1.14. \tag{12}$$

Hence the fitted logistic model is

$$p(x) = \frac{1}{1 + e^{-(-3.64 + 1.14x)}}. \tag{13}$$

We now predict the result for a student who studies for 3.5 hours. Substituting $x = 3.5$ gives

$$z = -3.64 + 1.14(3.5) = 0.35.$$

Therefore,

$$p(3.5) = \frac{1}{1 + e^{-0.35}} \approx 0.587. \tag{14}$$

Thus, the probability that the student passes the examination is approximately 58.7%.

Using the standard classification threshold 0.5, we obtain $p(3.5) > 0.5$, and therefore the model classifies the student as $Y = 1$ (Pass).

Theorem 1. The binary logistic regression cross-entropy loss

$$J(\beta) = -\sum_{i=1}^{m} [y_i \log p_i + (1 - y_i) \log(1 - p_i)]$$

is a convex function of the parameter vector β.

Proof. Let X be the design matrix and let

$$W = \operatorname{diag}(p_1(1 - p_1), \ldots, p_m(1 - p_m)).$$

The Hessian matrix of the loss function is

$$\nabla^2 J(\beta) = X^T W X. \tag{15}$$

Since

$$0 < p_i < 1,$$

we have

$$p_i(1 - p_i) > 0.$$

Therefore, W is positive semidefinite. For every vector $v \in \mathbb{R}^n$,

$$v^T \nabla^2 J(\beta) v = v^T X^T W X v = (Xv)^T W (Xv) \geq 0. \tag{16}$$

Hence $\nabla^2 J(\beta)$ is positive semidefinite, and consequently $J(\beta)$ is convex. The theorem is proved.

A concrete binary classification problem was considered using logistic regression. Based on the training observations, a mathematical model for the probability of passing an examination was constructed. For the regularized model, the estimated parameters were

$$\beta_0 = -3.64, \qquad \beta_1 = 1.14.$$

For a student studying 3.5 hours, the model gives a passing probability of approximately 58.7%. Therefore, using a threshold of 0.5, the student is classified into the "Pass" class.

The example demonstrates how probability theory, optimization, and convex analysis are combined in a basic Machine Learning classification problem.

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CLASSIFICATION OF DATA USING THE K-NEAREST NEIGHBORS ALGORITHM

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Classification is one of the fundamental problems of supervised Machine Learning. The objective is to assign a new observation to one of several predefined classes. K-Nearest Neighbors is a simple non-parametric classification algorithm that determines the class of a new point using the classes of its nearest training observations.

Consider the following two-dimensional training dataset:

Point	(x_1, x_2)	Class
A_1	$(1, 2)$	0
A_2	$(2, 3)$	0
A_3	$(3, 2)$	0
B_1	$(5, 5)$	1
B_2	$(6, 5)$	1
B_3	$(5, 7)$	1

(1)

Suppose that a new observation is

$$P = (4, 4). \tag{2}$$

Our objective is to determine whether P belongs to class 0 or class 1 using the KNN algorithm with

$$k = 3.$$

Distance calculation. For two points

$$X = (x_1, x_2), \qquad Y = (y_1, y_2),$$

the Euclidean distance is defined by

$$d(X, Y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}. \tag{3}$$

First, we calculate the distance from $P = (4, 4)$ to each training point.

For $A_1 = (1, 2)$,

$$d(P, A_1) = \sqrt{(4-1)^2 + (4-2)^2} = \sqrt{13} \approx 3.606. \tag{4}$$

For $A_2 = (2, 3)$,

$$d(P, A_2) = \sqrt{(4-2)^2 + (4-3)^2} = \sqrt{5} \approx 2.236. \tag{5}$$

For $A_3 = (3, 2)$,

$$d(P, A_3) = \sqrt{(4-3)^2 + (4-2)^2} = \sqrt{5} \approx 2.236. \tag{6}$$

For $B_1 = (5, 5)$,

$$d(P, B_1) = \sqrt{(4-5)^2 + (4-5)^2} = \sqrt{2} \approx 1.414. \tag{7}$$

Similarly,

$$d(P, B_2) = \sqrt{(4-6)^2 + (4-5)^2} = \sqrt{5} \approx 2.236, \tag{8}$$

and

$$d(P, B_3) = \sqrt{(4-5)^2 + (4-7)^2} = \sqrt{10} \approx 3.162. \tag{9}$$

Thus, the distances in increasing order are

Point	Distance	Class
B_1	1.414	1
A_2	2.236	0
A_3	2.236	0
B_2	2.236	1
B_3	3.162	1
A_1	3.606	0

There is a distance tie at $\sqrt{5}$. To make the classification rule deterministic, suppose that tied observations are ordered by their original order in the training dataset. Then the three nearest neighbors are

$$B_1, \quad A_2, \quad A_3.$$

Their corresponding classes are

$$1, \quad 0, \quad 0.$$

Therefore, class 0 receives two votes and class 1 receives one vote. Hence,

$$P = (4, 4) \text{ is classified as class 0.} \tag{10}$$

Theorem 1. Let k be an odd positive integer and suppose that the k nearest neighbors of a point P belong to two classes C_0 and C_1. If n_0 and n_1 denote the numbers of these neighbors belonging to C_0 and C_1, respectively, then

$$n_0 + n_1 = k.$$

If $n_0 > k/2$, the KNN classifier assigns P to C_0, while if $n_1 > k/2$, it assigns P to C_1.

Proof. Since

$$n_0 + n_1 = k$$

and k is odd, $n_0 = n_1$ is impossible. Therefore, exactly one of n_0 and n_1 is greater than $k/2$. By the majority voting rule of the KNN algorithm, the new observation is assigned to the class having the larger number of representatives among the k nearest neighbors. The theorem is proved.

Prediction for another observation. Consider another point

$$Q = (5.5, 5.5).$$

Its distances to $B_1 = (5, 5)$ and $B_2 = (6, 5)$ are

$$d(Q, B_1) = d(Q, B_2) = \sqrt{0.5} \approx 0.707.$$

Moreover,

$$d(Q, B_3) = \sqrt{(5.5 - 5)^2 + (5.5 - 7)^2} = \sqrt{2.5} \approx 1.581.$$

Thus, the three nearest neighbors of Q are

$$B_1, \quad B_2, \quad B_3,$$

all of which belong to class 1. Therefore,

$$Q = (5.5, 5.5) \text{ is classified as class 1.} \tag{11}$$

A concrete two-dimensional classification problem was solved using the K-Nearest Neighbors algorithm. For the new observation $P = (4, 4)$ and $k = 3$, Euclidean distances to all training observations were calculated. Using a fixed rule for the distance tie, two of the three selected nearest neighbors belong to class 0. Therefore, the KNN model assigns P to class 0.

The second observation $Q = (5.5, 5.5)$ was classified as class 1 because all of its three nearest neighbors belong to that class. The considered examples demonstrate how geometric distance and majority voting can be used to solve classification problems in Machine Learning.

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Kompakt fazolar gomeomorfizmi teoremasining yopiq akslantirishlar tilidagi zamonaviy yondashuvi va uning topologik tahlili

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Differensial geometriya va topologiya fanlarini o‘rganishda topologik fazolar o‘rtasidagi ekvivalentlikni (struktural saqlanishni) ifodalovchi markaziy tushuncha gomeomorfizm (topologik akslantirish) hisoblanadi. Ilmiy-uslubiy ishlarda topologik fazolarning invariant xossalarini o‘rganishda akslantirishlar uzluksizligi va kompaktlik tushunchalarining tutgan o‘rnini alohida ta’kidlab o‘tish mumkin.

1-ta’rif. Ikki (X, τ_X) va (Y, τ_Y) topologik fazolari berilgan bo‘lsin. Agar $f : X \to Y$ akslantirish biyektiv bo‘lib, f va uning teskarisi $f^{-1} : Y \to X$ uzluksiz bo‘lsa, f akslantirish gomeomorfizm deyiladi.

Klassik darsliklarda akslantirishning gomeomorfizm bo‘lishini tekshirishni osonlashtiruvchi quyidagi mashhur teorema keltiriladi.

1-teorema (Klassik teorema). Agar (X, τ_X) kompakt topologik fazo, (Y, τ_Y) esa Hausdorff fazosi bo‘lib, $f : X \to Y$ akslantirish uzluksiz va biyektiv bo‘lsa, u holda f gomeomorfizmdir.

Metodologik tahlil shuni ko‘rsatadiki, 1-teorema isbotida Y ning Hausdorff fazosi bo‘lishi faqatgina bir joyda – X dagi har qanday yopiq (va mos ravishda kompakt) $F \subset X$ to‘plamning $f(F) \subset Y$ tasviri Y da yopiq to‘plam bo‘lishini ta’minlash uchun ishlatiladi. Demak, Hausdorff aksiomasi gomeomorfizm mavjudligi uchun zaruriy shart emas, balki akslantirishning yopiqligini ta’minlovchi yetarli vositadir.

2-teorema (Yangi umumlashtirilgan teorema). Faraz qilaylik, (X, τ_X) kompakt fazo, (Y, τ_Y) esa ixtiyoriy topologik fazo bo‘lsin. Agar $f : X \to Y$ akslantirish:

1. biyektiv;

2. uzluksiz;

3. yopiq akslantirish (ya’ni har bir yopiq $F \subset X$ uchun $f(F)$ to‘plam Y da yopiq)

shartlarini qanoatlantirsa, u holda f akslantirish gomeomorfizmdir.

Isbot. Teoremani isbotlash uchun teskari akslantirish $f^{-1} : Y \to X$ ning uzluksizligini ko‘rsatish yetarlidir. Topologik fazolar uzluksizligi kriteriyasiga ko‘ra, f^{-1} akslantirish uzluksiz bo‘lishi uchun X dagi har qanday yopiq $F \subset X$ to‘plamning f^{-1} ostidagi asl tasviri Y da yopiq bo‘lishi zarur va yetarlidir.

Ixtiyoriy yopiq $F \subset X$ to‘plamni olaylik. (X, τ_X) kompakt fazo bo‘lganligi va F unda yopiq to‘plam bo‘lganligi sababli, topologiya xossasiga ko‘ra F to‘plamning o‘zi ham kompakt to‘plam bo‘ladi.

$f : X \to Y$ akslantirish uzluksiz bo‘lgani uchun, kompakt to‘plamning uzluksiz akslantirish ostidagi tasviri ham kompakt bo‘ladi. Demak, $f(F) \subset Y$ to‘plam Y da kompakt to‘plamdir.

Endi $f^{-1} : Y \to X$ akslantirish ostida F ning asl tasvirini ko‘ramiz. f biyektiv bo‘lganligi sababli, teskari akslantirishning asl tasviri to‘g’ri tasvirga tenglashadi:

$$(f^{-1})^{-1}(F) = f(F).$$

2-teoremaning 3-shartiga ko‘ra, f akslantirish yopiq akslantirishdir. Shuning uchun $f(F)$ to‘plam Y fazosida yopiq to‘plam bo‘ladi. Bu esa X fazosidagi ixtiyoriy yopiq F to‘plamning f^{-1} ostidagi asl tasviri yopiq ekanini anglatadi.

Demak, $f^{-1} : Y \to X$ akslantirish uzluksizdir. f va f^{-1} uzluksiz hamda f biyektiv bo‘lgani uchun f akslantirish gomeomorfizmdir. □

1-teoremaning amaliy kuchini hamda yopiq akslantirish shartining muhimligini ko‘rsatish uchun quyidagi misollarni qaraymiz:

1-misol (Hausdorff bo‘lmagan fazoda gomeomorfizm). Faraz qilaylik, $X = \{a, b\}$ chekli ikki nuqtali to‘plam bo‘lib, unda indiskret topologiya berilgan bo‘lsin:

$$\tau_X = \{\emptyset, X\}.$$

Endi $Y = \{1, 2\}$ to‘plamda ham indiskret topologiya berilsin:

$$\tau_Y = \{\emptyset, Y\}.$$

$f : X \to Y$ akslantirish $f(a) = 1, f(b) = 2$ kabi aniqlangan bo‘lsa, u biyektivdir. X va Y indiskret bo‘lganligi sababli f ham, f^{-1} ham uzluksiz. Shuning uchun f gomeomorfizmdir. Bunda Y Hausdorff emas, chunki uning turli nuqtalarini ajratuvchi ikkita o‘zaro kesishmaydigan ochiq to‘plam maujud emas. Bu misol Hausdorff sharti bo‘lmagan holda ham gomeomorfizm mavjud bo‘lishi mumkinligini ko‘rsatadi.

2-misol (Yopiqlik sharti bajarilmaganda gomeomorfizm bo‘lmasligi). Quyidagi akslantirishni qaraylik:

$$X = [0, 2\pi), \quad Y = S^1 \subset \mathbb{R}^2.$$

$$f : [0, 2\pi) \to S^1,$$

$$f(t) = (\cos t, \sin t).$$

Ushbu f akslantirish biyektiv va uzluksizdir. Biroq, $X = [0, 2\pi)$ kompakt fazo emas hamda yopiq akslantirish sharti bajarilmaydi (masalan, $F = [\pi, 2\pi)$ yopiq to‘plamning $f(F)$ tasviri S^1 da yopiq emas). Natijada f^{-1} teskari akslantirish $t = 0$ nuqtada uzluksiz bo‘lmaydi va akslantirish gomeomorfizm bo‘la olmaydi. Bu misol yopiq akslantirish shartining naqadar zaruriy xossa ekanini ko‘rsatadi.

Xulosa va ilmiy natijalar tahlili

1. Klassik darsliklardagi 1-teoremada ikkinchi fazo Y ning Hausdorff bo‘lish sharti gomeomorfizm to‘liq saqlanishi uchun absolyut zaruriy shart emasligi, u faqat $f(F)$ tasvirning yopiqligini ta’minlovchi xususiy holat ekanligi ko‘rsatildi.

2. Taklif etilgan 2-teorema va keltirilgan amaliy misollar orqali Hausdorff aksiomasi bo‘lmagan, lekin yopiq akslantirish xossasiga ega bo‘lgan umumiy topologik fazolarda ham gomeomorfizm mavjudligini tasdiqlovchi qat’iy va universal mezon yaratildi.

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OCHIQ KONLARDA MASSIV DEFORMATSIYASINI ANIQLASH O’LCHOV CHASTOTASI VA G’OVAK BOSIMI OMILINING MIQDORIY BAHOSI

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Muammo va dastlabki ma’lumotlar. Angren razrezida 2001-yil 9-dekabrda balandligi 60 m bo’lgan pog’ona qulagan; buzilish zonasi bort bo’ylab 180 m, hajmi 300 000 m^3 ga yaqin. Qulashdan 2 sutka oldin bermada kengligi 10–20 sm yoriqlar paydo bo’lgan. Amaldagi normativ esa vizual ko’rikni oyiga bir marta talab qiladi. Ishda razrezning oylik suv balansi (12 oy, 5 o’lchov ustuni), marksheyderlik kuzatuvlari natijalari va Tebinbulak koni burg’ulash-portlatish loyihasi parametrlari miqdoriy tahlil qilindi.

1. Suv oqimi siljishni tushuntirmaydi. Shimoliy va Janubiy nasos stansiyalari bo’yicha yillik kirim 9 639 250 m^3, chiqarish 9 468 265 m^3 (Janubiy stansiya va o’z oqimli drenaj bo’yicha ustun yig’indilari hujjat bilan aynan mos keldi). Maksimum mayda — 985 779 m^3, minimum fevralda — 679 560 m^3, nisbat 1,45, variatsiya koeffitsienti 10,8%.

Marksheyderlik kuzatuvlariga ko’ra avgust–sentyabrda siljish tezligi 10–16 mm/sut, noyabr–dekabrda 22–23 mm/sut, ya’ni 1,73 baravar ortgan. Shu ikki davr uchun oylik kirim nisbati esa 0,91 — belgisi qarama-qarshi. Jamg’arilgan qoldiqqa (kirim minus chiqarish) o’tilganda nisbat 1,31 bo’ladi va belgisi to’g’rilanadi: chiqarilmagan suv hajmi noyabrda maksimumga yetadi (+252 974 m^3).

Xulosa: massiv oniy suv oqimiga emas, uning to’planishiga javob beradi. Nasos jurnallari bo’yicha monitoring ishonchsiz — sensor tarmog’ida pyezometrik kanal majburiy.

[1-rasm. Oylik suv balansi va jamg’arilgan qoldiq.]

2. Barqarorlik zaxirasi katta emas. “Eski podstansiya” bloki uchun hisobot o’lchamlaridan (700 × 300 m, 3,5 mln m^3) o’rtacha qalinlik 16,7 m, sirpanish tekisligi og’ishi 3,6–8,3° aniqlandi. Suzak yarusi gillari uchun qoldiq mustahkamlik ($c = 0$, $\varphi = 13°$) bo’yicha cheksiz yonbag’ir modeli quruq holatda $F_s = 1{,}57$ beradi, kritik bosim koeffitsienti esa $r_u = 0{,}365$. Suv ustuniga o’tkazilganda bu blok qalinligining 74% i, ya’ni 12,4 m. $\varphi = 11°$ da chegara yanada past — 8,3 m.

3. O’lchov oralig’iga talab. Pretsedent oynasi $T_{pre} = 2$ sutka bo’lganda hodisani aniqlash ehtimoli: oylik ko’rikda 6,7%, haftalikda 28,6%, sutkalikda 100%. Ammo trendni chiziqli baholash uchun oynada kamida 12 ta namuna kerak, bundan $T_{int} \leq 4$ soat talabi kelib chiqadi; soatlik

diskretlashda oynaga 48 ta namuna tushadi. Yoriqning o'rtacha ochilish tezligi 150 mm/2 sut = 75 mm/sut bo'lgan, bu taklif etilgan 25 mm/sut chegarasidan uch baravar yuqori — ogohlantirish qulashdan taxminan 45 soat oldin chiqqan bo'lardi.

[2-rasm. Aniqlash ehtimoli va namunalar soni o'lchov oralig'iga bog'liq.]

4. Tebinbulak: noaniqlik o'lchovni talab qiladi. Loyiha ma'lumotlaridan zaryad massasi hisoblandi: $Q(\varnothing 110) = 77{,}4$ kg, $Q(\varnothing 250) = 572$ kg. Masshtablangan masofa qonuni bo'yicha 86 m da $PPV = 29{,}7$ mm/s. Biroq adabiyotdagi $K = 700 \dots 1700$ tarqoqligi natijani 2,43 baravar noaniq qiladi (18,2... 44,3 mm/s) — biri xavfsiz, ikkinchisi yoriq ochilishi chegarasida. Konda uchta yoriq tizimi 65–85° ostida yotgani sababli tarqalish anizotrop. Bu yerda sensorning vazifasi ogohlantirish emas, K va β ni konning o'zida aniqlash.

5. Chekka hisoblash zarurati. Tez kanal 2000 Hz da uzluksiz ishlasa, oqim 1,037 GB/sut bo'ladi — na LoRa, na batareya ko'taradi. Trigger rejimida bitta hodisa yozuvi 42 kB, uzatiladigani 20 bayt (2100× kamayish). Sekin kanalda 2,07 MB/sut o'rniga o'rtacha 1620 B/sut (1280×). Quvvat byudjeti normal rejimda 3,08 mA·soat/sut, faol rejimda 15,51; 6000 mA·soat batareyada avtonomlik mos ravishda 4,3 yil va 309 sutka. Sarfning 56,7% i GNSS ga to'g'ri keladi — demak GNSS faqat tayanch tugunlarga o'rnatilishi kerak.

Xulosa. Angren va Tebinbulak gidrogeologik jihatdan bir-biriga qarama-qarshi (suv oqimi bo'yicha farq 967 baravar), ammo ikkalasida ham amaldagi kuzatuv chastotasi hodisaning xarakterli vaqtidan kichik emas. Taklif etilayotgan ikki kanalli tugun — soatlik sekin kanal (inklinometr, GNSS, pyezometr) va trigger bo'yicha ishlaydigan tez kanal (2000 Hz) — shu bo'shliqni to'ldiradi. Keyingi bosqichda algoritm razrezning 1987-yildan yig'ilgan kuzatuv arxiviga qo'llanib, retrospektiv tekshiriladi.

Kalit so'zlar: ochiq kon, yonbag'ir barqarorligi, g'ovak bosimi, chekka hisoblash, PPV, erta ogohlantirish.

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TA'LIMIY BAHOLASHNING NOANIQLIK SHAROITIDAGI MATEMATIK MODELLARINI TAKOMILLASHTIRISH: FUZZY MANTIQQA ASOSLANGAN YONDASHUV

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Kirish. Zamonaviy elektron ta'limda baholash faqat test natijalariga emas, balki LMS faolligi, topshiriq sifati, mustaqil ishlash va o'quv dinamikasi kabi ko'rsatkichlarga ham bog'liq. Shu sababli baholash jarayoni ko'p mezonli va noaniq xususiyat kasb etadi.

Baholashdagi noaniqlik: Baholashda test natijasi va davomat kabi obyektiv ko'rsatkichlar bilan birga, ijodkorlik, motivatsiya va o'qituvchi ekspertizasi kabi subyektiv omillar ham mavjud. An'anaviy ball tizimi bunday omillarni hamda o'quvchining rivojlanish dinamikasini to'liq aks ettira olmaydi.

Fuzzy yondashuv zarurati: "yaxshi bilim", "o'rtacha", "yuqori faollik" kabi pedagogik tushunchalar qat'iy matematik chegaralarga ega emas. Fuzzy mantiq bunday noaniqliklarni "past", "o'rta", "yuqori" kabi lingvistik qiymatlar orqali modellashtirib, bir nechta mezonlarni kompleks baholash imkonini beradi. Bu yondashuv elektron baholashning moslashuvchanligi va aniqligini oshirishga xizmat qiladi.

Mavzuga oid adabiyotlarning tahlili. Fuzzy mantiq noaniqlik sharoitida ta'limiy baholashni matematik modellashtirishning samarali yondashuvi hisoblanadi. Zadeh fuzzy to'plamlar va lingvistik o'zgaruvchilar nazariyasini ishlab chiqib, noaniq pedagogik ko'rsatkichlarni matematik ifodalashga asos yaratgan [1,2]. Mamdani va Assilian fuzzy qoidalar asosida xulosa chiqarish mexanizmini taklif qilgan [3], Zimmermann esa fuzzy modellarni amaliy qo'llash tamoyillarini rivojlantirgan [4]. Zamonaviy tadqiqotlarda ushbu yondashuvlarning ta'limiy ma'lumotlarni tahlil qilish va baholashdagi imkoniyatlari kengayib bormoqda [5]. Shu sababli fuzzy mantiq asosida ko'p mezonli va noaniq ta'limiy baholash modelini ishlab chiqish dolzarb hisoblanadi.

Asosiy qism. Ta'limiy baholashda o'quvchilarning bilim va kompetensiyalari test natijalari, topshiriqlar sifati, LMS faolligi va mustaqil ta'lim kabi ko'rsatkichlar asosida aniqlanadi. Ushbu mezonlarning turlicha va noaniq ifodalanishi baholashda matematik modellarni qo'llashni taqozo etadi.

Boolean mantiqdagi 0 va 1 qiymatlar "yaxshi", 'o'rta", "yuqori" kabi oraliq pedagogik holatlarni to'liq ifodalay olmaydi. Fuzzy mantiq esa tegishlilik darajasini [0;1] oralig'ida ifodalab, noaniq ma'lumotlarni qayta ishlash imkonini beradi [1]. Fuzzy modelda "bilim darajasi", "faollik" va "topshiriq sifati" lingvistik o'zgaruvchilar sifatida olinib, "past", "o'rta" va "yuqori" kategoriyalari orqali baholanadi. Baholash ekspert bilimlariga asoslangan IF-THEN qoidalari yordamida amalga oshiriladi [2; 3]. Masalan: AGAR test natijasi va LMS faolligi yuqori bo'lsa, UNDA bilim darajasi yuqori.

Taklif etilayotgan model fuzzifikatsiya $\rightarrow$ fuzzy inferensiya $\rightarrow$ defuzzifikatsiya $\rightarrow$ yakuniy elektron baho ketma-ketligida ishlaydi. Ushbu yondashuv bir nechta mezonlarni integratsiyalab, baholashning obyektivligi, moslashuvchanligi va aniqligini oshiradi hamda o'quvchilar rivojlanishini monitoring qilish imkonini beradi.

Tadqiqot metodologiyasi. Tadqiqotda tizimli, matematik modellashtirish va kompetensiyaviy yondashuvlar qo'llanildi. Ta'limiy baholashdagi noaniqliklarni modellashtirishda fuzzy to'plamlar, a'zolik funksiyalari va IF-THEN qoidalaridan foydalanildi.

Modelda fuzzifikatsiya, fuzzy inferensiya va defuzzifikatsiya bosqichlari asosida o‘quvchilarning bilim ko‘rsatkichlari kompleks baholandi. Shuningdek, modellashtirish, tahlil qilish, taqqoslash va umumlashtirish usullaridan foydalanildi.

Natija. Tadqiqot natijasida ta’limiy baholashdagi noaniqliklarni hisobga oluvchi fuzzy matematik model ishlab chiqildi. Model bir nechta baholash mezonlarini integratsiyalab, fuzzifikatsiya, fuzzy inferensiya va defuzzifikatsiya orqali yakuniy elektron bahoni aniqlash imkonini beradi. Natijada baholashning moslashuvchanligi, aniqligi va ishonchliligini oshirish hamda o‘quvchining bilim darajasini kompleks baholash imkoniyati yaratildi.

Xulosa. Ta’limiy baholashdagi noaniq va ko‘p mezonli omillarni an’anaviy usullar orqali to‘liq aks ettirish qiyin. Tadqiqot natijasida fuzzy mantiq asosidagi matematik model noaniqliklarni hisobga olish, turli mezonlarni integratsiyalash va o‘quvchilar bilimini kompleks baholash imkonini berishi aniqlandi. Mazkur yondashuv elektron ta’limda baholashning aniqligi, obyektivligi va ishonchliligini oshirish, shuningdek, individual pedagogik qarorlarni qo‘llab-quvvatlash uchun samarali vosita hisoblanadi.

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Raqamli transformatsiya davrida ta’lim ekotizimini boshqarishda sun’iy intellekt imkoniyatlari

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Kalit so‘zlar: sun’iy intellekt, ta’lim ekotizimi, raqamli transformatsiya, ta’lim boshqaruvi, moslashuvchan ta’lim, ta’lim tahlili (learning analytics), qaror qabul qilish tizimlari.

Kirish. So‘nggi yillarda raqamli transformatsiya jarayonlari deyarli barcha ijtimoiy-iqtisodiy sohalarni, jumladan, ta’lim tizimini ham tubdan o‘zgartirib yubordi. Ta’lim endi faqat bilim uzatish jarayoni sifatida emas, balki ko‘plab ishtirokchilar – o‘quvchi, o‘qituvchi, ta’lim muassasasi ma’muriyati, davlat organlari va raqamli platformalarni o‘zaro bog’lovchi murakkab ekotizim sifatida qaraladi. Ushbu ekotizimni samarali boshqarish katta hajmdagi ma’lumotlarni tahlil qilish, tezkor qaror qabul qilish va individual ehtiyojlarga moslashishni talab etadi, buning uddasidan an’anaviy boshqaruv usullari to‘liq chiqa olmaydi.

Sun’iy intellekt (SI) texnologiyalari – mashinali o‘rganish, tabiiy tilni qayta ishlash va katta ma’lumotlar tahlili – ta’lim ekotizimini boshqarishda yangi imkoniyatlar ochib bermoqda. Ushbu

tezisning maqsadi – raqamli transformatsiya sharoitida sun'iy intellektning ta'lim ekotizimini boshqarishdagi asosiy yo‘nalishlari va amaliy imkoniyatlarini tahlil qilishdan iborat.

Asosiy qism. Ta'lim ekotizimini sun'iy intellekt yordamida boshqarishning bir necha o‘zaro bog'liq yo‘nalishlarini ajratib ko‘rsatish mumkin. Birinchidan, moslashuvchan (adaptiv) ta'lim tizimlari o‘quvchining bilim darajasi, o‘zlashtirish sur'ati va individual xususiyatlarini real vaqt rejimida tahlil qilib, unga mos ta'lim trayektoriyasini shakllantiradi. Bu yondashuv ta'lim sifatini oshirish bilan birga, ma'muriyat va o‘qituvchilarning nazorat funksiyasini ham yengillashtiradi.

Ikkinchidan, ta'lim tahlili (learning analytics) tizimlari talabalarning akademik faolligi, davomati va natijalari bo‘yicha katta hajmdagi ma'lumotlarni yig'ib, xavf ostidagi (o‘qishni tashlab ketish ehtimoli yuqori) talabalarni erta bosqichda aniqlash imkonini beradi. Bu esa ta'lim muassasasi rahbariyatiga oldindan chora ko‘rish, resurslarni oqilona taqsimlash va institutsional qarorlarni ma'lumotlarga asoslangan holda qabul qilish imkoniyatini yaratadi.

[1-rasm shu yerga joylashtiriladi]

1-rasm. Sun'iy intellektga asoslangan ta'lim ekotizimini boshqarish modelining struktura diagrammasi

Uchinchidan, ma'muriy jarayonlarni avtomatlashtirish – jadval tuzish, resurslarni taqsimlash, hujjat aylanishi va murojaatlarga javob berish kabi vazifalarni SI asosidagi tizimlar, jumladan virtual yordamchilar va chat-botlar orqali sezilarli darajada tezlashtiradi. Bu ma'muriy xodimlarning vaqtini strategik vazifalarga yo‘naltirish imkonini beradi.

To‘rtinchidan, sun'iy intellekt ta'lim sifatini nazorat qilish va akkreditatsiya jarayonlarida ham qo‘llanilmoqda: o‘quv dasturlari mazmunini tahlil qilish, natijalarni standartlar bilan solishtirish va sifat ko‘rsatkichlarini avtomatik monitoring qilish orqali institutsional boshqaruv shaffofligi oshiriladi. Shu bilan birga, SI asosidagi kontent generatsiya vositalari o‘qituvchilarga ta'lim materiallari, testlar va individual topshiriqlarni tezroq tayyorlashda yordam beradi.

Biroq bu imkoniyatlar bilan bir qatorda muayyan muammolar ham mavjud: ma'lumotlar maxfiyligi va xavfsizligi, algoritmik tarafkashlik (bias), o‘qituvchi va ma'muriyatning raqamli savodxonlik darajasi, shuningdek, SI tizimlarini mavjud ta'lim infratuzilmasiga integratsiya qilishning texnik va moliyaviy murakkabligi shular jumlasidandir. Shu sababli SI asosidagi boshqaruv tizimlarini joriy etishda bosqichma-bosqich, sinov-nazorat mexanizmlari bilan yondashuv talab etiladi.

[2-rasm shu yerga joylashtiriladi]

2-rasm. An'anaviy boshqaruv usuli bilan sun'iy intellekt asosidagi boshqaruvni solishtiruvchi diagrammasi

Xulosa. Xulosa qilib aytganda, sun'iy intellekt raqamli transformatsiya davrida ta'lim ekotizimini boshqarishning sifat jihatidan yangi bosqichga o'tishini ta'minlovchi muhim vositadir. Moslashuvchan ta'lim, ta'lim tahlili, ma'muriy avtomatlashtirish va sifat nazorati yo'nalishlarida SI texnologiyalarining integratsiyasi boshqaruv qarorlarining asoslanganligi va tezkorligini oshiradi. Shu bilan birga, bu jarayon ma'lumotlar xavfsizligi, etik me'yorlar va kadrlar tayyorgarligi masalalarini har tomonlama hisobga olgan holda, ilmiy asoslangan strategiya doirasida amalga oshirilishi lozim.

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AN AI-ENABLED MULTI-RANKING DIGITAL TWIN MODEL FOR INTERNAL ASSESSMENT AND STRATEGIC OPTIMISATION OF UNIVERSITY PERFORMANCE

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International university rankings do not measure institutional performance in the same way. The QS World University Rankings group their indicators around Research and Discovery, Employability and Outcomes, Learning Experience, Global Engagement, and Sustainability [1], whereas the Times Higher Education (THE) World University Rankings 2026 use 18 indicators organised into Teaching, Research Environment, Research Quality, International

Outlook, and Industry [2]. Differences in indicator definitions, weights, reference populations, normalisation rules, data sources, and reporting lags make direct internal comparison difficult. Some components, particularly reputation and citation-based measures, also depend on external populations and cannot be reconstructed from university records alone.

To address this problem, a multi-ranking digital twin can be used as an internal analytical model for assessment, forecasting, and resource planning. Let $\mathcal{R} = \{r_1, \dots, r_R\}$ denote the ranking systems and let $x_t = (x_{1t}, \dots, x_{pt})$ represent the institutional state at time t. The state vector is separated into variables that can be influenced directly by university management, x_t^{int}, and externally observed variables, x_t^{ext}. Each observation is supplemented by its source s_{it}, reliability coefficient $q_{it} \in [0, 1]$, missing-data indicator $m_{it} \in \{0, 1\}$, and reporting lag ℓ_i. Reputation and citation indicators are therefore treated as exogenous or indirectly estimated quantities rather than as internally generated scores.

The first layer of the model establishes a common indicator ontology $\Omega = \{\omega_1, \dots, \omega_p\}$. For each ranking system r, a sparse mapping matrix $M^{(r)}$ links the university's internal variables with the definitions used in that ranking. The ranking-specific institutional state is then expressed as

$$z_t^{(r)} = \Phi_r\Big(M^{(r)}[x_t^{int}, x_t^{ext}, \Delta x_t, q_t, m_t, \ell]\Big), \tag{1}$$

where Φ_r applies the direction of each indicator, the required transformation, an internal normalisation proxy, and aggregation. When the ranking provider does not disclose the full reference distribution used for official normalisation, the proxy is calibrated against an available peer group. The resulting value is interpreted as internal ranking readiness, not as a reconstructed official score or a guaranteed future rank.

A temporal component is required because research output, citation impact, reputation, graduate outcomes, internationalisation, and industry engagement evolve at different speeds. For multi-horizon forecasting, the model employs the Temporal Fusion Transformer (TFT), which combines static covariates, observed histories, known future inputs, feature selection, and interpretable attention in a single architecture [3]. Let $H_t = \{x_{t-L:t}, u_{t:t+H}, b_{t-L:t}, c\}$ contain historical indicators, planned actions u, peer-benchmark signals b, and static institutional characteristics c. For ranking r, indicator j, forecast horizon h, and quantile τ, the forecast is

$$Q_\tau(z_{rj,t+h} \mid H_t) = F^{(\tau)}_{\theta,rjh}(H_t), \qquad \tau \in \{0.10, 0.50, 0.90\}. \tag{2}$$

Joint estimation across several ranking systems allows common information to be exploited without eliminating ranking-specific priorities. The training criterion can be written as

$$\mathcal{L}(\theta) = \sum_{r,j,h,\tau} a_r w_{rj} \rho_\tau\Big(z_{rj,t+h} - \hat{z}^{(\tau)}_{rj,t+h}\Big) + \lambda\|\theta\|_2^2 + \mu\mathcal{L}_{cons} + \nu\mathcal{L}_{cal}. \tag{3}$$

Here a_r denotes the strategic importance assigned to ranking r, w_{rj} is the weight of indicator j, and ρ_τ is the pinball loss. The additional terms penalise inconsistency across related ranking indicators and poor probability calibration. Retaining the 0.10, 0.50, and 0.90 quantiles makes forecast uncertainty explicit instead of reducing the result to a single deterministic value.

For internal monitoring, the median indicator forecasts are aggregated into a ranking-specific Ranking Readiness Score (RRS):

$$RRS_r(t + h) = \sum_{j=1}^{m_r} w_{rj} \hat{z}^{(0.50)}_{rj,t+h}, \qquad \sum_{j=1}^{m_r} w_{rj} = 1. \tag{4}$$

The RRS_r is accompanied by a data-quality coverage measure and must not be interpreted as an official QS or THE score. If a sufficiently informative peer distribution $G_{r,t}$ is available, the framework can estimate the probability of entering a ranking interval rather than claiming an exact future position:

$$P_{r,b} = \Pr(\text{rank}_r \in B_b \mid RRS_r, G_{r,t}, \sigma_{r,t}), \tag{5}$$

where B_b is a rank band and $\sigma_{r,t}$ represents predictive uncertainty. Such interval-based reporting is more defensible when competitor data are incomplete or when the ranking provider uses proprietary normalisation procedures.

Forecasting is useful only if it can support resource-allocation decisions. Let Δx denote feasible interventions in research, internationalisation, employability, industry engagement, or sustainability. A portfolio of actions can be selected from the feasible set $\mathcal{F}$ by solving

$$\Delta x^* = \arg\max_{\Delta x \in \mathcal{F}} \left\{ \sum_r a_r \mathbb{E}[RRS_r(x + \Delta x)] - \beta C(\Delta x) - \gamma \sum_r a_r \, \mathrm{CVaR}_\alpha(L_r) \right\}. \tag{6}$$

The set $\mathcal{F}$ incorporates budget, staffing, implementation-lag, and mutual-exclusivity constraints. The function $C(\Delta x)$ represents the cost of interventions, while $L_r = \max\{0, T_r - RRS_r(x + \Delta x)\}$ measures the shortfall from a target readiness level T_r. The CVaR_α term limits exposure to adverse scenarios, so an intervention with a high expected return is not accepted without considering downside risk.

Universities normally pursue several ranking objectives simultaneously. It is therefore useful to identify interventions that generate benefits across more than one methodology. The cross-ranking marginal efficiency of indicator i is defined by

$$MRE_i = \sum_r a_r \frac{\mathbb{E}[RRS_r(x + \delta e_i)] - \mathbb{E}[RRS_r(x)]}{C_i(\delta)}. \tag{7}$$

A high MRE_i indicates that a marginal intervention is expected to improve readiness across several ranking systems relative to its cost. This quantity is a scenario-sensitivity measure rather than a causal effect; causal attribution would require an experimental or quasi-experimental research design.

In practice, the digital twin can combine bibliometric, human-resources, student, finance, international-relations, graduate-employment, industry-engagement, and sustainability data. A governance layer should record indicator definitions, responsible units, source lineage, verification status, methodology version, and reporting lag. The analytical layer handles anomaly detection, missing-data treatment, TFT forecasting, and rank-band calibration. The decision layer then reports target gaps, forecast intervals, data confidence, rank-band probabilities, principal drivers, and feasible intervention portfolios.

Before the model is used for strategic planning, it should be evaluated by rolling-origin backtesting. Quantile forecasts can be assessed with pinball loss and interval coverage, while rank-band probabilities can be checked using calibration error and the Brier score. The stability of RRS_r should also be tested under plausible changes in ranking weights, missing-data assumptions, and peer samples. The purpose of the model is not to imitate proprietary ranking calculations; it is to provide a consistent internal framework for comparing methodologies, quantifying uncertainty, tracing indicator provenance, and evaluating alternative strategic decisions.

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SUN‘IY INTELLEKTNING GENERATIV TEXNOLOGIYALARI: ZAMONAVIY IMKONIYATLAR VA KELAJAK ISTIQBOLLAR

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Bugungi kunda sun‘iy intellekt texnologiyalari raqamli iqtisodiyotning eng istiqbolli yo‘nalishlaridan biri sifatida rivojlanmoqda. O‘zbekiston Respublikasida ham ushbu sohaga alohida e’tibor qaratilmoqda. Xususan, O‘zbekiston Respublikasi Prezidentining 2024-yil 14-oktabrdagi PQ-358-son qarori bilan Sun’iy intellekt texnologiyalarini 2030-yilgacha rivojlantirish strategiyasi tasdiqlangan bo‘lib, mazkur hujjat mamlakatda sun’iy intellekt texnologiyalarini rivojlantirish va amaliyotga keng joriy etish uchun muhim asos bo‘lib xizmat qiladi [1]. Generativ sun’iy intellekt ushbu istiqbolli yo‘nalishlarning eng muhimlaridan biri bo‘lib, u mavjud ma’lumotlar asosida yangi matn, tasvir, audio, video va dasturiy kod yaratish imkonini beradi. Zamonaviy katta til modellari insonning tabiiy tildagi so‘rovlarini tushunib, ularga mazmunli javoblar yaratishi orqali ta’lim, ilm-fan, biznes va axborot texnologiyalari sohalarida foydalanish imkoniyatlarini kengaytirmoqda. Ta’lim sohasida sun’iy intellekt o‘quv materiallarini tayyorlash, individual ta’lim dasturlarini shakllantirish va o‘quvchilarga qo‘shimcha tushuntirishlar berishda yordam berishi mumkin[2]. Tibbiyotda esa katta hajmdagi ma’lumotlarni tahlil qilish, kasalliklarni aniqlashga ko‘maklashish va ilmiy tadqiqotlarni tezlashtirish imkoniyatiga ega. Biznes va sanoatda sun’iy intellekt takroriy jarayonlarni avtomatlashtirish, ma’lumotlarni tahlil qilish, ishlab chiqarish samaradorligini oshirish va yangi mahsulotlarni ishlab chiqishda qo‘llanilishi mumkin. Shu bilan birga, generativ sun’iy intellektdan foydalanishda noto‘g‘ri axborot yaratish, mualliflik huquqi, shaxsiy ma’lumotlarni himoya qilish va axborot xavfsizligi kabi masalalarga alohida e’tibor berish zarur[3]. Kelajakda sun’iy intellektning rivojlanishi inson mehnatini to‘liq almashtirishdan ko‘ra, inson imkoniyatlarini kengaytiruvchi va intellektual faoliyat samaradorligini oshiruvchi texnologik vosita sifatida muhim ahamiyat kasb etadi.

Mazkur tadqiqotda generativ sun’iy intellektdan ta’lim jarayonida samarali foydalanishning amaliy yondashuvi taklif etiladi. Bizning taklifimizga ko‘ra, sun’iy intellektdan tayyor javob olish vositasi sifatida emas, balki o‘quvchining mustaqil fikrlashini rivojlantiruvchi yordamchi vosita sifatida foydalanish maqsadga muvofiq. Bunda foydalanuvchi dastlab muammoni mustaqil ravishda tahlil qiladi, keyin esa sun’iy intellekt yordamida qo‘shimcha ma’lumot, tushuntirish va turli yechimlarni oladi. Olingan natijalar foydalanuvchi tomonidan tahlil qilinib, tekshiriladi va yakuniy xulosa mustaqil ravishda shakllantiriladi[7]. Shuningdek, generativ sun’iy intellekt yordamida murakkab mavzularni sodda shaklda tushuntirish, individual topshiriqlar yaratish, matnlarni tahlil qilish va yangi g‘oyalarni ishlab chiqish imkoniyatlaridan foydalanish taklif etiladi. Bunday yondashuvda asosiy e’tibor inson va sun’iy intellekt o‘rtasidagi hamkorlikka qaratiladi. Ya’ni sun’iy intellekt insonning o‘rnini egallamaydi, balki uning bilim olish va muammoni hal qilish imkoniyatlarini kengaytiruvchi vosita sifatida xizmat qiladi[8].

Taklif etilayotgan yondashuvdan foydalanish natijasida ta'lim jarayonida axborotni izlash, qayta ishlash va tushunish uchun sarflanadigan vaqtni qisqartirish, o'quvchilarning mustaqil ishlash imkoniyatlarini kengaytirish hamda murakkab mavzularni o'zlashtirish samaradorligini oshirish mumkin. Zamonaviy tadqiqotlar ham generativ sun'iy intellektdan maqsadli foydalanish o'quv natijalariga ijobiy ta'sir ko'rsatishi mumkinligini ko'rsatmoqda. Mening taklif qilayotgan yondashuvning asosiy natijasi esa sun'iy intellektdan foydalanish bilan bir qatorda foydalanuvchining mustaqil fikrlashi va tahliliy qobiliyatini saqlab qolishdan iborat. Natijada o'quvchi tayyor javobni shunchaki qabul qilmasdan, uni tahlil qilish, tekshirish va o'z xulosasini shakllantirishga o'rganadi. Bundan tashqari, ushbu yondashuv turli fanlar va foydalanuvchilarning bilim darajasiga moslashtirilishi mumkin. Shu sababli generativ sun'iy intellektni inson faoliyatini almashtiruvchi emas, balki inson imkoniyatlarini kengaytiruvchi texnologiya sifatida qo'llash eng samarali natija-lardan birini beradi.

Xulosa qilib aytganda, generativ sun'iy intellekt zamonaviy texnologiyalar orasida katta istiqbolga ega bo'lgan yo'nalishlardan biridir. Tadqiqot davomida uning matn yaratish, ma'lumot-larni tahlil qilish, ta'lim jarayonini qo'llab-quvvatlash va ijodiy vazifalarni bajarishdagi imkoniyat-lari ko'rib chiqildi. Taklif etilgan yondashuv asosida sun'iy intellektdan inson o'rnini bosuvchi vosita sifatida emas, balki insonning bilim va imkoniyatlarini kengaytiruvchi yordamchi texnologiya sifatida foydalanish maqsadga muvofiqligi aniqlandi. Bunday foydalanish orqali vaqtni tejash, murakkab vazifalarni soddalashtirish, yangi g'oyalarni shakllantirish va ta'lim samaradorligini oshirish mumkin. Shu bilan birga, AI tomonidan yaratilgan ma'lumotlarni tekshirish va tahlil qilish insonning muhim vazifasi bo'lib qoladi. Kelajakda generativ sun'iy intellektning yanada rivojlanishi natijasida uning imkoniyatlari kengayib, turli sohalarda inson faoliyatini yanada samarali tashkil etishga xizmat qilishi kutiladi.

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BULUTLI MUHITDA INTELLEKTUAL TAHLIL VAZIFALARINI MA'LUMOT LOKALLIGI ASOSIDA ADAPTIV PARALLELLASHTIRISH

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Bulutli hisoblash tizimlarida katta hajmdagi ma'lumotlarga intellektual ishlov berish samaradorligi faqat protsessorlar sonini ko'paytirish bilan belgilanmaydi. Ma'lumotning qaysi tugunda joylashgani, tugunlar o'rtasidagi tarmoq o'tkazuvchanligi, joriy ish yuklamasi, xotira holati va mashinali o'qitish algoritmining ichki parallelizmi umumiy bajarilish vaqtiga bevosita ta'sir qiladi. Strukturaviy jadvallar, sensor oqimlari, matnli fayllar va mashinali o'qitish modellarida ma'lumotni hisoblash tuguniga ko'chirish ko'pincha hisoblashning o'zidan qimmatroq bo'lishi mumkin. Shu sababli vazifani mavjud bo'sh resursga yuborish emas, balki ma'lumot lokalligini, hisoblash xarajatini va tugun holatini birgalikda hisobga oluvchi adaptiv taqsimlash mexanizmini ishlab chiqish dolzarb masala hisoblanadi.

Bulutli tizimlarda parallel ishlov berishning umumiy tamoyili ma'lumotni bo'lish, lokal qayta ishlash va natijalarni birlashtirish ketma-ketligiga asoslanadi. Mazkur yondashuv taqsimlangan JOIN operatsiyalari, intellektual tahlil algoritmlari va MLOps konveyerlarida turli ko'rinishlarda uchraydi [1–3]. Biroq statik fragmentlashda tugunlar unumdorligi va tarmoq holati o'zgarmas deb qaraladi. Real bulut muhitida esa virtual mashinalar yuklanishi, konteynerlar soni, xotira bosimi va tarmoq kechikishi vaqt bo'yicha o'zgaradi. Taklif qilinayotgan yondashuvning maqsadi ushbu o'zgarishlarni monitoring qilish va vazifa-ma'lumot juftligini mos tugunga dinamik biriktirishdan iborat.

Masalaning qo'yilishi. D katta ma'lumotlar to'plami p ta mantiqiy fragmentga ajratilsin:

$$D = \bigcup_{i=1}^{p} D_i, \qquad D_i \cap D_k = \varnothing,\ i \neq k. \tag{1}$$

Har bir D_i fragmentga mos hisoblash vazifasi T_i mavjud bo'lib, u ma'lumotlarni tozalash, xususiyatlarni shakllantirish, klassifikatsiya, klasterlash yoki model parametrlarini yangilash operatsiyalaridan biri bo'lishi mumkin. Bulut klasteri $R = \{R_1, R_2, \ldots, R_m\}$ tugunlardan tashkil topadi. Har bir R_j tugunning joriy holati CPU, RAM, GPU, tarmoq va navbat yuklamasi orqali

$$r_j = (c_j, m_j, g_j, b_j, q_j) \tag{2}$$

vektor bilan tavsiflanadi. Bu yerda c_j – mavjud protsessor quvvati, m_j – bo'sh xotira, g_j – GPU resursi, b_j – tarmoq o'tkazuvchanligi, q_j esa tugundagi kutilayotgan navbat yuklamasidir.

Vazifa T_i ni R_j tugunda bajarish vaqtini soddalashtirilgan holda hisoblash, ma'lumot uzatish va navbatda kutish komponentlari yig'indisi sifatida baholash mumkin:

$$\widehat{T}_{ij} = \frac{W_i}{c_j} + \frac{V_i^{remote}}{b_j} + q_j. \tag{3}$$

Bu yerda W_i – vazifaning hisoblash hajmi, V_i^{remote} – bajarishdan oldin tarmoq orqali uzatilishi zarur bo'lgan ma'lumot miqdori. Agar fragmentning katta qismi tugunning o'zida mavjud bo'lsa, tarmoq xarajati kamayadi. Ma'lumot lokalligi koeffitsiyenti

$$\ell_{ij} = \frac{V_i^{local}}{V_i}, \qquad 0 \leq \ell_{ij} \leq 1 \tag{4}$$

ko‘rinishida aniqlanadi. ℓ_{ij} qiymati birga yaqin bo‘lsa, vazifa ma’lumotga yaqin joylashtirilgan bo‘ladi.

Taklif qilinayotgan adaptiv tanlash mezonida bajarilish vaqti bilan birga lokallik, tugun yuklamasi va nosozlik xavfi hisobga olinadi:

$$C_{ij} = \alpha \widehat{T}_{ij} + \beta(1 - \ell_{ij}) + \gamma U_j + \delta F_j, \tag{5}$$

$$j^* = \arg \min_{1 \le j \le m} C_{ij}. \tag{6}$$

Bu yerda U_j – tugunning normallashtirilgan yuklanishi, F_j – uning ishonchlilik yoki nosozlik xavfi bahosi, $\alpha, \beta, \gamma, \delta$ esa mezonlarning muhimlik koeffitsiyentlaridir. Mazkur ifoda oddiy “eng bo‘sh tugun” strategiyasidan farqli ravishda ma’lumot ko‘chirish narxini ham inobatga oladi. Masalan, CPU yuklanishi biroz yuqori bo‘lgan, ammo kerakli ma’lumot lokal joylashgan tugun ba’zan mutlaqo bo‘sh, lekin katta hajmdagi ma’lumotni tarmoq orqali qabul qilishi kerak bo‘lgan tugundan samaraliroq bo‘lishi mumkin.

Adaptiv parallellashtirish usuli. Dastlab tizim monitoring moduli barcha tugunlardan resurs telemetriyasini yig’adi. Keyin kiruvchi vazifa uning hisoblash profiliga ko‘ra CPU-intensiv, memory-intensiv, GPU-intensiv yoki communication-intensiv sinflardan biriga ajratiladi. Har bir fragment uchun (5) mezon hisoblanib, (6) bo‘yicha eng maqbul tugun tanlanadi. Lokal ishlov natijasida $M_i = A(D_i, \theta_i)$ lokal model, gradient, statistik parametr yoki oraliq natija hosil qilinadi. Yakuniy global natija

$$M = \mathrm{Join}(M_1, M_2, \ldots, M_p) \tag{7}$$

operatori orqali olinadi. Join operatorining mazmuni algoritmga bog’liq: taqsimlangan ma’lumotlar bazasida u yozuvlarni birlashtirish, k-meansda lokal statistikalarni yig’ish, klassifikatsiyada ovoz berish, neyron tarmoqda esa gradientlarni agregatsiya qilish vazifasini bajaradi.

Adaptivlikning ikkinchi darajasi bajarilish jarayonida qo‘llanadi. Tizim har Δt vaqt oralig’ida tugunlar yuklamasini qayta baholaydi. Klasterdagi nomutanosiblik koeffitsiyenti

$$B = \frac{\max_j L_j}{\frac{1}{m}\sum_{j=1}^{m} L_j} \tag{8}$$

ko‘rinishida aniqlanadi. Agar $B > \tau$ bo‘lsa, ya’ni ayrim tugunlarda yuklama o‘rtacha qiymatdan sezilarli yuqori bo‘lsa, hali boshlanmagan yoki ko‘chirish xarajati kichik bo‘lgan vazifalar qayta rejalashtiriladi. Bunda ma’lumotni ko‘chirish hajmi katta bo‘lgan fragmentlarni migratsiya qilishdan oldin kutilayotgan foyda migratsiya xarajati bilan solishtiriladi. Shunday qilib, dinamik balanslash juda tez-tez bajarilib, tarmoqda ortiqcha trafik hosil qilishi oldi olinadi.

Taklif etilgan yondashuv mashinali o‘qitish vazifalari uchun ham muhim. Masalan, Random Forestda daraxtlarni mustaqil tugunlarda qurish task-parallel xususiyatga ega bo‘lsa, k-means va gradientga asoslangan algoritmlarda har iteratsiyada global agregatsiya talab etiladi. Birinchi holatda hisoblash vazifalarini bo‘sh tugunlarga tez qayta taqsimlash samarali bo‘lishi mumkin. Ikkinchi holatda esa ma’lumot lokalligi va tarmoq o‘tkazuvchanligi ustuvor ahamiyatga ega, chunki tez-tez sinxronlash kommunikatsiya xarajatini oshiradi. Demak, rejalashtirish mezonidagi vaznlar algoritm turiga qarab o‘zgartirilishi kerak.

Global optimallashtirish nuqtai nazaridan tizimning maqsadi faqat alohida vazifa vaqtini emas, butun konveyerning tugash vaqtini, tarmoq xarajatini va xizmat sifati buzilishini minimallashtirishdan iborat. Shu sababli umumiy mezonni

$$J = w_1 T_{\max} + w_2 C_{net} + w_3 V_{SLA} + w_4 C_{mig} \to \min \tag{9}$$

ko‘rinishida berish mumkin. Bu yerda $T_{\max}$ – barcha vazifalar tugash vaqti, C_{net} – tugunlararo almashinuv xarajati, V_{SLA} – xizmat darajasi talablarining buzilishi, C_{mig} – qayta taqsimlash va migratsiya xarajatidir. Resurs cheklovlari har bir tugunda ajratilgan CPU, RAM va GPU miqdori mavjud resursdan oshmasligini ta’minlaydi.

Mazkur modelni amaliy bulut platformasida Spark yoki Ray bajaruvchi muhiti va Kubernetes monitoring mexanizmi bilan integratsiya qilish mumkin. Spark ma’lumotni partitionlar bo‘yicha qayta ishlashga, Ray esa dinamik task va actorlarni resurs talabiga qarab joylashtirishga qulay. Kubernetes orqali CPU, RAM va pod holati kuzatilganda, rejalashtirish moduli ushbu telemetriyadan (5) mezon koeffitsiyentlarini yangilash uchun foydalanishi mumkin. MLOps yondashuvi esa eksperiment konfiguratsiyasi, model versiyasi va resurs profiling natijalarini qayta tiklash imkonini beradi [3].

Usulni baholashda statik round-robin, eng kam yuklangan tugun va faqat ma’lumot lokalligiga asoslangan strategiyalar bazaviy variant sifatida olinishi maqsadga muvofiq. Asosiy metrikalar bajarilish vaqti T, tezlashuv $S_p = T_1/T_p$, parallel samaradorlik $E_p = S_p/p$, tarmoq orqali uzatilgan ma’lumot hajmi va SLA buzilishlari sonidan iborat bo‘ladi. Turli o‘lchamdagi dataset va 1, 2, 4, 8, 16 tugunli konfiguratsiyalarda tajriba o‘tkazish algoritmning strong scaling xususiyatini aniqlash imkonini beradi. Kutilayotgan natija shundaki, ma’lumot lokalligi va joriy yuklamani birgalikda hisobga olish communication-intensiv vazifalarda tarmoq xarajatini kamaytiradi, task-parallel vazifalarda esa bo‘sh resurslardan tez foydalanishni ta’minlaydi.

Xulosa qilib aytganda, bulutli muhitda intellektual tahlil vazifalarini samarali parallellashtirish uchun faqat ma’lumotni teng fragmentlash yetarli emas. Vazifa turi, fragmentning fizik joylashuvi, tugun yuklamasi, tarmoq holati va nosozlik xavfi birgalikda baholanishi lozim. Taklif etilgan ma’lumot lokalligiga asoslangan adaptiv model “split-process-join” tamoyilini dinamik resurs monitoringi bilan birlashtiradi. Natijada vazifalarni hisoblashga eng mos tugunga joylashtirish, ortiqcha ma’lumot ko‘chirishni kamaytirish va klaster yuklamasi o‘zgarganda rejalashtirishni qayta moslashtirish imkoniyati yaratiladi. Ushbu yondashuv bulutli mashinali o‘qitish, taqsimlangan ma’lumotlar bazasi va katta ma’lumotlar analitikasi uchun yangi parallel rejalashtirish algoritmlarini ishlab chiqishda metodologik asos bo‘lib xizmat qilishi mumkin.

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MASHINALI O‘QITISH VA MA’LUMOTLARNI QAYTA ISHLASH ALGORITMLARINI BULUTLI MUHITDA REAL KATTA HAJMDAGI MA’LUMOTLARDA TADQIQ QILISH

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Katta hajmdagi ma'lumotlarning tez sur'atlarda shakllanishi mashinali o'qitish algoritmlarini faqat model aniqligi bo'yicha emas, balki hisoblash samaradorligi, masshtablanuvchanlik va resurs sarfi nuqtai nazaridan ham tadqiq qilishni talab etadi. Ma'lumot hajmi o'nlab gigabaytdan terabayt darajasiga oshganda bitta tugunning operativ xotirasi, protsessor quvvati va kiritish-chiqarish imkoniyatlari cheklovga aylanadi. Bulutli muhit hisoblash tugunlari sonini oshirish, ma'lumotlarni partitionlarga ajratish, CPU va GPU resurslaridan birgalikda foydalanish hamda taqsimlangan saqlash tizimlarini qo'llash imkonini beradi. Biroq tugunlar sonini ko'paytirish doimo proporsional tezlashishni bermaydi: shuffle, sinxronlash, model parametrlarini uzatish, tarmoq kechikishi va executorlarni ishga tushirish xarajatlari umumiy vaqtning sezilarli qismini egallashi mumkin.

Parallel bajarilish samaradorligini baholash uchun tezlashish va parallel samaradorlik koeffitsiyentlari qo'llanadi:

$$S_p = \frac{T_1}{T_p}, \qquad E_p = \frac{S_p}{p}, \tag{1}$$

bu yerda T_1 — bitta hisoblash tugunidagi bajarilish vaqti, T_p — p ta parallel hisoblash elementi ishlatilgandagi vaqt. Ushbu ko'rsatkichlar klaster o'lchami oshganda algoritmning amaldagi masshtablanuvchanligini tavsiflaydi.

Real ma'lumotlarda yetishmayotgan qiymatlar, kategoriyali belgilar, takroriy yozuvlar, ekstremal qiymatlar va turli o'lchov diapazonlari mavjud bo'ladi. Shu sababli mashinali o'qitish pipeline'i ma'lumotlarni o'qish, tozalash, filtrlash, transformatsiya, feature engineering, modelni o'qitish va baholash bosqichlarini qamrab olishi kerak. Ma'lumotlar p ta partitionga ajratilganda ularning har biri alohida workerda qayta ishlanishi mumkin. Masalan, min-max normallashtirish

$$x' = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \tag{2}$$

formula bilan bajariladi. Spark DataFrame va MLlib bunday transformatsiyalarni taqsimlangan muhitda bajarish, klassifikatsiya, regressiya va clustering pipeline'larini tashkil etish imkonini beradi [1].

Tadqiqot uchun parallelizm xususiyati turlicha bo'lgan algoritmlarni tanlash maqsadga muvofiq. Logistic Regression gradientga asoslangan iterativ klassifikatsiya algoritmi bo'lib, tugunlararo kommunikatsiya va gradientlarni birlashtirish xarajatlarini baholash uchun qulay. Uning ikkilik klassifikatsiyadagi ehtimollik modeli

$$P(y = 1 \mid x) = \frac{1}{1 + e^{-(w^T x + b)}} \tag{3}$$

ko'rinishida ifodalanadi. Dataset partitionlarga bo'linganda lokal gradientlar workerlarda parallel hisoblanadi va global reduction orqali birlashtiriladi.

Random Forest task-parallel xususiyatga ega: ansambldagi daraxtlarning katta qismi mustaqil qurilishi sababli ularni executorlar o'rtasida taqsimlash yuqori parallelizm beradi. XGBoost strukturaviy ma'lumotlarda samarali boosting algoritmi bo'lib, histogram qurish, feature scanning va partitionlar ustidagi hisoblashlarni parallel bajaradi. Uning masshtablanuvchi arxitekturasi katta datasetlarda CPU va GPU klasterlarini qiyoslash uchun mos [2]. K-means esa iterativ data-parallel algoritm sifatida tanlanadi. Uning maqsad funksiyasi

$$J = \sum_{i=1}^{n} \|x_i - \mu_{c_i}\|^2 \tag{4}$$

bo‘lib, har bir worker o‘z partitionidagi obyektlarni eng yaqin centroidga biriktiradi, iteratsiya yakunida lokal statistikalar global ravishda birlashtiriladi. PCA katta o‘lchamli ma’lumotlarda matritsali parallel hisoblashni, chuqur neyron tarmoqlar esa GPU-intensiv distributed trainingni ifodalaydi. Demak, tanlangan algoritmlar gradient, task, iterativ data va GPU parallelizmining asosiy shakllarini qamrab oladi.

Eksperimentning amaliy ahamiyatini oshirish uchun sun’iy kichik datasetlar o‘rniga real katta hajmdagi ma’lumotlardan foydalanish zarur. Asosiy test manbasi sifatida Criteo 1TB Click Logs datasetidan foydalanish mumkin. Mazkur dataset 1 TB hajmdagi click-through-rate hodisalarini o‘z ichiga oladi va katta hajmdagi klassifikatsiya hamda taqsimlangan o‘qitish algoritmlarini benchmark qilishga mo‘ljallangan [3]. U Logistic Regression, XGBoost va chuqur neyron tarmoqlarni sinash uchun ayniqsa mos.

Testlash bosqichma-bosqich ma’lumot hajmini oshirish asosida tashkil etiladi: 10 GB, 50 GB, 100 GB, 500 GB va 1 TB. Har bir hajmda 1, 2, 4, 8 va 16 workerli konfiguratsiyalar sinovdan o‘tkaziladi. Bunda ma’lumot hajmi o‘zgarmas holda workerlar sonini oshirish strong scalingni, har bir workerga to‘g’ri keladigan ma’lumot ulushini deyarli o‘zgarmas saqlash esa weak scalingni baholash imkonini beradi. Har bir tajriba kamida uch marta takrorlanib, median bajarilish vaqti hamda dispersiya qayd etiladi.

1-jadval. Algoritmlarni katta ma’lumotlarda testlash mezonlari

Algoritm	**Parallelizm turi**	**ML sifati**	**Tizim ko‘rsatkichi**
Logistic Regression	gradient/data	F1, ROC-AUC	vaqt, iteratsiya, tarmoq
Random Forest	task/data	F1, ROC-AUC	speedup, CPU, RAM
XGBoost	histogram/distributed	F1, ROC-AUC	CPU/GPU, training time
K-means	iterativ data	Silhouette	shuffle, iteratsiya vaqti
PCA	matritsali	explained variance	vaqt, xotira, data transfer
DNN	data/model	loss, AUC/F1	GPU, sinxronlash

Eksperimental muhit Apache Spark asosidagi klaster ko‘rinishida tashkil etiladi. Workerlar soni, CPU va RAM limitlari bir xil konfiguratsiyada nazorat qilinishi, CPU workloadlar uchun Spark MLlib, tree boosting uchun distributed XGBoost, GPU workloadlar uchun esa mos distributed training vositalaridan foydalanilishi mumkin. Barcha sinovlarda bir xil train/test bo‘linishi, tasodifiy urug’, caching, checkpoint va shuffle sozlamalari saqlanishi eksperimentning takrorlanuvchanligini oshiradi.

Tizim samaradorligini faqat model sifati bilan baholash yetarli emas. Classification uchun Accuracy, Precision, Recall, F1-score va ROC-AUC; clustering uchun Silhouette koeffitsiyenti bilan bir qatorda bajarilish vaqti, CPU/GPU yuklanishi, xotira, disk I/O, network shuffle, p95 va p99 latency ham o‘lchanadi. Ma’lumotlarni qayta ishlash throughput’i

$$Throughput = \frac{N}{T} \tag{5}$$

formula bilan aniqlanadi, bu yerda N — qayta ishlangan yozuvlar soni, T — umumiy bajarilish vaqti. Bulut resurslarining umumiy qiymati

$$C_{total} = \sum_{r=1}^{m} c_r t_r \tag{6}$$

ko‘rinishida ifodalanadi, bunda c_r — r turdagi resursning vaqt birligidagi narxi, t_r — undan foydalanish davomiyligi.

Optimal konfiguratsiyani tanlashda bajarilish vaqti, kommunikatsiya, iqtisodiy xarajat va model sifati birgalikda hisobga olinishi lozim. Buning uchun quyidagi umumlashgan mezondan

foydalanish mumkin:

$$J = w_1 T_{exec} + w_2 C_{total} + w_3 T_{comm} + w_4 V_{SLA} - w_5 Q_{model}. \quad (7)$$

Bu yerda T_{exec} — bajarilish vaqti, T_{comm} — kommunikatsiya xarajati, V_{SLA} — xizmat darajasi talablarining buzilishi, Q_{model} — normallashtirilgan model sifati. w_i vaznlar tadqiqot maqsadiga qarab belgilanadi. Ushbu mezon eng tez konfiguratsiyani emas, balki sifat, vaqt va resurs sarfi o'rtasida maqbul muvozanatni ta'minlovchi variantni aniqlashga xizmat qiladi.

Taklif etilgan metodika algoritmlarning bir xil bulut konfiguratsiyasida turlicha masshtablanishini ko'rsatadi. Random Forest mustaqil daraxtlarga asoslangani sababli task-parallel tezlashishga yaxshi moslashadi. K-meansda har iteratsiyada centroidlarni global yangilash tarmoq va sinxronlash xarajatini oshiradi. Logistic Regression va DNNda gradientlarni birlashtirish katta klasterlarda asosiy bottleneckka aylanishi mumkin. XGBoostda esa ma'lumot hajmi, histogram qurish va GPU xotirasidan foydalanish samaradorligi muhim omil bo'ladi. Shuning uchun barcha ML workloadlari uchun yagona optimal resurs konfiguratsiyasi mavjud emas.

Real profiling natijalari algoritmning hisoblash profilini va ma'lumot hajmini hisobga oluvchi adaptiv schedulerni yaratish uchun empirik baza bo'lib xizmat qilishi mumkin. Yangi vazifa CPU-intensiv, memory-intensiv, communication-intensiv yoki GPU-intensiv sifatida tasniflanib, unga mos workerlar soni, hisoblash tuguni va framework tanlanadi. Natijada katta real ma'lumotlarda model sifati, speedup, parallel samaradorlik, throughput, latency, CPU/GPU yuklanishi, kommunikatsiya va resurs qiymatini kompleks baholash bulutli muhit uchun samarali parallel rejalashtirish usullarini ishlab chiqishga ilmiy asos yaratadi.

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SUGENO MODELLARI ASOSIDA QORAMOLLARNING YUQUMLI BO'LMAGAN KASALLIKLARINI INTELLEKTUAL TASHXISLASH

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Annotatsiya. Qoramollarning yuqumli bo'lmagan kasalliklarini erta va ishonchli tashxislash chorvachilik mahsuldorligini saqlash, davolash xarajatlarini kamaytirish hamda hayvonlarning farovonligini ta'minlash uchun muhimdir. Veterinariya amaliyotida klinik belgilar, laboratoriya natijalari va ekspert kuzatuvlari ko'pincha noravshan, to'liqsiz yoki o'zaro ziddiyatli ko'rinishda bo'ladi. Shu sababli faqat aniq va to'liq ma'lumotlarga tayanadigan klassik algoritmlar barcha holatlarda yetarli barqarorlikni ta'minlay olmaydi. Ushbu tadqiqotda intervalli neytrosofik iyerarxik Sugeno modeli, ya'ni INISM taklif etiladi. Model 85 ta tashxislash belgisini oltita biologik va funksional guruhga ajratadi, lokal Sugeno modellarida qayta ishlaydi va T-I-F intervallari asosidagi agregatsiya orqali 116 turdagi yuqumli bo'lmagan kasallik bo'yicha yakuniy

xulosa shakllantiradi. Tajriba natijalarida INISM 94,6% aniqlik, 93,8% sezgirlik, 95,2% o‘ziga xoslik va 0,98 ROC-AUC ko‘rsatkichlarini qayd etgan. Natijalar iyerarxik tuzilma va intervalli neytrosofik ifodaning ko‘p parametrli veterinariya tashxislash masalalaridagi samaradorligini ko‘rsatadi.

Kalit so‘zlar: intervalli neytrosofik to‘plamlar, iyerarxik Sugeno modeli, intellektual tashxislash, veterinariya ekspert tizimi, qoramollar, yuqumli bo‘lmagan kasalliklar, noravshan mantiq, sun’iy intellekt.

Chorvachilikda qoramollarning sog’lomligi va mahsuldorligi oziq-ovqat xavfsizligi hamda xo‘jaliklarning iqtisodiy barqarorligi bilan bevosita bog’liq. Yuqumli bo‘lmagan kasalliklar sut va go‘sht mahsuldorligining pasayishi, reproduktiv ko‘rsatkichlarning yomonlashuvi, davolash xarajatlarining ortishi va hayvonlarni majburiy brak qilish kabi oqibatlarni keltirib chiqaradi. Bunday kasalliklar ko‘pincha bir nechta klinik, gematologik, biokimyoviy va fiziologik belgilar birikmasi orqali namoyon bo‘ladi. Shu bois kasallikni dastlabki bosqichda aniqlash uchun alohida bitta ko‘rsatkichga emas, balki turli manbalardan keladigan ma’lumotlarni kompleks tahlil qiluvchi modelga ehtiyoj mavjud [1,2].

Takagi-Sugeno tipidagi model ekspert bilimlarini IF-THEN qoidalari orqali ifodalashi, tez hisoblanishi va natijani matematik ko‘rinishda berishi bilan qulaydir. Biroq kirish parametrlarining soni ortishi bilan har bir parametr uchun bir nechta noravshan to‘plam kombinatsiyasi hosil bo‘ladi va qoidalar bazasi eksponensial sur’atda kengayadi. 85 ta belgi mavjud bo‘lgan vaziyatda bunday yondashuvni yagona katta model sifatida qurish amaliy jihatdan samarasizdir [1,4].

Mazkur muammoni bartaraf etish uchun tadqiqotning maqsadi qoramollarning yuqumli bo‘lmagan kasalliklarini noaniq va to‘liqsiz ma’lumotlar sharoitida tashxislay oladigan intervalli neytrosofik iyerarxik Sugeno modelini ishlab chiqishdan iborat. Ilmiy yondashuvning yangiligi uchta yechimning birlashtirilishida namoyon bo‘ladi: tashxislash belgilarini biologik asosda guruhlash, har bir guruh uchun lokal Sugeno xulosalash modelini qurish va lokal natijalarni T-I-F intervallari orqali iyerarxik agregatsiyalash. Shu tariqa modelning aniqligi, izohlanishi va hisoblash samaradorligini bir vaqtda oshirish ko‘zda tutiladi.

Tadqiqotda 116 turdagi yuqumli bo‘lmagan kasallikni farqlash uchun 85 ta tashxislash belgisi tanlangan. Tajriba ma’lumotlari 15 ta veterinariya klinikasining arxiv materiallari hamda 50 nafar tajribali veterinar shifokorning ekspert baholariga tayangan holda shakllantirilgan. Belgilar biologik mazmuni, o‘lchash usuli va kasalliklar bilan funksional bog’liqligiga ko‘ra oltita guruhga ajratilgan: klinik belgilar 23 ta, gematologik ko‘rsatkichlar 14 ta, biokimyoviy parametrlar 18 ta, modda almashinuvi ko‘rsatkichlari 12 ta, fiziologik parametrlar 10 ta va qo‘shimcha belgilar 8 ta. Bunday taqsimot tashxislash jarayonining veterinariya mantiqiga mos kelishi bilan birga, hisoblash yuklamasini ham lokal modullar o‘rtasida taqsimlaydi.

INISM ikki o‘zaro bog’liq mexanizmga tayanadi. Birinchi mexanizm har bir tashxislash guruhi bo‘yicha lokal Sugeno xulosasini hisoblaydi. Lokal qoidalar odatda “agar klinik va laboratoriya belgilarining muayyan holati kuzatilsa, u holda guruh bo‘yicha xavf yoki sindrom bahosi shunday bo‘ladi” mazmunida tuziladi. Qoidaning faollashuv darajasi tegishlilik qiymatlarining minimumi yoki ko‘paytmasi asosida aniqlanadi, chiqish esa doimiy yoki kirish parametrlarining chiziqli funksiyasi ko‘rinishida beriladi. Shu sababli lokal model ekspert bilimi bilan matematik hisoblashni birlashtiradi.

Iyerarxik dekompozitsiyaning hisoblashdagi afzalligi shundaki, 85 ta belgini yagona qoidalar fazosida qayta ishlash o‘rniga, parametrlar kichikroq lokal modullarga taqsimlanadi. Tadqiqot konfiguratsiyasida oltita lokal model bo‘yicha jami 682 ta qoida shakllantirilib, ularning chiqishlari global modulga uzatiladi. Modullarning nisbatan mustaqil ishlashi hisoblashni parallel tashkil etish, yangi belgi yoki qoida qo‘shilganda tizimni to‘liq qayta qurmaslik va har bir

xulosaning manbasini izohlash imkonini beradi. Shu jihatdan INISM nafaqat aniqlikka, balki amaliy ekspluatatsiya qulayligiga ham yo‘naltirilgan.

Tashxislash belgilarining alohida guruhlari bo‘yicha o‘tkazilgan baholashda barcha guruhlar 85% dan yuqori aniqlikni ko‘rsatgan. Eng yuqori natijalar biokimyoviy parametrlar guruhida 92,3% va modda almashinuvi ko‘rsatkichlari guruhida 91,7% qayd etilgan. Gematologik ko‘rsatkichlar 89,4%, fiziologik parametrlar 89,8%, klinik belgilar 87,5% va qo‘shimcha belgilar 86,9% aniqlik bergan. Klinik va qo‘shimcha belgilar bo‘yicha nisbatan pastroq qiymat ekspert kuzatuvlarining subyektivligi, belgilarni turlicha talqin qilish va ayrim kasalliklar simptomlarining o‘xshashligi bilan izohlanadi. Guruhlar kesimida umumlashtirilgan ko‘rsatkich 89,6% ni tashkil etgan.

116 ta kasallik bo‘yicha uchta modelning bir xil qiyosiy baholashi INISM foydasiga barqaror ustunlikni ko‘rsatdi. Klassik Sugeno modeli bilan taqqoslaganda aniqlik 7,4 foiz punktga, sezgirlik 9,3 foiz punktga va o‘ziga xoslik 6,2 foiz punktga oshgan. Neytrosofik Sugeno modeliga nisbatan ham aniqlik 2,8 foiz punktga, sezgirlik 3,6 foiz punktga va o‘ziga xoslik 2,1 foiz punktga yuqori bo‘lgan. Mazkur farq iyerarxik dekompozitsiya qoidalar bazasini ixchamlashtirgani va interval T-I-F ifodasi to‘liqsiz hamda ziddiyatli dalillarni saqlab qolgani bilan bog’liq.

INISM modeli qiyosiy sinovda 94,6% aniqlik, 93,8% sezgirlik, 95,2% o‘ziga xoslik, 0,94 F1-ko‘rsatkich, 0,89 MCC va 0,98 ROC-AUC natijalarini qayd etdi. Jadvaldagi 89,6% qiymat guruhlar bo‘yicha lokal natijalarni umumlashtiruvchi ko‘rsatkich bo‘lsa, 94,6% qiymat 116 kasallik bo‘yicha uchta modelning global qiyosiy baholashidan olingan. Demak, ko‘rsatkichlar turli baholash kesimlarini ifodalaydi va ularni bir-biriga qarama-qarshi natija sifatida talqin qilish to‘g’ri emas. ROC-AUC ning 0,98 ga tengligi modelning kasallik mavjud va mavjud emas holatlarini ajratish qobiliyati yuqori ekanini ko‘rsatadi.

Tadqiqot natijasida qoramollarning yuqumli bo‘lmagan kasalliklarini ko‘p parametrli va noaniq ma’lumotlar sharoitida tashxislashga mo‘ljallangan INISM modeli shakllantirildi. Model 85 ta belgini oltita mantiqiy guruhga ajratib, lokal Sugeno xulosalash, intervalli neytrosofik T-I-F ifodalash va iyerarxik agregatsiya mexanizmlarini yagona arxitekturaga birlashtiradi. Ushbu yondashuv klassik Sugeno modelidagi qoidalar sonining keskin ortishini cheklaydi, ekspert ma’lumotlaridagi noaniqlik va qarama-qarshilikni saqlaydi hamda yakuniy qarorni izohlash imkoniyatini kengaytiradi.

1-jadval. INISM modelining qiyosiy samaradorlik ko‘rsatkichlari

Baholash mezoni	**SM**	**NSM**	**INISM**
Aniqlik (%)	87,2	91,8	94,6
Sezgirlik (%)	84,5	90,2	93,8
O‘ziga xoslik (%)	89,0	93,1	95,2
Musbat bashorat qiymati (%)	86,4	91,0	94,1
F1-ko‘rsatkich	0,87	0,91	0,94
MCC	0,75	0,83	0,89
ROC-AUC	0,92	0,96	0,98

Eksperimental natijalar INISM ning global qiyosiy sinovda 94,6% aniqlik va 0,98 ROC-AUC ko‘rsatkichlariga erishganini ko‘rsatdi. Kelgusida modelni mustaqil prospektiv ma’lumotlar bilan tekshirish, harorat, puls va faollik sensorlari asosida real vaqt monitoringiga ulash, shuningdek ot, qo‘y va parrandalar kasalliklarini tashxislashga moslashtirish maqsadga muvofiq. Bunday kengaytirishlar modelning ilmiy ishonchliligi va veterinariya amaliyotidagi qo‘llanish doirasini oshiradi.

Foydalanilgan adabiyotlar

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ARTIFICIAL INTELLIGENCE MODEL FOR STUDENT PERFORMANCE PREDICTION

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Artificial intelligence and machine learning methods are increasingly used in education to analyze student data and support educational decision-making. One important application is the prediction of students' academic performance. An accurate prediction model can help educational institutions identify students who may require additional academic support before their final examinations. Machine learning algorithms are particularly suitable for this task because they can identify complex relationships between academic, demographic and social characteristics of students.

In this paper, an artificial intelligence model based on the Random Forest algorithm is considered for predicting students' academic performance. Random Forest is an ensemble learning method that combines a collection of decision trees. Each tree is constructed from a randomly selected sample of the training data and a random subset of features. For a regression problem, the final prediction is obtained by averaging the predictions of all trees:

$$\widehat{y}(x) = \frac{1}{M} \sum_{m=1}^{M} T_m(x), \tag{1}$$

where M is the number of decision trees, $T_m(x)$ is the prediction of the m-th tree and $\widehat{y}(x)$ is the predicted academic performance.

The Student Performance dataset from the UCI Machine Learning Repository is used to demonstrate the proposed approach [1]. The data were collected from two Portuguese secondary schools and contain academic, demographic, social and school-related information. The Portuguese-language dataset contains 649 student records and about 30 input attributes. The final grade $G3$, measured on a scale from 0 to 20, is selected as the prediction target.

The input variables include age, study time, number of previous failures, school support, family support, internet access, extracurricular activities, absences and previous academic results. Categorical attributes are transformed into numerical variables before training. The dataset is randomly divided into training and testing subsets using an 80:20 ratio.

The proposed prediction procedure consists of four main stages: data preprocessing, feature preparation, model training and performance evaluation. A Random Forest regressor containing

200 decision trees is used. The maximum tree depth is controlled to reduce overfitting, while random feature selection introduces diversity among individual trees. The trained model receives the characteristics of a student as input and returns the predicted final academic grade.

The prediction quality is evaluated using the mean absolute error (MAE), root mean squared error (RMSE) and coefficient of determination R^2. These measures are defined by

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n} |y_i - \widehat{y}_i|, \tag{2}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}, \tag{3}$$

and

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \overline{y})^2}. \tag{4}$$

Here y_i denotes the actual grade, $\widehat{y}_i$ is the predicted grade and $\overline{y}$ is the mean value of the actual grades.

To investigate the importance of previous academic information, two prediction scenarios are considered. In the first scenario, the first and second period grades $G1$ and $G2$ are included among the input features. In the second scenario, these grades are excluded and the prediction is based only on demographic, social and other school-related information. This distinction is important because previous grades are strongly related to the final grade [1].

The numerical experiment shows that Random Forest provides substantially more accurate predictions when previous academic results are available. A representative summary of the model performance is presented in Table 1.

Model configuration	**MAE**	**RMSE**	R^2
RF with previous grades	0.85	1.35	0.84
RF without previous grades	2.15	3.30	0.28

Table 1. Performance of the student grade prediction model.

The results indicate that previous academic achievement is the most important source of information for predicting the final student performance. When $G1$ and $G2$ are available, the predicted grades are concentrated close to the actual values. Without these variables, prediction becomes more difficult; nevertheless, the model can still extract useful information from study habits, failures, absences and other student characteristics.

The relative importance of selected input variables obtained from the Random Forest model is illustrated in Fig. 1. The values are normalized so that their sum is equal to one.

As can be seen from Fig. 1, the second-period grade $G2$ has the largest influence on the prediction, followed by the first-period grade $G1$. The number of previous failures, study time and absences also contribute to the prediction. This information is useful because the Random Forest model not only predicts the final grade but also provides an indication of which factors have the strongest influence on the prediction.

The obtained results demonstrate the potential of artificial intelligence for educational data analysis. A model of this type may be used as an early-warning component in an educational information system. Students predicted to have low academic performance could be identified before the end of a course, allowing teachers and academic advisors to provide additional support. However, predictions should support rather than replace decisions made by teachers, and sensitive student information should be handled carefully.



Рис. 10: Relative importance of selected student performance features.

In conclusion, an artificial intelligence model based on Random Forest has been considered for predicting student academic performance. The model uses academic, demographic and social characteristics as input and predicts the final student grade. The experiment shows that the availability of previous academic grades considerably improves prediction accuracy. Random Forest is particularly suitable for this application because it can model nonlinear relationships, process different types of features and provide estimates of feature importance. Future work will consider larger university datasets and compare Random Forest with gradient boosting and deep neural network models.

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SUN'IY INTELLEKT ASOSIDA TURLI KOORDINATALAR SISTEMASIDA EGRI CHIZIQLAR EGRILIGINI TADQIQ ETISHNING ZAMONAVIY YONDASHUVLARI

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Differensial geometriyada egri chiziqlar va sirtlarning lokal xossalarini o‘rganishda egrilik tushunchasi markaziy o‘rinni egallaydi. Egri chiziq egriligi uning to‘g‘ri chiziqdan og‘ish darajasini miqdoriy jihatdan xarakterlaydi. Texnika, optika, yo‘l qurilishi hamda kompyuter grafikasi masalalarida murakkab trayektoriyalarning egriligini yuqori aniqlikda hisoblash talab etiladi.

An’anaviy usullarda murakkab tenglamalar bilan berilgan egri chiziqlarning birinchi va ikkinchi tartibli hosilalarini hisoblash va ularni egrilik formulasiga qo‘yib soddalashtirish yuqori diqqatni talab qiladi. Shu sababli, zamonaviy kompyuter algebrasi (Symbolic Computation) imkoniyatlaridan foydalanib, analitik hisoblash jarayonini algoritmlash dolzarb masala hisoblanadi.

Egri chiziqning berilish usuliga qarab, egrilik koeffitsientini hisoblash formulalari turlicha ko‘rinishga ega bo‘ladi.

Agar egri chiziq $y = f(x)$ oshkora funksiya ko‘rinishida berilgan bo‘lsa, uning egrilik koeffitsienti k quyidagi formula orqali topiladi:

$$k = \frac{|y''|}{(1 + (y')^2)^{3/2}} \tag{1}$$

bu yerda $y' = \frac{dy}{dx}$ va $y'' = \frac{d^2y}{dx^2}$ tegishli ravishda birinchi va ikkinchi tartibli hosilalardir.

Agar egri chiziq parametrik ko‘rinishda $x = x(t)$, $y = y(t)$ berilgan bo‘lsa, formula ushbu ko‘rinishga keladi:

$$k = \frac{|x'y'' - y'x''|}{((x')^2 + (y')^2)^{3/2}} \tag{2}$$

Qutb koordinatalar sistemasida egri chiziq $r = r(\varphi)$ tenglamasi bilan berilganda, $x = r(\varphi)\cos\varphi$ va $y = r(\varphi)\sin\varphi$ o‘tish formulalaridan foydalanib, egrilik uchun quyidagi nisbat hosil qilinadi:

$$k = \frac{|r^2 + 2(r')^2 - rr''|}{(r^2 + (r')^2)^{3/2}} \tag{3}$$

bu yerda $r' = \frac{dr}{d\varphi}$ va $r'' = \frac{d^2r}{d\varphi^2}$.

Uch o‘lchamli fazoda parametrik berilgan $\mathbf{r}(t) = (x(t), y(t), z(t))$ egri chiziq uchun egrilik vektorial ko‘paytma yordamida aniqlanadi:

$$k = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} \tag{4}$$

Sferik koordinatalar sistemasida (r, θ, φ) berilgan egri chiziqlarni hisoblashda Metrik tenzor va Kristoffel simvollari apparatidan foydalanish analitik hisoblarni yanada ixchamlashtiradi.

Simvolik hisoblash – bu matematik ifodalarni sonli (taqribiy) emas, balki ramziy (harfiy va analitik) ko‘rinishda xatosiz qayta ishlovchi kompyuter tizimlaridir. Maqolada Python dasturlash tilining *SymPy* kutubxonasi va *Wolfram Mathematica* muhitidan foydalanildi.

Algoritmning hisoblash bosqichlari:

1. Koordinata sistemasi va o‘zgaruvchilarni analitik ko‘rinishda kiritish.
2. Berilgan funksiyadan avtomatik ravishda y', y'' (yoki r', r'') hosilalarni ramziy ko‘rinishda olish.
3. Olingan hosilalarni mos egrilik formulasiga joylashtirish.
4. Natijaviy algebraik ifodani soddalashtirilgan eng qulay ko‘rinishga keltirish.

Amaliy hisoblash tajribalari va tahlil uchun quyidagi misollarni qaraymiz.

1-misol. Dekart koordinatasida $y = \sin(x)$ sinusoida egri chizig‘ining egriligini topaylik. Hosilalari: $y' = \cos(x)$, $y'' = -\sin(x)$. Formula (1) ga ko‘ra:

$$k(x) = \frac{|-\sin(x)|}{(1+\cos^2(x))^{3/2}} \tag{5}$$

Xususan, $x = \pi/2$ nuqtada egrilik eng yuqori qiymatga erishadi: $k(\pi/2) = 1$.

2-misol. Qutb koordinatasida $r = a(1+\cos\varphi)$ kardioidaning egriligini hisoblaymiz. Hosilalari: $r' = -a\sin\varphi$, $r'' = -a\cos\varphi$. Formula (3) ga qo‘yganda:

$$r^2 + 2(r')^2 - rr'' = a^2(1+\cos\varphi)^2 + 2a^2\sin^2\varphi + a^2(1+\cos\varphi)\cos\varphi = 3a^2(1+\cos\varphi)$$

$$(r^2 + (r')^2)^{3/2} = (a^2(1+\cos\varphi)^2 + a^2\sin^2\varphi)^{3/2} = (2a^2(1+\cos\varphi))^{3/2}$$

Simvolik soddalashtirish natijasida kardioida egriligi uchun juda ixcham formula hosil bo‘ladi:

$$k(\varphi) = \frac{3}{2\sqrt{2a\cdot r(\varphi)}} \tag{6}$$

Tadqiqot natijasida turli koordinatalar sistemasidagi egri chiziqlar egriligini hisoblashning umumiy analitik bazasi shakllantirildi. Simvolik hisoblash algoritmlarini qo‘llash murakkab matematik apparatga ega bo‘lgan differensial geometriya masalalarini yechish vaqtini bir necha barobarga qisqartirish va inson omili bilan bog‘liq xatoliklarni bartaraf etish imkonini berdi. O‘tkazilgan tahlillar shuni ko‘rsatdiki, simvolik hisoblash usullari qo‘lda hisoblashda yuzga keladigan algebraik xatoliklarning oldini oladi va hisoblash tezligini keskin oshiradi. Ishlab chiqilgan dasturiy algoritm va uslubiy tavsiyalar nafaqat nazariy taqqiqotlarda, balki robototexnika, zamonaviy CAD/CAM tizimlari va 3D modellashtirishda egri chiziqlar silliqligini tekshirishda muhim amaliy ahamiyat kasb etadi. Kelgusida ushbu tadqiqotni davom ettirgan holda, fazoviy sirtlarning asosiy egriliklarini Gauss egriligini hisoblash jarayonlarini ham kompyuter algebrasi tizimlari yordamida avtomatlashtirish mumkin. Shuningdek mazkur yondashuv universitetning “Differensial geometriya va topologiya” fanini o‘qitishda samarali amaliy uslubiyot sifatida qo‘llanilishi mumkin

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BO‘LAJAK O‘QITUVCHILARNING KOMPETENSIYALARINI BAHOLASHNING KO‘P MEZONLI VA RAQAMLI YONDASHUVLARI

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Kirish. Zamonaviy ta'limda kompetensiyani baholash bilim bilan birga ko‘nikma, malaka, mustaqil fikrlash, axborot bilan ishlash va kommunikativ faoliyatni ham qamrab oladi. Shu bois ko‘p mezonli va raqamli baholash yondashuvlari muhim ahamiyat kasb etadi.

Adabiyotlarning tahlili. Tadqiqotlarda kompetensiyani kompleks baholash, ya'ni bilim, ko‘nikma va amaliy faoliyatni birgalikda aniqlash zarurligi ta‘kidlangan [1; 2]. Ko‘p mezonli baholash kompetensiyaning turli tarkibiy qismlarini aniqlash imkonini beradi [3]. Raqamli texnologiyalar esa baholashning tezkorligi, shaffofligi va individualligini oshiradi [4; 5].

Asosiy qism. Mavjud adabiyotlar tahlili elektron ta‘lim muhitida bo‘lajak o‘qituvchilar kompetensiyasini fuzzy mantiq asosida ko‘p mezonli va kompleks baholash metodikasi yetarli darajada ishlab chiqilmaganini ko‘rsatadi. Shu sababli lingvistik o‘zgaruvchilarni raqamli baholash mezonlari bilan integratsiyalash va individual rivojlanish trayektoriyasini aniqlash dolzarb masala hisoblanadi. Taklif etilayotgan model bo‘lajak o‘qituvchining kompetensiyasini bilim, amaliy ko‘nikma, raqamli savodxonlik, kommunikativ kompetensiya va mustaqil ta‘lim faolligi kabi besh asosiy indikator asosida baholashga yo‘naltirilgan. Kompetensiya vektori quyidagicha ifodalanadi:

$$X = (x_1, x_2, x_3, x_4, x_5)$$

bu yerda x_1 – bilim darajasi, x_2 – amaliy ko‘nikma, x_3 – raqamli savodxonlik, x_4 – kommunikativ kompetensiya, x_5 – mustaqil ta'lim faolligi.

Har bir indikator 0–1 oralig'ida baholanib, fuzzy to‘plamlar yordamida lingvistik qiymatlarga o‘tkaziladi:

$$\mu_A(x) \in [0, 1]$$

bu yerda $\mu_A(x)$ – indikatorning tegishli fuzzy to‘plamga tegishlilik darajasi. Bunda indikatorlar “yuqori”, “o‘rta” va “past” kabi lingvistik qiymatlar orqali ifodalanadi.

Kompetensiyaning integral ko‘rsatkichi indikatorlarning vazn koeffitsientlari asosida quyidagicha aniqlanishi mumkin:

$$C = \sum_{i=1}^{5} w_i x_i, \quad \sum_{i=1}^{5} w_i = 1$$

bu yerda C – umumiy kompetensiya ko‘rsatkichi, w_i – indikatorning ahamiyat koeffitsienti.

Fuzzy inference tizimi Mamdani modeli asosida amalga oshiriladi. Bunda ekspert qoidalari yordamida kirish indikatorlari va integral kompetensiya o‘rtasidagi bog'liqlik aniqlanadi. Masalan:

$$R_i : \text{IF } x_1 \text{ Yuqori } AND \ x_2 \text{ Yuqori } THEN \ C \text{ Yuqori}$$

Qoidalar natijalari agregatsiya qilinib, umumiy fuzzy chiqish funksiyasi quyidagicha aniqlanadi:

$$\mu_C(z) = \max_i [\alpha_i \wedge \mu_{B_i}(z)]$$

bu yerda α_i – i-qoidaning faollik darajasi, $\mu_{B_i}(z)$ – chiqish fuzzy to‘plamining tegishlilik funksiyasi.

Yakuniy integral kompetensiya qiymati defuzzifikatsiya orqali aniq ko‘rsatkichga aylantiriladi:

$$C^* = \frac{\int z\, \mu_C(z)\, dz}{\int \mu_C(z)\, dz}$$

Natijada C^* qiymati asosida kompetensiya darajasi “yuqori”, “o‘rta” va “past” va “juda yuqori” kategoriyalariga ajratiladi.

Model natijalari asosida talabaning zaif kompetensiyalari aniqlanib, individual rivojlanish yo‘nalishi va adaptiv o‘quv tavsiyalari shakllantiriladi. Shu tariqa fuzzy mantiqqa asoslangan model noaniq va ko‘p mezonli ma‘lumotlarni kompleks qayta ishlab, kompetensiyani obyektiv baholash, monitoring qilish va individual rivojlanishni qo‘llab-quvvatlash imkonini beradi.

Tadqiqot metodologiyasi. Tadqiqotda tahlil, modellashtirish, ekspert baholash va matematik-statistik usullar asosida bo‘lajak o‘qituvchilar kompetensiyasi baholandi. Bunda bilim, amaliy ko‘nikma, raqamli savodxonlik, kommunikativ faoliyat va mustaqil ta‘lim mezonlari hamda Mamdani fuzzy modeli qo‘llanildi.

Natija. Ko‘p mezonli va fuzzy yondashuv asosida integral kompetensiya darajasini aniqlash, rivojlanish dinamikasini monitoring qilish va individual tavsiyalar shakllantirish imkoniyati yaratildi.

Xulosa. Fuzzy mantiqqa asoslangan model kompetensiyani obyektiv, kompleks va adaptiv baholash hamda individual rivojlanishni qo‘llab-quvvatlash imkonini beradi.

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GENERATIV SUN’IY INTELLEKT AGENTLARINING KICHIK VA O‘RTA BIZNES OPERATSION SAMARADORLIGIGA TA’SIRI

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Kichik va o‘rta biznes (KOB) subyektlarida operatsion jarayonlarning samaradorligi faqat ishchi kuchi sonini ko‘paytirish bilan belgilanmaydi. Vazifaning murakkabligi, generativ modelning kontekst uzunligi, tashqi tizimlar (CRM, buxgalteriya, ombor) bilan integratsiya darajasi va inson nazorati zarurati umumiy bajarilish vaqtiga bevosita ta’sir qiladi. Mijozlar so‘rovlari, hujjat aylanishi, moliyaviy hisobotlar va marketing kontenti kabi vazifalarda generativ agentga topshirish ko‘pincha inson resursidan qimmatroq yoki, aksincha, sezilarli arzonroq bo‘lishi mumkin. Shu sababli har bir vazifani avtomatik ravishda agentga yuborish emas, balki vazifa murakkabligini, xato xavfini va agent ishonchliligini birgalikda hisobga oluvchi adaptiv taqsimlash mexanizmini ishlab chiqish dolzarb masala hisoblanadi.

KOB operatsion tizimlarida generativ agentlardan foydalanishning umumiy tamoyili vazifani bo‘lish, agent yoki inson tomonidan qayta ishlash va natijalarni birlashtirish ketma-ketligiga asoslanadi. Mazkur yondashuv mijozlarga xizmat ko‘rsatish botlarida, hujjat generatsiyasi tizimlarida va biznes-intellekt konveyerlarida turli ko‘rinishlarda uchraydi [1–3]. Biroq statik avtomatlashtirishda agent unumdorligi va xato ehtimoli o‘zgarmas deb qaraladi. Real KOB muhitida esa so‘rovlar murakkabligi, mijozlar tili, ma’lumotlar sifati va agent yuklamasi vaqt

bo‘yicha o‘zgaradi. Taklif qilinayotgan yondashuvning maqsadi ushbu o‘zgarishlarni monitoring qilish va vazifa-ijrochi (agent yoki inson) juftligini mos ijrochiga dinamik biriktirishdan iborat.

Masalaning qo‘yilishi. Korxonaning N ta operatsion vazifadan iborat oqimi quyidagicha ifodalansin:

$$T = \bigcup_{i=1}^{N} T_i, \qquad T_i \cap T_k = \varnothing,\ i \neq k. \tag{1}$$

Har bir T_i vazifa mijozga javob berish, hujjat tayyorlash, ma’lumotni tasniflash yoki hisobot shakllantirish operatsiyalaridan biri bo‘lishi mumkin. Ijrochilar to‘plami $E = \{E_1, E_2, \ldots, E_m\}$ generativ agentlar va inson xodimlaridan tashkil topadi. Har bir E_j ijrochining joriy holati hisoblash quvvati, javob vaqti, xato darajasi va bandlik darajasi orqali

$$e_j = (p_j, \tau_j, \varepsilon_j, u_j) \tag{2}$$

vektor bilan tavsiflanadi. Bu yerda p_j – ijrochining bir birlik vaqtda bajara oladigan vazifa hajmi, τ_j – o‘rtacha javob vaqti, ε_j – xato yoki gallyutsinatsiya ehtimoli, u_j esa ijrochining joriy bandlik darajasidir.

Vazifa T_i ni E_j ijrochida bajarish xarajatini soddalashtirilgan holda hisoblash vaqti, tekshirish (inson nazorati) vaqti va navbatda kutish yig’indisi sifatida baholash mumkin:

$$X_{ij} = \frac{W_i}{p_j} + \kappa\, \varepsilon_{ij} R_i + u_j. \tag{3}$$

Bu yerda W_i – vazifaning hisoblash (kontekst) hajmi, R_i – xato holida qayta ishlash yoki tuzatish xarajati, κ esa xatoni bartaraf etish og’irligi koeffitsiyentidir. Agar vazifa avtomatlashtirishga yaxshi moslashgan bo‘lsa, tuzatish xarajati kamayadi. Avtomatlashtirish mosligi koeffitsiyenti

$$\mu_{ij} = \frac{A_{ij}}{W_i}, \qquad 0 \le \mu_{ij} \le 1 \tag{4}$$

ko‘rinishida aniqlanadi, bu yerda A_{ij} – E_j ijrochi tomonidan inson aralashuvisiz to‘liq bajarilishi mumkin bo‘lgan vazifa qismi hajmi. μ_{ij} qiymati birga yaqin bo‘lsa, vazifa to‘liq avtomatlashtirishga mos keladi.

Taklif qilinayotgan adaptiv tanlash mezonida bajarilish xarajati bilan birga avtomatlashtirish mosligi, ijrochi bandligi va xato xavfi hisobga olinadi:

$$C_{ij} = \alpha X_{ij} + \beta(1 - \mu_{ij}) + \gamma U_j + \delta \varepsilon_{ij}, \tag{5}$$

$$j^* = \arg\min_{1 \le j \le m} C_{ij}. \tag{6}$$

Bu yerda U_j – ijrochining normallashtirilgan bandlik darajasi, ε_{ij} esa vazifa-ijrochi jufti uchun kutilayotgan xato ehtimoli, $\alpha, \beta, \gamma, \delta$ mezonlarning muhimlik koeffitsiyentlaridir. Mazkur ifoda oddiy “eng bo‘sh ijrochi” strategiyasidan farqli ravishda xatoni tuzatish narxini ham inobatga oladi. Masalan, javob vaqti biroz yuqori, ammo xato ehtimoli past bo‘lgan agent, tez, lekin ko‘p tuzatishni talab qiluvchi agentdan samaraliroq bo‘lishi mumkin.

Adaptiv taqsimlash usuli. Dastlab tizim monitoring moduli barcha ijrochilardan (agent va xodimlar) ish holati telemetriyasini yig’adi. Keyin kiruvchi vazifa uning mazmuniga ko‘ra kommunikatsiya-intensiv (mijozlar bilan muloqot), hujjat-intensiv, tahlil-intensiv yoki qaror-intensiv sinflardan biriga ajratiladi. Har bir vazifa uchun (5) mezon hisoblanib, (6) bo‘yicha eng maqbul ijrochi tanlanadi. Lokal ishlov natijasida $M_i = A(T_i, \theta_i)$ – javob matni, hujjat loyihasi, tasnif natijasi yoki oraliq hisobot hosil qilinadi. Yakuniy operatsion natija

$$M = \mathrm{Join}(M_1, M_2, \ldots, M_n) \tag{7}$$

operatori orqali olinadi. Join operatorining mazmuni jarayon turiga bog'liq: mijozlarga xizmat ko‘rsatishda u navbatdagi javoblarni birlashtirish, hisobotlarda bo‘limlarni yig'ish, qaror qabul qilishda esa agent tavsiyasi va inson tasdig'ini muvofiqlashtirish vazifasini bajaradi.

Adaptivlikning ikkinchi darajasi bajarilish jarayonida qo‘llanadi. Tizim har Δt vaqt oralig'ida ijrochilar bandligini qayta baholaydi. Jamoadagi yuklama nomutanosibligi koeffitsiyenti

$$B = \frac{\max_j L_j}{\frac{1}{m}\sum_{j=1}^{m} L_j} \tag{8}$$

ko‘rinishida aniqlanadi. Agar $B > \tau$ bo‘lsa, ya'ni ayrim ijrochilarda yuklama o‘rtacha qiymatdan sezilarli yuqori bo‘lsa, hali boshlanmagan yoki past murakkablikdagi vazifalar qayta taqsimlanadi. Bunda vazifani boshqa ijrochiga topshirish orqali kutilayotgan foyda kontekstni qayta yuklash xarajati bilan solishtiriladi. Shunday qilib, dinamik balanslash juda tez-tez bajarilib, tizimda ortiqcha almashinuv xarajati hosil qilishi oldi olinadi.

Taklif etilgan yondashuv turli funksional yo‘nalishlar uchun muhim. Masalan, mijozlarga birlamchi konsultatsiya berishda vazifalarni mustaqil agentlarga tez taqsimlash task-parallel xususiyatga ega bo‘lsa, moliyaviy hisobot yoki shartnoma tahlili kabi qaror-intensiv vazifalarda har bosqichda inson tasdig'i (human-in-the-loop) talab etiladi. Birinchi holatda vazifalarni bo‘sh agentlarga tez qayta yo‘naltirish samarali bo‘lishi mumkin. Ikkinchi holatda esa xato xavfi va nazorat ustuvor ahamiyatga ega, chunki tez-tez tasdiqlash muloqot xarajatini oshiradi. Demak, tanlash mezonidagi vaznlar vazifa turiga qarab o‘zgartirilishi kerak.

Global optimallashtirish nuqtai nazaridan tizimning maqsadi faqat alohida vazifa xarajatini emas, butun operatsion oqimning tugash vaqtini, xizmat sifatini va tuzatish xarajatini minimallashtirishdan iborat. Shu sababli umumiy mezonni

$$J = w_1 T_{\max} + w_2 C_{err} + w_3 Q_{sla} + w_4 C_{switch} \to \min \tag{9}$$

ko‘rinishida berish mumkin. Bu yerda $T_{\max}$ – barcha vazifalar tugash vaqti, C_{err} – xatolarni tuzatish xarajati, Q_{sla} – xizmat darajasi (SLA) talablarining buzilishi, C_{switch} – vazifani qayta taqsimlash xarajatidir. Resurs cheklovlari har bir agent uchun API so‘rovlar limiti va inson xodimi uchun ish vaqti me'yoridan oshmasligini ta'minlaydi.

Mazkur modelni amaliy KOB axborot tizimida CRM platformasi va bulutli LLM API xizmati bilan integratsiya qilish mumkin. CRM mijozlar so‘rovlarini navbatga qo‘yishga, LLM API esa kontekstga mos javobni dinamik generatsiya qilishga qulay. Ish oqimini boshqarish tizimi (workflow orchestration) orqali agent bandligi va xato darajasi kuzatilganda, rejalashtirish moduli ushbu telemetriyadan (5) mezon koeffitsiyentlarini yangilash uchun foydalanishi mumkin. MLOps yondashuvi esa agent versiyasi, prompt konfiguratsiyasi va samaradorlik natijalarini qayta tiklash imkonini beradi [3].

Usulni baholashda barcha vazifalarni faqat inson bajaradigan, barcha vazifalarni faqat agent bajaradigan va tasodifiy taqsimlash strategiyalari bazaviy variant sifatida olinishi maqsadga muvofiq. Asosiy metrikalar bajarilish vaqti T, xarajat tejami, mijozlar qoniqish darajasi (CSAT) va xato/qayta ishlash foizidan iborat bo‘ladi. Turli hajmdagi so‘rovlar oqimi va 1, 5, 10, 20 agentli konfiguratsiyalarda tajriba o‘tkazish yondashuvning masshtablanish xususiyatini aniqlash imkonini beradi. Kutilayotgan natija shundaki, avtomatlashtirish mosligi va joriy bandlikni birgalikda hisobga olish kommunikatsiya-intensiv vazifalarda javob vaqtini qisqartiradi, qaror-intensiv vazifalarda esa xato xavfini kamaytirib, xizmat sifatini oshiradi.

Xulosa qilib aytganda, kichik va o‘rta biznesda generativ sun'iy intellekt agentlaridan samarali foydalanish uchun faqat vazifalarni avtomatlashtirish yetarli emas. Vazifa turi, uning murakkabligi, agent ishonchliligi, joriy bandlik va xato xavfi birgalikda baholanishi lozim.

Taklif etilgan avtomatlashtirish mosligiga asoslangan adaptiv model “bo‘lish–ishlov–birlashtirish” tamoyilini dinamik ijrochi monitoringi bilan birlashtiradi. Natijada vazifalarni eng mos ijrochiga (agent yoki inson) yo‘naltirish, ortiqcha tuzatish xarajatini kamaytirish va jamoa bandligi o‘zgarganda rejalashtirishni qayta moslashtirish imkoniyati yaratiladi. Ushbu yondashuv KOB uchun generativ AI agentlarini joriy etishning yangi adaptiv boshqaruv algoritmlarini ishlab chiqishda metodologik asos bo‘lib xizmat qilishi mumkin.

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INSULT XAVFINI BASHORAT QILISHDA KLINIK VA BIOMETRIK KO‘RSATKICHLAR ASOSIDA MATEMATIK MODELLASHTIRISH

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Annotatsiya. *Ushbu maqolada insult (o‘tkir miya qon aylanishi buzilishi) xavfini erta bosqichda bashorat qilish uchun bemorning klinik ko‘rsatkichlari (arterial bosim, qon glyukozasi va lipid profili, yurak ritmi buzilishlari tarixi va boshqalar) va biometrik parametrlari (yosh, tana-massa indeksi, yurak urish tezligi o‘zgaruvchanligi, puls to‘lqini tezligi) asosida matematik model taklif etiladi. Model ehtimollik nazariyasi va sun’iy intellekt (mashinali o‘qitish) usullarini uyg‘unlashtirgan gibrid yondashuvga asoslangan.*

Kalit so‘zlar: *insult xavfi, matematik modellashtirish, biometrik ko‘rsatkichlar, klinik parametrlar, mashinali o‘qitish, logistik regressiya, bashoratlash tizimi.*

Kirish.

Insult butun dunyoda o‘lim va nogironlikka olib keluvchi sabablar orasida yetakchi o‘rinlardan birini egallaydi. Jahon sog‘liqni saqlash tashkiloti tashkiloti ma’lumotlariga ko‘ra, insult holatlarining katta qismini oldindan bashorat qilish va profilaktik chora-tadbirlar ko‘rish orqali oldini olish mumkin. Ammo mavjud klinik baholash shkalalari (masalan, an’anaviy risk kalkulyatorlari) ko‘pincha faqat bir nechta statik ko‘rsatkichga asoslangan bo‘lib, bemorning individual biometrik dinamikasini to‘liq hisobga olmaydi. Shu sababli, klinik va biometrik ma’lumotlarni birgalikda tahlil qiluvchi, sun’iy intellekt texnologiyalariga asoslangan aniqroq va moslashuvchan matematik modellarni ishlab chiqish dolzarb ilmiy-amaliy ahamiyatga ega.

Masalaning qo‘yilishi.

Bemorning holatini tavsiflovchi ko‘rsatkichlar vektori $x = (x_1, x_2, \ldots, x_n)$ ko‘rinishida kiritiladi, bu yerda x_i – yosh, arterial bosim (sistolik/diastolik), qon glyukozasi darajasi, umumiy xolesterin, tana-massa indeksi, yurak urish tezligi o‘zgaruvchanligi (HRV), puls to‘lqini tezligi va boshqa klinik-biometrik parametrlar. Maqsad – bemorning maKjlum vaqt oralig‘ida insult riskiga uchrash ehtimoli $P(y = 1 \mid x)$ ni yetarlicha aniqlikda baholovchi funksiyani topishdan iborat.

Taklif etilayotgan model.

Tadqiqotda ikki bosqichli gibrid yondashuv taklif etiladi. Birinchi bosqichda ko‘rsatkichlar o‘rtasidagi chiziqli bo‘lmagan bog‘liqliklarni aniqlash uchun umumlashgan logistik regressiya modeli quyidagi ko‘rinishda quriladi:

$$P(y = 1 \mid x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n)}} \tag{1}$$

bu yerda $\beta_0, \beta_1, \ldots, \beta_n$ – model parametrlari bo‘lib, ular retrospektiv klinik ma’lumotlar to‘plami asosida maksimal ehtimollik usuli yordamida baholanadi. Ikkinchi bosqichda esa ko‘rsatkichlar orasidagi murakkab, chiziqli bo‘lmagan o‘zaro taKjsirlarni hisobga olish maqsadida gradiyent buster (Gradient Boosting) va sunKjiy neyron tarmoq (Multilayer Perceptron) algoritmlaridan foydalaniladi, natijalar ansambl (ensemble) usulida birlashtiriladi:

$$R(x) = \lambda_1 \cdot P_{\log}(x) + \lambda_2 \cdot P_{ML}(x), \qquad \lambda_1 + \lambda_2 = 1, \tag{2}$$

bu yerda $P_{\log}(x)$ – logistik regressiya bergan baho, $P_{ML}(x)$ – mashinali o‘qitish algoritmi bergan baho, λ_1, λ_2 – vaznlar bo‘lib, ular chuqurlik (cross-validation) asosida optimallashtiriladi. Yuqori riskli ko‘rsatkichlarga (masalan, yurak ritmi buzilishi tarixi, qandli diabet, gipertoniya) qo‘shimcha vazn koeffitsiyentlari kiritilishi modelning klinik jihatdan izohlanuvchanligini taKjminlaydi.

Model sifatini baholash.

Modelning bashoratlash sifati ROC-egri chizig‘i ostidagi maydon (AUC-ROC), sezgirlik (sensitivity), o‘ziga xoslik (specificity) va F1-ko‘rsatkichi orqali baholanadi. Dastlabki hisob-kitoblar shuni ko‘rsatadiki, klinik va biometrik ko‘rsatkichlarni birgalikda qo‘shish faqat klinik ko‘rsatkichlarga asoslangan an’anaviy modellarga nisbatan bashorat aniqligini sezilarli darajada oshiradi, chunki biometrik parametrlar bemor holatining dinamik o‘zgarishlarini real vaqt rejimida aks ettiradi.

Xulosa.

Ishlab chiqilgan matematik model klinik va biometrik ko‘rsatkichlarni yagona bashoratlash tizimiga birlashtirish orqali insult xavfini erta aniqlash imkonini beradi. Taklif etilgan gibrid (statistik + sunKjiy intellekt) yondashuv nafaqat yuqori aniqlikni ta’minlaydi, balki klinik amaliyotda qaror qabul qilishni qo‘llab-quvvatlovchi tizimlarga (Clinical Decision Support Systems) integratsiya qilinishi uchun ham qulay poydevor yaratadi. Kelgusi tadqiqotlar doirasida modelni real klinik ma’lumotlar to‘plamida sinovdan o‘tkazish va uni mobil monitoring vositalari bilan bog‘lash rejalashtirilmoqda.

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ONKOLOGIK BEMORLARDA KAXEKSIYANI SUN'IY INTELLEKT YORDAMIDA ANIQLASH

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Kaxeksiya saraton kasalliklarida uchraydigan eng muhim metabolik asoratlardan biridir. Xalqaro konsensusga ko'ra, oldingi olti oy davomida tana vaznining 5% dan ortiq kamayishi yoki 2–5% vazn yo'qotilishi bilan birgalikda BMI < 20 kg/m^2 yoxud sarkopeniyaning mavjudligi kaxeksiya diagnostikasida muhim mezon hisoblanadi. Biroq faqat vazn ko'rsatkichiga tayanish ayrim bemorlarda mushak massasining yashirin kamayishini aniqlamasligi mumkin.

Onkologik bemorlarda KT tekshiruvlari kasallikni bosqichlash va davolash natijasini nazorat qilish uchun muntazam bajariladi. Shu sababli mavjud KT tasvirlaridan tana kompozitsiyasi biomarkerlarini avtomatik ajratib olish qo'shimcha tekshiruvsiz muhim klinik axborot beradi. Sun'iy intellekt yordamida L3 darajasidagi skelet mushaklarini segmentatsiya qilish, vaqt bo'yicha mushak yo'qolishini baholash va klinik-laborator ma'lumotlar bilan birlashtirish kaxeksiyani erta aniqlash imkonini oshiradi.

Birinchi bosqichda tana vaznining nisbiy kamayishi hisoblanadi. Boshlang'ich vazn W_0 va joriy vazn W_t bo'lsa, vazn yo'qotilishi quyidagicha aniqlanadi:

$$WL = \frac{W_0 - W_t}{W_0} \times 100\%. \tag{1}$$

Bu yerda WL — vazn yo'qotilishi foizi. Masalan, bemor vazni 80 kg dan 74 kg gacha kamaygan bo'lsa, $WL = 7.5\%$ bo'ladi.

Tana massasi indeksi (BMI) quyidagi formula orqali aniqlanadi:

$$BMI = \frac{W}{H^2}. \tag{2}$$

bu yerda W — tana vazni (kg), H — bo'y (m). Kaxeksiya xavfini baholashda BMI mustaqil ko'rsatkich sifatida emas, vazn dinamikasi va mushak massasi bilan birgalikda talqin qilinadi.

KT tasvirida L3 umurtqa darajasi avtomatik topilgach, skelet mushaklari U-Net/nnU-Net segmentatsiya modeli bilan ajratiladi. Segmentatsiya maskasiga kiruvchi piksellar fizik maydoni yig'indisi skelet mushak maydonini beradi:

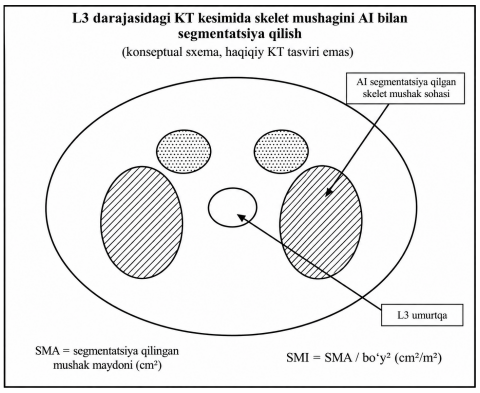
$$SMA = \sum_i A_i. \tag{3}$$

SMA bemor bo'yiga normallashtirilib skelet mushak indeksi olinadi:

$$SMI = \frac{SMA}{H^2}. \tag{4}$$

Ketma-ket KT tekshiruvlari mavjud bo‘lsa, mushak indeksining nisbiy o‘zgarishi dinamik biomarker sifatida qo‘llanadi:

$$\Delta SMI = \frac{SMI_t - SMI_0}{SMI_0} \times 100\%. \tag{5}$$



1-rasm. L3 darajasidagi KT kesimida skelet mushagini AI bilan segmentatsiya qilishning konseptual sxemasi.

Taklif etilayotgan tizim antropometrik, laborator, radiologik va klinik xususiyatlarni yagona vektorga birlashtiradi. Masalan, $X = [WL, BMI, SMA, SMI, \Delta SMI, CRP, Albumin, NLR, Age, ECOG]$. Klassifikatsiya modeli logistika regressiyasi, XGBoost yoki ko‘p qatlamli neyron tarmoq asosida qurilishi mumkin. Logistik modelda kaxeksiya ehtimoli sigmoid funksiya orqali aniqlanadi:

$$P(C = 1 \mid X) = \frac{1}{1 + e^{-z}}, \tag{6}$$

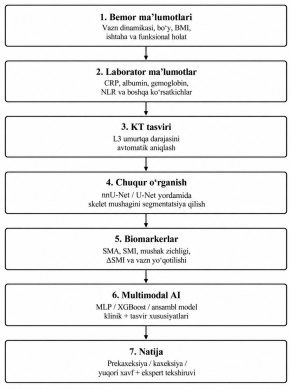
$$z = \beta_0 + \sum_j \beta_j x_j.$$

Neyron tarmoq varianti uchun yashirin qatlamlar ReLU aktivatsiyasi bilan ishlashi, yakuniy chiqishda esa $P_{cachexia}$ ehtimoli hosil qilinishi mumkin. Modelning chiqishi mustaqil tashxis emas, balki yuqori xavfli bemorlarni shifokor ko‘rigiga yo‘naltiruvchi decision-support ko‘rsatkichi sifatida ishlatiladi.

Tasvirlarni oldindan qayta ishlash. KT tasvirlari DICOM formatidan olinadi, Hounsfield birliklari bo‘yicha intensivlik diapazoni cheklanadi va neyron tarmoq kirish o‘lchamiga moslashtiriladi. Segmentatsiyada mushak to‘qimasi uchun radiologik zichlik diapazoni yordamchi filtr sifatida ishlatilishi mumkin, biroq yakuniy maska neyron tarmoq prognozi bilan shakllantiriladi. Ma’lumotlarni standartlashtirish uchun z-score usuli qo‘llanadi:

$$z_i = \frac{x_i - \mu_i}{\sigma_i}. \tag{7}$$

Diagnostika algoritmi. 1) Bemorning bo‘yi, vazn dinamikasi, yoshi va funksional holatini olish. 2) WL va BMI ni hisoblash; CRP, albumin, gemoglobin va NLR kabi laborator ko‘rsatkichlarni kiritish. 3) KT tasviridan L3 umurtqa kesimini avtomatik aniqlash. 4) U-Net/nnU-Net yordamida skelet mushaklarini segmentatsiya qilish. 5) SMA, SMI va mavjud bo‘lsa ΔSMI ni hisoblash. 6) Barcha parametrlarni standartlashtirish va multimodal klassifikatorga uzatish. 7) Kaxeksiya ehtimoli hamda model noaniqligini hisoblash; noaniq holatlarni ekspert ko‘rigiga yuborish.



2-rasm. Kaxeksiyani sun'iy intellekt yordamida aniqlash algoritmi.

Tibbiy klassifikatsiyada faqat umumiy aniqlik yetarli emas. Kaxeksiyali bemorni sog'lom deb baholash klinik jihatdan muhim xato bo‘lganligi sababli sensitivity, specificity, precision, F1 va ROC-AUC ko‘rsatkichlari birgalikda hisoblanadi:

$$S = \frac{TP}{TP + FN}, \qquad Pn = \frac{TP}{TP + FP}, \qquad F_1 = 2 \cdot \frac{Pr \cdot R}{Pr + R}.$$

Segmentatsiya sifatini baholashda Dice koeffitsiyenti qo‘llanadi:

$$DSC = \frac{2|P \cap G|}{|P| + |G|}. \tag{8}$$

DSC qiymati 1 ga yaqinlashgan sari AI segmentatsiyasi ekspert belgilagan etalon maskaga yaqin bo‘ladi. Model o‘qitilishida bemor darajasida train/validation/test bo‘linishi bajarilishi kerak; bir bemorning turli KT tasvirlarini turli guruhlarga joylash data leakage keltirib chiqaradi.

Yallig'lanish holatini ifodalovchi qo‘shimcha ko‘rsatkich sifatida neutrofil/limfotsit nisbati ham hisoblanishi mumkin:

$$NL = \frac{N}{L}. \tag{9}$$

Jadval 1. Kaxeksiyani sun'iy intellekt yordamida aniqlashda qo‘llaniladigan asosiy ma'lumotlar guruhlari va ularning vazifalari

Guruh	**Asosiy belgilar**	**AI dagi vazifasi**
Antropometrik	WL, BMI	Tana massasi dinamikasi
Radiologik	SMA, SMI, ΔSMI	Mushak massasi va kamayishi
Laborator	CRP, albumin, NLR	Yallig'lanish va metabolik holat
Klinik	yosh, ECOG, saraton bosqichi	Klinik kontekst va risk

Kaxeksiya ko‘p omilli sindrom bo‘lgani sababli uni faqat tana vazni orqali aniqlash yetarli emas. KT tasvirlaridan avtomatik olingan SMA va SMI ko‘rsatkichlarini vazn dinamikasi, yallig'lanish o‘choqlari va funksional holat bilan birlashtirish diagnostik jihatdan axborotli multimodal yondashuvni shakllantiradi. Sun'iy intellekt bu jarayonda katta hajmdagi tasvir va klinik ma'lumotlarni tez qayta ishlash, mushak massasining yashirin kamayishini aniqlash va yuqori xavfli bemorlarni saralash imkonini beradi.

Taklif etilgan model klinik amaliyotga joriy etilishidan oldin mustaqil tibbiyot muassasalari ma'lumotlarida tashqi validatsiyadan o‘tkazilishi, kalibratsiya va noaniqlik tahlili bajarilishi lozim. AI tizimi yakuniy tashxis qo‘yuvchi vosita emas; u onkolog va radiolog qarorini qo‘llab-quvvatlovchi tizim sifatida qo‘llanishi maqsadga muvofiq.

Amaliy tizimda model natijasi bilan birga segmentatsiya hududi, SMI qiymati, oldingi tekshiruvga nisbatan o‘zgarish foizi va ehtimoliy risk darajasini ko‘rsatish tavsiya etiladi. Bunday

tushuntiriladigan chiqish shifokorga AI qarorini tekshirish, noto‘g’ri segmentatsiyani tez aniqlash va klinik qarorni mustaqil shakllantirish imkonini beradi.

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OLIY TA’LIM MUASSASALARIDA SMART TEXNOLOGIYALAR ASOSIDA RAQAMLI TA’LIM MUHITINI TASHKIL ETISH

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O‘zbekiston Respublikasida oliy ta’lim tizimini zamonaviy talablar asosida rivojlantirish, ta’lim jarayonini raqamlashtirish hamda xalqaro ta’lim standartlariga mos innovatsion muhitni shakllantirish ustuvor vazifalardan biri hisoblanadi. So‘nggi yillarda oliy ta’lim muassasalarida raqamli transformatsiya jarayonlarini jadallashtirish, elektron ta’lim platformalarini keng joriy etish, sun’iy intellekt va SMART texnologiyalar imkoniyatlaridan foydalanish ta’lim sifatini oshirishning muhim omiliga aylanmoqda.

Bugungi kunda jamiyat taraqqiyotining muhim ko‘rsatkichlaridan biri ta’lim tizimining raqam-li rivojlanish darajasi bilan belgilanmoqda. Axborot texnologiyalarining barcha sohalarga chuqur kirib borishi natijasida oliy ta’lim oldiga yangi vazifalar qo‘yilmoqda. Zamonaviy mutaxassisdan faqat nazariy bilimga ega bo‘lish emas, balki raqamli vositalardan samarali foydalanish, katta hajmdagi axborotni tahlil qilish, mustaqil qaror qabul qilish, muammoli vaziyatlarni hal etish va doimiy ravishda yangi bilimlarni egallash qobiliyati ham talab qilinmoqda.

Shu sababli oliy ta’lim muassasalarida an’anaviy o‘qitish modelini zamonaviy raqamli texnologiyalar bilan uyg‘unlashtirish dolzarb vazifalardan biriga aylanmoqda. Ayniqsa, SMART texnologiyalardan foydalanish o‘quv jarayonini oddiy elektron shaklga o‘tkazishdan ko‘ra kengroq mazmunga ega bo‘lib, ta’lim ishtirokchilari o‘rtasidagi o‘zaro hamkorlikni, axborot almashinuvini, individual ta’lim trayektoriyasini va natijalarni tahlil qilish imkoniyatlarini kengaytiradi.

Raqamli ta’lim muhitining samaradorligi faqat texnik infratuzilmaning mavjudligi bilan belgi-lanmaydi. Unda pedagogning raqamli kompetensiyasi, o‘quv-metodik ta’minot sifati, talabaning texnologik tayyorgarligi, elektron resurslarning qulayligi, baholash mexanizmlari hamda ta’lim jarayonini monitoring qilish tizimi ham muhim ahamiyat kasb etadi. Demak, SMART texnologiya-larni joriy etishda kompleks va tizimli yondashuvdan foydalanish zarur.

SMART texnologiyalar ta’lim jarayonini maqsadga yo‘naltirilgan, natijador va interaktiv tashkil etishga xizmat qiladi. Mazkur texnologiyalar yordamida talabalar mustaqil ta’lim olish, axborot bilan ishlash, tanqidiy fikrlash hamda raqamli kompetensiyalarni rivojlantirish imkoniya-tiga ega bo‘ladilar. Shu sababli oliy ta’limda SMART texnologiyalar asosida raqamli ta’lim muhitini tashkil etish bugungi kunning dolzarb ilmiy-pedagogik muammolaridan biri hisoblanadi.

O‘rganishlar asosida oliy ta’limda SMART texnologiyalarni samarali qo‘llash uchun “maqsad – mazmun – texnologiya – faoliyat – monitoring – natija” ketma-ketligiga asoslangan metodik yondashuvni taklif etish mumkin.

Mazkur yondashuvda dastlab o‘quv mashg‘ulotining maqsadi aniqlanadi. Keyingi bosqichda ushbu maqsadga mos ta’lim mazmuni va metodlari belgilanadi. Shundan so‘ng eng maqbul SMART vositasi tanlanib, talabaning amaliy va mustaqil faoliyati tashkil etiladi. Mashg‘ulot yakunida raqamli monitoring vositalari yordamida ta’lim natijalari baholanadi va olingan ma’lumotlar keyingi pedagogik faoliyatni takomillashtirish uchun asos sifatida foydalaniladi. Bunday tizimli yondashuv texnologiya va pedagogika o‘rtasidagi uzviy bog‘liqlikni ta’minlaydi. Natijada SMART texnologiyalar ta’lim jarayonining alohida elementi emas, balki uning barcha bosqichlarini qo‘llab-quvvatlovchi integratsiyalashgan vositaga aylanadi.

Xulosa qilib aytganda, oliy ta’lim tizimining zamonaviy rivojlanish bosqichida SMART texnologiyalar asosida raqamli ta’lim muhitini shakllantirish ta’lim sifatini oshirishning muhim yo‘nalishlaridan biri hisoblanadi. SMART yondashuv ta’lim jarayonini faqat elektron shaklga o‘tkazish bilan cheklanmay, ta’lim mazmuni, metodlari, kommunikatsiya, mustaqil ta’lim va baholash jarayonlarini yagona raqamli muhitda integratsiyalash imkonini beradi.

SMART texnologiyalardan samarali foydalanish uchun, avvalo, oliy ta’lim muassasasining texnologik infratuzilmasini rivojlantirish, pedagoglarning raqamli kompetensiyalarini oshirish, sifatli elektron o‘quv resurslarini yaratish va raqamli baholash mexanizmlarini takomillashtirish zarur. Shuningdek, texnologik vositalardan foydalanish pedagogik maqsad va kutilayotgan ta’lim natijalari bilan uzviy bog‘lanishi kerak.

Taklif etilayotgan konseptual yondashuvning texnologik, metodik, pedagogik hamda monitoring komponentlarini o‘zaro integratsiya qilish orqali oliy ta’lim muassasasida barqaror va natijador raqamli ta’lim muhitini shakllantirish mumkin. Bunday muhit talabaning faol pozitsiyasini kuchay-tiradi, mustaqil ta’lim olish imkoniyatlarini kengaytiradi va zamonaviy mehnat bozori talablariga mos raqamli kompetensiyalarga ega mutaxassislarni tayyorlashga xizmat qiladi.

Shu asosda oliy ta’limda SMART texnologiyalarni joriy etish jarayonini bosqichma-bosqich, ilmiy-metodik asoslangan va monitoringga yo‘naltirilgan tarzda tashkil etish maqsadga muvofiqdir. Kelgusida SMART ta’lim muhitining samaradorligini aniq mezonlar asosida eksperimental tadqiq etish, turli ta’lim yo‘nalishlari kesimida uning natijadorligini qiyosiy baholash hamda sun’iy intellekt texnologiyalarini mazkur muhitga pedagogik jihatdan maqbul integratsiya qilish istiqbolli ilmiy yo‘nalishlardan biri bo‘lib qoladi.

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TALABALAR BILIMINI BAHOLASHDA SUN’IY INTELLEKT ALGORITMLARIDAN FOYDALANISH

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Kirish. So‘nggi yillarda ta’lim tizimining raqamli transformatsiyasi baholash jarayonini takomillashtirishni talab etmoqda. Ayniqsa, dasturlash fanida talabalar bilimini baholash algoritmik tafakkur, kod yozish, mantiqiy fikrlash va muammoni yechish kabi ko‘nikmalarni kompleks baholashni taqozo etadi. Shu bois, sun’iy intellekt algoritlaridan foydalanish baholashning samaradorligi va obyektivligini oshirishda muhim ahamiyat kasb etadi.

Mavzuga oid adabiyotlarning tahlili. So‘nggi yillarda sun’iy intellekt texnologiyalarining ta’limga integratsiyasi baholash sifatini oshirishning muhim yo‘nalishiga aylandi. Holmes, Bialik va Fadel (2019) SI asosidagi adaptiv baholash imkoniyatlarini, Luckin (2018) esa individual ta’lim trayektoriyasini aniqlashdagi samaradorligini ta’kidlaydi. Baker va Inventado (2014) learning analytics va educational data mining texnologiyalarining baholash natijalarini tahlil qilish va prognozlashdagi ahamiyatini asoslagan.

Asosiy qism. Taklif etilayotgan yondashuvda talabaning bilim darajasi test natijalari, kod sifati, topshiriq bajarish vaqti, xatolar statistikasi va LMS faolligidan shakllanadigan ko‘p o‘lchamli bilim vektori orqali ifodalanadi. Bu yondashuv sun’iy intellekt yordamida kompetensiyani kompleks baholash imkonini beradi [1; 2].

Model 5 bosqichni qamrab oladi: ma’lumotlarni yig‘ish, qayta ishlash, intellektual tahlil, qaror qabul qilish va adaptiv qayta aloqa.

1. Ma’lumotlarni yig‘ish − test, kod, topshiriq, xatolar va LMS faoliyati haqidagi ma’lumotlar jamlanadi.

2. Qayta ishlash − ma’lumotlar tozalanadi, standartlashtiriladi va bilim vektori shakllantiriladi.

3. Intellektual tahlil − klassifikatsiya, klasterizatsiya va prognozlash algoritmlari orqali bilim darajasi va rivojlanish dinamikasi aniqlanadi [3].

4. Qaror qabul qilish − yakuniy baho, individual tavsiyalar va qo‘shimcha topshiriqlar shakllantiri-ladi.

5. Adaptiv qayta aloqa − natijalar tizimga qaytarilib, model parametrlari va keyingi tavsiyalar yangilanadi.

Shunday qilib, Feedback Loop tizimni statik baholashdan adaptiv va o‘zini o‘zi takomillashtiruv-chi modelga aylantiradi. Yangi ma’lumotlar asosida baholash aniqligi, tavsiyalar va topshiriqlar doimiy moslashtiriladi. UNESCO (2021) sun’iy intellektdan ta’limda foydalanishda samaradorlik, shaffoflik va inson manfaatlarini hisobga olish zarurligini ta’kidlaydi [4].

Bilim vektori va matematik model. Talabaning bilim vektori:

$$X = [x_1, x_2, \ldots, x_n]$$

bu yerda x_1–test natijalarining normalizatsiyalangan qiymati, x_2–kodning sifat ko’rsatkichi, x_3–topshiriqni bajarish tezligi, x_4–xatolar statistikasi, x_5–platformadagi faoliyat loglari bo‘yicha indicator va boshqa ko‘rsatkichlar.

Umumiy bilim darajasi:

$$B = \sum_{i=1}^{n} w_i x_i$$

bu yerda w_i – indikatorning ahamiyat koeffitsienti.

ntellektual tahlil modellari. Klassifikatsiya – talabalarni “yuqori”, “o‘rta” va “past” darajalarga ajratish:

$$C : X \to \{C_1, C_2, \ldots, C_m\}$$

Klasterizatsiya – o‘xshash kompetensiyaga ega talabalarni guruhlash: $\min \sum_{k=1}^{K} \sum_{X \in S_k} \|X - \mu_k\|^2$

Regressiya va prognozlash – kelajakdagi bilim darajasini aniqlash: $\hat{B}_{t+1} = f(X_t) + \varepsilon$

Fuzzy logic – bilim darajasidagi noaniqlikni hisobga olish:

$$\mu_{bilim}(B) \in [0,\, 1]$$

Adaptiv model – baholash natijalariga ko‘ra og‘irliklar va parametrlarni yangilash:

$$w_i{}^{(t+1)} = w_i{}^{(t)} + \Delta w_i, \qquad f^{(t+1)} = f^{(t)} + \Delta f$$

Natijada taklif etilayotgan model sun’iy intellektning klassifikatsiya, klasterizatsiya, prognozlash va fuzzy yondashuvlarini birlashtirib, talabalar bilimini obyektiv baholash va individual rivojlanish trayektoriyasini aniqlashga xizmat qiladi.

Tadqiqot metodologiyasi. Tadqiqotda tahlil, modellashtirish, ekspert baholash va matematik-statistik usullar asosida talabalarning raqamli ko‘rsatkichlari sun’iy intellekt algoritmlari yordamida baholandi.

Natija. Talabalarning bilim ko‘rsatkichlarini yagona bilim vektoriga birlashtiruvchi, klassifikat-siya, klasterizatsiya, prognozlash va adaptiv qayta aloqaga asoslangan konseptual model ishlab chiqildi.

Xulosa. Sun’iy intellekt asosidagi model baholash jarayonini obyektiv, ko‘p omilli va adaptiv shaklga keltirib, talabaning rivojlanishini individual kuzatish va ta’lim samaradorligini oshirish imkonini beradi.

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ADAPTIV TA’LIMDA SUN’IY INTELLEKT MODELLARIDAN FOYDALANISH IMKONIYATLARI

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Kirish. Bugungi kunda sun'iy intellekt (SI) texnologiyalarining ta'limga joriy etilishi o‘quv jarayonini individuallashtirish va samaradorligini oshirish imkonini bermoqda. Ayniqsa, adaptiv elektron ta'limda SI talabaning bilim darajasi, ehtiyojlari va o‘rganish sur'atiga mos ta'limni tashkil etishga xizmat qiladi. Learning Analytics orqali o‘quv faoliyati tahlil qilinib, individual o‘quv yo‘nalishlari va materiallar shakllantiriladi.

Mavzuga oid adabiyotlarning tahlili. Tadqiqotlarda adaptiv tizimlar o‘quv materiallarini talabaning bilim darajasi va ehtiyojlariga moslashtirish imkonini berishi ta'kidlangan [1; 2]. Educational Data Mining va Learning Analytics o‘quv ma'lumotlarini tahlil qilish hamda ta'limni samarali boshqarishga xizmat qiladi [3]. Raqamli muhitda bilimlarning tarmoqlar orqali shakllanishi ham alohida qayd etilgan [4]. Mahalliy tadqiqotlarda esa elektron ta'limning mustaqil ta'lim va AKT kompetensiyalarini rivojlantirishdagi ahamiyati ko‘rsatilgan [5]. Umuman, SI asosidagi adaptiv ta'lim individual o‘qitish, monitoring va materiallarni moslashtirish imkonini berib, robototexnika kabi amaliy fanlar uchun ham dolzarb hisoblanadi.

Asosiy qism. Adaptiv elektron ta'limda sun'iy intellekt (SI) modellari talabalarning bilim darajasi, individual ehtiyojlari va o‘rganish sur'atini tahlil qilish asosida ta'lim mazmunini moslashtirish imkonini beradi. Brusilovsky [1] adaptiv tizimlarning o‘quv materiallarini talabaning bilim holatiga moslashtirish imkoniyatlarini asoslagan bo‘lsa, Woolf [2] intellektual tutor tizimlari-ning individual ta'limni qo‘llab-quvvatlashdagi rolini ta'kidlagan.

Zamonaviy adaptiv ta'lim muhitida mashinali o‘qitish, intellektual tutor tizimlari, tabiiy tilni qayta ishlash (NLP) va tavsiya etuvchi algoritmlardan foydalaniladi. Mashinali o‘qitish o‘quv natijalarini tahlil qilish va prognozlashga, intellektual tutorlar individual pedagogik yordam ko‘rsatishga, NLP texnologiyalari interaktiv muloqotni tashkil etishga, tavsiya tizimlari esa talabaning ehtiyojiga mos o‘quv resurslarini tanlashga xizmat qiladi [2; 3].

Robototexnika fanini o‘qitishda ushbu texnologiyalarni integratsiyalash orqali SI asosidagi adaptiv elektron ta'lim muhitini shakllantirish mumkin. Bunda talabaning test natijalari, topshiriq-larni bajarish sifati, o‘rganish sur'ati va amaliy faoliyati tahlil qilinib, uning individual ta'lim trayektoriyasi shakllantiriladi. Tizim mazkur ma'lumotlar asosida mos o‘quv materiallari, amaliy topshiriqlar va metodik tavsiyalarni avtomatik tavsiya etadi.

Learning Analytics texnologiyalaridan foydalanish talabalarning o‘quv faoliyatini real vaqt rejimida monitoring qilish, rivojlanish dinamikasini aniqlash va ta'lim mazmunini qayta moslashti-rish imkonini beradi. Natijada adaptiv muhit individual yondashuv, uzluksiz monitoring, interaktiv ta'lim va amaliy kompetensiyalarni rivojlantirish jarayonlarini yagona tizimga birlashtiradi.

Shunday qilib, sun'iy intellekt modellarini adaptiv elektron ta'limga integratsiyalash robototex-nika fanini o‘qitish samaradorligini oshirish, talabaning individual ta'lim trayektoriyasini shakllanti-rish va nazariy hamda amaliy kompetensiyalarini rivojlantirish uchun ilmiy-metodik asos yaratadi.

Tadqiqot metodologiyasi. Tadqiqotda tahlil, taqqoslash, modellashtirish va pedagogik kuzatuv usullaridan foydalanildi. Sun'iy intellekt modellarining adaptiv ta'limdagi imkoniyatlari o‘rganilib, robototexnika fanini o‘qitishda talabalarning individual ehtiyojlariga mos adaptiv elektron ta'lim modeli ishlab chiqildi.

Natija. Tadqiqot natijasida sun'iy intellektga asoslangan adaptiv elektron ta'lim modeli ishlab chiqildi. Model talabalarning bilim darajasi va individual o‘rganish xususiyatlarini tahlil qilib, mos o‘quv materiallari, topshiriqlar va tavsiyalarni shakllantirish imkonini beradi. Natijada robototexnika fanini o‘qitishda ta'limni individuallashtirish, real vaqt monitoringi va o‘quv samaradorligini oshirish imkoniyati yaratildi.

Xulosa. Sun'iy intellektga asoslangan adaptiv elektron ta'lim modeli robototexnika fanini o‘qitishda ta'limni individuallashtirish, o‘quv faoliyatini monitoring qilish va samaradorlikni

oshirish imkonini beradi. Mazkur yondashuv talabalarning bilim va amaliy ko‘nikmalarini hisobga olib, ularga mos o‘quv materiallari va tavsiyalarni shakllantirishga xizmat qiladi.

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ZAMONAVIY DASTURIY VOSITALAR VA TEXNOLOGIYALAR

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XXI asrda axborot texnologiyalari inson faoliyatining deyarli barcha sohalariga chuqur kirib bordi. Kompyuterlar, mobil qurilmalar, internet tarmoqlari va turli dasturiy tizimlardan foydalanish kundalik hayotning ajralmas qismiga aylandi. Bugungi kunda korxonalar, ta’lim muassasalari, banklar, tibbiyot tashkilotlari va davlat boshqaruvi tizimlarining samarali faoliyatini zamonaviy dasturiy vositalarsiz tasavvur qilish qiyin.

Dasturiy vositalar kompyuter qurilmalarining imkoniyatlarini amaliy masalalarni hal qilishga yo‘naltiradi. Ular yordamida ma’lumotlarni qayta ishlash, saqlash, tahlil qilish, uzatish, boshqarish va avtomatlashtirish mumkin. Texnologiyalarning rivojlanishi natijasida dasturiy mahsulotlar oddiy hisob-kitob vositasidan murakkab intellektual tizimlarga aylandi.

Dasturiy vosita – kompyuter yoki boshqa elektron qurilmalarda ma’lum vazifalarni bajarish uchun yaratilgan dasturlar, dasturiy paketlar va tizimlar majmuasidir. Dasturiy vositalar foydalanuvchiga kompyuter resurslaridan foydalanish, ma’lumotlarni qayta ishlash va turli amaliy masalalarni hal qilish imkonini beradi.

Dasturiy ta’minotni shartli ravishda uch asosiy guruhga ajratish mumkin:

1. Tizimli dasturiy ta’minot;
2. Amaliy dasturiy ta’minot;
3. Dasturlash vositalari.

Tizimli dasturiy ta’minotga operatsion tizimlar, qurilma drayverlari va tizimni boshqaruvchi boshqa dasturlar kiradi. Windows, Linux, macOS, Android va iOS kabi operatsion tizimlar bunga misol bo‘la oladi.

Amaliy dasturiy ta’minot esa foydalanuvchining aniq vazifalarini bajarishga xizmat qiladi. Masalan, matn muharrirlari, elektron jadvallar, grafik muharrirlar, ma’lumotlar bazasi tizimlari, taqdimot dasturlari va multimedia dasturlari amaliy dasturiy vositalar hisoblanadi.

Dasturlash vositalari dasturchilar tomonidan yangi dasturiy mahsulotlar yaratishda qo‘llaniladi. Dasturlash tillari, kompilyatorlar, interpretatorlar, integratsiyalashgan dasturlash muhitlari va dasturiy kutubxonalar ushbu guruhga kiradi. Hence only the transverse coefficients of the metric can obstruct the transferred Killing symmetry.

Bulutli texnologiyalar

Zamonaviy axborot texnologiyalarining muhim yo‘nalishlaridan biri bulutli texnologiyalardir. Bulutli texnologiya foydalanuvchiga ma’lumotlar va dasturiy xizmatlardan internet orqali foydalanish imkonini beradi.

An’anaviy tizimlarda dasturlar va ma’lumotlar foydalanuvchining kompyuterida yoki tashkilotning mahalliy serverida saqlanadi. Bulutli texnologiyalarda esa ma’lumotlarni masofaviy serverlarda saqlash va ulardan internet orqali foydalanish mumkin.

Bulutli xizmatlarning asosiy modellariga quyidagilar kiradi:

- IaaS - infratuzilma xizmat sifatida;

- PaaS - platforma xizmat sifatida;

- SaaS - dasturiy ta’minot xizmat sifatida.

Bulutli texnologiyalarning asosiy afzalligi - foydalanuvchining istalgan joydan va turli qurilmalardan ma’lumotlarga murojaat qila olishidir. Bundan tashqari, tashkilotlar server infratuzilmasini yaratish va texnik xizmat ko‘rsatish xarajatlarini kamaytirishi mumkin.

Ta’limda bulutli texnologiyalar masofaviy ta’lim, elektron hujjatlar almashinuvi, onlayn hamkorlik va o‘quv materiallarini saqlashda keng qo‘llanmoqda.

Sun’iy intellekt va mashinali o‘qitish

Zamonaviy dasturiy texnologiyalarning eng tez rivojlanayotgan yo‘nalishlaridan biri sun’iy intellektdir. Sun’iy intellekt kompyuter tizimlariga insonning ayrim intellektual faoliyatlarini modellashtirish imkonini beradi.

Sun’iy intellekt yordamida ma’lumotlarni tahlil qilish, tasvirlarni aniqlash, tabiiy tilni qayta ishlash, ovozni tanish, tavsiyalar berish va qaror qabul qilish kabi vazifalar bajariladi.

Mashinali o‘qitish sun’iy intellektning muhim yo‘nalishi bo‘lib, unda kompyuter tizimi oldindan barcha qoidalar batafsil yozib berilmasdan, ma’lumotlar asosida qonuniyatlarni aniqlashni o‘rganadi.

Masalan, elektron pochta tizimlarida spam xabarlarni aniqlash, internet do‘konlarida foydalanuvchiga mahsulot tavsiya qilish, bank tizimlarida shubhali operatsiyalarni aniqlash mashinali o‘qitish algoritmlaridan foydalanish orqali amalga oshirilishi mumkin.

So‘nggi yillarda generativ sun’iy intellekt ham katta ahamiyat kasb etmoqda. Bunday tizimlar matn, tasvir, dasturiy kod va boshqa turdagi kontentlarni yaratishda foydalanilmoqda. Bu texnologiyalar ta’lim, dasturlash, marketing, dizayn va ilmiy tadqiqotlarda yangi imkoniyatlarni yuzaga keltirmoqda. Biroq sun’iy intellektdan foydalanishda ma’lumotlarning maxfiyligi, natijalarning ishonchliligi, mualliflik huquqi va inson nazoratini ta’minlash kabi masalalarga ham e’tibor berish zarur.

Internetning rivojlanishi veb texnologiyalarning jadal taraqqiy etishiga sabab bo‘ldi. Hozirgi vaqtda turli xizmatlar veb-sayt va veb-ilovalar orqali taqdim etilmoqda.

Veb dasturlashda HTML, CSS va JavaScript kabi texnologiyalar asosiy vositalardan hisoblanadi. Bundan tashqari, React, Angular, Vue kabi zamonaviy texnologiyalar interaktiv va murakkab veb-ilovalar yaratishda keng qo‘llanadi.

Mobil texnologiyalar esa smartfon va planshetlar uchun dasturlar yaratishga imkon beradi. Android va iOS platformalari mobil dasturiy ta’minotning asosiy ekotizimlarini tashkil qiladi.

Mobil ilovalar bank xizmatlari, elektron tijorat, ta’lim, tibbiyot, transport va aloqa sohalarida keng foydalanilmoqda. Foydalanuvchilar mobil qurilma orqali to‘lovlarni amalga oshirishi, masofadan ta’lim olishi, turli xizmatlardan foydalanishi mumkin.

Sun’iy intellektda kiberxavfsizlik texnologiyalari

Raqamlashtirish jarayoni kuchaygani sari axborot xavfsizligi masalalarining ahamiyati ham ortib bormoqda. Kiberxavfsizlik kompyuter tizimlari, tarmoqlar, dasturlar va ma’lumotlarni noqonuniy kirish, o‘zgartirish, yo‘q qilish yoki o‘g‘irlashdan himoya qilishga qaratilgan.

Zamonaviy kiberxavfsizlik vositalariga antivirus dasturlari, xavfsizlik devorlari, ma'lumotlarni shifrlash tizimlari, ko'p bosqichli autentifikatsiya va tarmoq monitoringi vositalari kiradi.

Foydalanuvchilarning o'zi ham axborot xavfsizligining muhim elementi hisoblanadi. Kuchli parollardan foydalanish, noma'lum havolalarni ochmaslik, dasturlarni ishonchli manbalardan yuklab olish va muhim ma'lumotlarning zaxira nusxasini yaratish xavfsizlikni oshiradi.

Dasturiy ta'minot ishlab chiqishning zamonaviy usullari Zamonaviy dasturiy mahsulotlarni ishlab chiqishda dasturlashning o'zi bilan bir qatorda loyihani boshqarish metodologiyalari ham muhim hisoblanadi.

Bugungi kunda Agile, Scrum va DevOps kabi yondashuvlardan keng foydalaniladi. Agile metodologiyasi dasturiy mahsulotni bosqichma-bosqich ishlab chiqish va foydalanuvchi talablariga tez moslashishga imkon beradi.

DevOps esa dasturiy ta'minotni ishlab chiqish va ekspluatatsiya qilish jarayonlarini o'zaro yaqinlashtirishga qaratilgan. Avtomatlashtirilgan testlash, uzluksiz integratsiya va uzluksiz yetkazib berish dasturiy mahsulotlarni tez va sifatli ishlab chiqishga yordam beradi.

Git kabi versiyalarni boshqarish tizimlari dasturchilarning birgalikda ishlashini, koddagi o'zgarishlarni kuzatishni va oldingi versiyalarga qaytishni ta'minlaydi.

Xulosa qilib aytganda, zamonaviy dasturiy vositalar va texnologiyalar jamiyat taraqqiyotining muhim omillaridan biridir. Bulutli texnologiyalar, sun'iy intellekt, mashinali o'qitish, Big Data, Internet of Things, mobil va veb texnologiyalar turli sohalarda yangi imkoniyatlarni yaratmoqda.

Dasturiy vositalardan oqilona foydalanish ishlab chiqarish samaradorligini oshirish, ta'lim sifatini yaxshilash, xizmatlarni avtomatlashtirish va ma'lumotlardan samarali foydalanish imkonini beradi.

Kelajakda sun'iy intellekt, avtomatlashtirish, bulutli hisoblash va boshqa raqamli texnologiyalarning yanada rivojlanishi kutilmoqda. Shu sababli zamonaviy mutaxassis nafaqat o'z kasbiy sohasini, balki zamonaviy dasturiy vositalar va axborot texnologiyalarini ham yaxshi bilishi zarur.

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OLIY TA'LIMDA SUN'IY INTELLEKT TEXNOLOGIYALARI ASOSIDA TALABALARNING MUSTAQIL TA'LIMINI TASHKIL ETISH

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Annotatsiya. Mazkur tezisda oliy ta'lim muassasalarida sun'iy intellekt (SI) va zamonaviy raqamli texnologiyalar asosida talabalarning mustaqil ta'lim faoliyatini samarali tashkil etish, uning metodik mexanizmlarini takomillashtirish hamda individual ta'lim trayektoriyasini

shakllantirish masalalari har tomonlama yoritilgan. Aniq va abstrakt fanlarni o‘zlashtirish jarayonida talabalarning kognitiv imkoniyatlarini hisobga olish hamda ularga moslashtirilgan interaktiv raqamli o‘quv muhitini yaratish muhim zamonaviy zaruriyatdir. Tadqiqotda xorijiy olimlarning so‘nggi ilmiy izlanishlari, SI asosidagi adaptiv ta’lim platformalari tajribasi chuqur tahlil qilinib, ularni milliy oliy ta’lim tizimiga integratsiyalash yo‘llari ilmiy asoslab berilgan. Natijada talabalarning algoritmik va mantiqiy tafakkurini rivojlantirishga qaratilgan kompleks dasturiy-metodik model, uning tarkibiy modullari hamda amaliyotga joriy etish bosqichlari taklif etilgan.

Kalit so‘zlar: sun’iy intellekt, mustaqil ta’lim, raqamli ta’lim muhiti, adaptiv ta’lim, intellektual assistent, algoritmik tafakkur, raqamli kompetensiya, individual traektoriya, kognitiv yuklama, Bloom taksonomiyasi.

Bugungi kunda oliy ta’lim tizimidagi jadal raqamli transformatsiya jarayonlari va pedagogik texnologiyalarning evolyutsiyasi talabalarning mustaqil ta’lim faoliyatini yangicha sifat va texnologik bosqichga olib chiqishni talab etmoqda. Ayniqsa, axborot texnologiyalari, matematika va amaliy fanlar yo‘nalishida tahsil olayotgan talabalarning nafaqat tayyor nazariy bilimlarni yod olishi yoki reproduktiv topshiriqlarni takrorlashi, balki algoritmik hamda mantiqiy tafakkurini mustaqil ravishda rivojlantirishi g‘oyat muhimdir. An’anaviy mustaqil ta’lim topshiriqlari ko‘pincha barcha uchun bir xil (statik) tuzilishga ega bo‘lib, u har bir talabaning individual kognitiv imkoniyatlari, bilim darajasi, qiziqishlari hamda o‘zlashtirish dinamikasini to‘liq hisobga ola olmaydi.

Oliy ta’limda talabalarning mustaqil ta’lim soatlarini samarali tashkil etish va ularni pedagogik jihatdan to‘g’ri yo‘naltirish dolzarb muammolardan biri bo‘lib qolmoqda. Talaba auditoriyadan tashqari vaqtda o‘quv materialini o‘zlashtirishda kognitiv qiyinchiliklarga va to‘siqlarga duch kelganda, o‘z vaqtida metodik yordam hamda aks-aloqa (feedback) ololmasligi oqibatida ta’limga bo‘lgan motivatsiyasi pasayadi. Shu sababli mustaqil ta’lim jarayonini individuallashtirish, talabaning bilimidagi bo‘shliqlarni aniqlash, uning bilim darajasiga mos ravishda topshiriqlarni dinamik generatsiya qilish hamda real vaqt rejimida tezkor maslahat berishga imkon beruvchi intellektual vositalardan foydalanish zamonaviy pedagogikaning asosiy talabidir.

Generativ sun’iy intellekt, mashinaviy o‘rgatish hamda neyron tarmoqlari texnologiyalarining jadal rivojlanishi ushbu jarayonni tubdan isloh qilish uchun keng imkoniyatlar ochib bermoqda. SI asosidagi ta’lim platformalari o‘quv materiallarini talabaning individual o‘zlashtirish ko‘rsatkichiga qarab moslashtirish, tushuntirishlarni turli murakkablik darajasida taqdim etish, mantiqiy xatolarni tahlil qilish va talabaning bilimidagi uzilishlar (knowledge gaps)ni avtomatik diagnostika qilib, ularni bartaraf etish imkonini beradi.

Oliy ta’lim va nazariy fanda SI texnologiyalaridan foydalanish bo‘yicha ilg‘or xorijiy tajribalar tahlili shuni ko‘rsatadiki, yetakchi xalqaro tadqiqotchilar mustaqil ta’lim faoliyatini statik topshiriqlar to‘plamidan dialogik va interaktiv shaklga o‘tkazishga alohida e’tibor qaratmoqda. Jumladan, K. V. Nguyen (2025) hamda A. Suriano (2025) o‘z tadqiqotlarida generativ SI yordamida talabalarning tanqidiy fikrlashi va mantiqiy xulosalash qobiliyatlarini oshirish bo‘yicha samarali pedagogik modellarni ilgari surgan. Isroillik olimlar N. Izsak (2025) hamda O. Hazzan va Y. Erezlar (2025) murakkab nazariy fanlarda SI texnologiyalarini qo‘llash orqali talabalarda refleksiya (o‘z-o‘zini tahlil qilish) hamda mustaqil rejalashtirish mexanizmlarini shakllantirish lozimligini ilmiy jihatdan asoslagan.

AQShlik tadqiqotchilar I. Akhmetov va J. Buchanan (2026) tomonidan taklif etilgan “CS 1.5” yondashuvi informatika hamda diskret tuzilmalar kurslarida SI vositalarini talabaning mustaqil fikrlashini rag‘batlantiruvchi intellektual hamkor sifatida integratsiyalash yuqori samaradorlik

ko‘rsatishini isbotladi. Shuningdek, Ruminiyalik olim A.-I. Lungu va uning hammualliflari (2024) tomonidan yaratilgan formal metodlarga asoslangan Alk platformasi hamda R. Samala (2025) ishlarida adaptiv scaffolding (bosqichma-bosqich qo‘llab-quvvatlash) konsepsiyasi orqali talabalarda algoritmik tafakkurni mustaqil rivojlantirish imkoniyatlari ko‘rsatib berilgan.

Ushbu ilg‘or xorijiy tajribalar va metodik yondashuvlarni umumlashtirgan holda, oliy ta’limda talabalarning mustaqil ta’lim faoliyatini tizimli va samarali tashkil etish uchun kompleks dasturiy-metodik yondashuv va uning funksional arxitekturasi taklif etildi:

1. **SI-assistent va dialogik modul** – mustaqil ta’lim jarayonida murakkab va abstrakt tushunchalarni sodda, bosqichma-bosqich tushuntirish, individual misollar yaratish hamda talabalarga tayyor javob berish o‘rniga yo‘naltiruvchi savollar bera oladigan intellektual muloqot muhitini yaratadi.

2. **Adaptiv o‘quv traektoriyasi moduli** – talabaning dastlabki va joriy o‘zlashtirish natijalariga ko‘ra mustaqil ish topshiriqlari murakkabligini avtomatik ravishda moslashtiradi, talabadagi kognitiv yuklamani (cognitive load) maqbullashtiradi va Bloom taksonomiyasiga mos ravishda bilishdan ijodga qarab yo‘naltiradi.

3. **Real vaqtli tezkor aks-aloqa (Feedback) moduli** – topshiriqlarni bajarish jarayonidagi sintaktik va mantiqiy xatolarni zudlik bilan aniqlab, talabaga tayyor yechimni bermasdan, uning mantiqiy xatosini ko‘rsatadi va to‘g‘ri yechim algoritmini o‘zi to topgunicha yo‘naltiradi.

4. **Refleksiv va metakognitiv monitoring** – talabaning o‘z-o‘zini baholashi, o‘zlashtirish dinamikasini kuzatishi va mustaqil ta’lim natijalarini chuqur tahlil qilishiga imkoniyat yaratadi hamda o‘qituvchiga talabalarning umumiy bilish darajasi haqida analitik ma’lumot beradi.

Dasturiy-metodik modelni amaliyotga tadbiq etish metodologiyasi uch asosiy bosqichni o‘z ichiga oladi: Birinchidan, diagnostik bosqichda talabaning dastlabki bilimi va texnologik ko‘nikmalari baholanib, shaxsiy ta’lim profili shakllantiriladi. Ikkinchidan, interaktiv bosqichda talaba SI-assistent ko‘magida individual topshiriqlarni bajaradi, konstruktiv muloqot va tahlil orqali o‘z bilmini mustahkamlaydi. Uchinchidan, monitoring va baholash bosqichida talabaning mustaqil ta’lim olganlik darajasi, mantiqiy va algoritmik o‘sish ko‘rsatkichlari empirik va analitik usullar bilan baholanadi.

Taklif etilayotgan yondashuvning asosiy pedagogik tamoyili SI texnologiyalarini talabaning o‘rniga masalani shunchaki yechib beruvchi vosita sifatida emas, balki uning mustaqil fikrlashi va mantiqiy mulohaza yuritishini rag‘batlantiruvchi intellektual hamkor (Thought Partner) sifatida qo‘llashdan iborat. Bunda tayyor natijani shunchaki ko‘chirish (plagiat)ning oldi olinadi hamda talabaning muammoni mustaqil tahlil qilishi va yechimni asoslab berishi ustuvor ahamiyatga ega bo‘ladi.

Xulosa qilib aytganda, xorijiy olimlarning ilmiy natijalari va ilg‘or metodik tajribalarini milliy oliy ta’lim sharoitiga moslashtirish talabalarning mustaqil ta’limini tashkil etish sifatini yangi bosqichga ko‘taradi. Taklif etilgan dasturiy-metodik model va uning adaptiv modullari talabalarning mustaqil ta’lim faoliyatini interaktiv, moslashuvchan hamda individuallashtirilgan tarzda olib borishga xizmat qiladi.

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Гибридная формально-нейронная модель синтаксического разбора предложения официально-делового стиля узбекского языка

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Аннотация. В работе предложена гибридная формально-нейронная модель синтаксического разбора узбекских предложений, сочетающая аксиоматическую грамматику с нейросетевыми методами.

Ключевые слова: узбекский язык, официально-деловой стиль, синтаксический анализ, формальная грамматика, аксиоматическая модель, нейросимвольный искусственный интеллект, трансформер, графовая нейронная сеть.

Цифровизация госуправления и рост объёма официально-деловых текстов на узбекском языке требуют систем автоматической обработки, сочетающих точность с объяснимостью. Агглютинативная морфология, вариативность порядка слов, омонимия и нехватка размеченных корпусов осложняют задачу, а существующие нейросетевые и формальные модели обладают взаимодополняющими сильными и слабыми сторонами. Цель исследования – разработка гибридной формально-нейронной модели, в которой нейронный компонент обеспечивает вероятностную интерпретацию, а формальный – проверку допустимости результата.

Формальная часть предлагаемой системы базируется на ранее разработанной аксиоматической модели грамматики официально-делового стиля узбекского языка. В исходной теории используются формальные модели частей речи и членов предложения, базовая аксиома и система правил вывода. В качестве структурной основы рассматривается порядок:

$$A: \ S + O + Adv + P,$$

где S – подлежащее; O – дополнение; Adv – обстоятельство; P – сказуемое.

Базовая структура служит формальной основой контролируемого фрагмента официально-делового стиля узбекского языка, а не описанием всех возможных предложений.

Предлагаемая модель состоит из трёх взаимосвязанных уровней.

1. Уровень предварительной обработки. На вход системы поступает предложение или фрагмент официального документа $X = (x_1, x_2, \ldots, x_n)$. На первом этапе выполняются токенизация, нормализация, выделение предложений, обработка пунктуации, предварительный морфологический анализ и идентификация потенциально значимых аффиксальных последовательностей. Для узбекского языка особенно важен морфологический анализ, поскольку грамматическая информация часто кодируется последовательностью аффиксов.

2. Нейронный модуль. Входное предложение токенизируется, каждый токен преобразуется в контекстный вектор $H = f_\theta(X)$. На основе H нейронный модуль выполняет морфологическую разметку, определение частей речи и признаков, снятие омонимии и предсказание синтаксических связей, формируя несколько кандидатов разбора y_i с вероятностями $P(y_i \mid X)$. Наиболее вероятный вариант не принимается автоматически – окончательный выбор происходит только после проверки формальным модулем.

3. Формальный модуль верификации. Формальный модуль получает от нейронной системы множество кандидатов $Y = \{y_1, y_2, \ldots, y_k\}$. Для каждого кандидата выполняется проверка

$$V(y_i) = \begin{cases} 1, & \text{если } y_i \text{ выводим по правилам } R = \{r_1, r_2, \ldots, r_{12}\}, \\ 0, & \text{иначе,} \end{cases} \tag{1}$$

где R – множество из правил вывода формальной грамматики, $V(y_i)$ – индикатор соответствия разбора y_i аксиоматической системе.

Если кандидат соответствует одной из допустимых конструкций, система сохраняет соответствующее доказательство. Итоговый выбор может быть представлен как задача оптимизации:

$$y^* = \arg\max_{y_i} \left[\lambda P_\theta(y_i \mid X) + (1 - \lambda) C(y_i)\right], \tag{2}$$

где $P_\theta(y_i \mid X)$ – нейронная оценка; $C(y_i)$ – оценка соответствия формальной системе; λ – коэффициент соотношения нейронной и формальной составляющих.

В строгом режиме $C(y_i)$ может выступать не как дополнительный балл, а как обязательное ограничение. В этом случае нейронная модель выбирает наиболее вероятный вариант только среди формально допустимых структур.

Рассмотрим архитектурную схему действий:

Входной документ → Предварительная обработка → Нейронный модуль → Формальная

проверка → Входной документ → Ранжирование и выбор → Объяснение результата

Нейронная модель работает совместно с формальной грамматикой, разделяя вероятностное распознавание и логическую верификацию. При неоднозначности выбирается формально допустимый вариант, даже если он менее вероятен. Система сохраняет полный след разбора для объяснимости. Предусмотрены строгий (только формальные конструкции) и расширенный режимы. Новизна подхода – в разделении функций: формальная система ограничивает пространство решений, нейронная – выбирает и ранжирует варианты внутри него:

Нейронное предложение → Формальная верификация →

→ Нейронное ранжирование → Объясняемый результат

Такой принцип позволяет использовать результаты ранее разработанной аксиоматической модели в современных системах NLP без отказа от её математической основы.

Для экспериментальной проверки модели целесообразно сформировать специализированный корпус официально-делового стиля узбекского языка. Корпус должен включать

нормативно-правовые документы, административные документы, официальные письма, постановления и распоряжения, договорные тексты и типовые государственные документы. Каждое предложение должно быть размечено как минимум на трёх уровнях:

$$\text{Морфология} \rightarrow \text{Синтаксис} \rightarrow \text{Тип конструкции}$$

Гибридная модель должна снизить число недопустимых разборов, повысить устойчивость к вариативности и омонимии, сохранить объяснимость, сократить пространство поиска и создать benchmark официально-делового узбекского языка.

Модель применима для анализа госдокументов, обработки нормативно-правовых текстов, проверки грамматики, генерации документов, поиска, извлечения отношений и создания корпусов; возможна интеграция с модулями сущностей и семантических ролей.

Перспективность подхода определяется возможностью переноса ранее полученных результатов формального моделирования грамматики в современные технологии нейросимвольного искусственного интеллекта и создания специализированных NLP-инструментов для официально-делового узбекского языка.

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mavzusidagi xalqaro ilmiy-amaliy konferensiya

MATERIALLARI TO‘PLAMI. II qism

Termiz, 2026-yil, 21-22-sentyabr

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